Establishment and verification of the prediction model of soil wetting pattern size in vertical moistube irrigation

Yanwei Fan, Zhiwei Yang and Hujun Wei

ABSTRACT

To solve the difficulty of observing the soil wetting pattern of vertical moistube irrigation, the HYDRUS-2D software is used to simulate wetting pattern data under different conditions. Based on the analysis of Origin 9.0 and summarizing the law of simulation data, an empirical model for predicting wetting pattern size is constructed, with the reliability of the model verified by experimental data. The results show that the power function is used to fit the relationship between the size of wetting pattern and irrigation time. The power function exponent has small changes in three directions (vertical upward, horizontal, and vertical downward) of the moistube. Further analysis shows that the power function coefficient is in accordance with the power function relationship with soil-saturated hydraulic conductivity, the steady permeability of the moistube, and the difference between soil-saturated moisture content and initial moisture content. The average absolute error of statistical indicators of the built model is between 0.30 and 1.42 cm, the root mean square error is between 0.42 and 1.65 cm, and the Nash-Sutcliffe efficiency coefficient is not less than 0.93. The model has a good prediction effect and can provide a scientific basis for the design, operation, and management of the moistube irrigation engineering.

Key words | HYDRUS-2D, predictive model, soil, vertical moistube irrigation, wetting pattern size

HIGHLIGHTS

- Based on Origin 9.0 analysis and summarizing the law of the simulation data, a simplified empirical model for predicting the size of the soil wetting pattern is constructed, and was verified by experimental data.
- The model can well predict the size of soil wetting pattern under vertical moistube irrigation, and provide scientific basis for the optimization of the moistube irrigation engineering.

INTRODUCTION

Vertical moistube irrigation is a new type of moistube irrigation technology that uses a polymer semi-permeable membrane to make a moistube, through which water seeps out for continuous underground irrigation (Quinones-Bolanos et al. 2005; Yang 2010). Vertical moistube irrigation has the advantages over underground drip irrigation of deeper wetting depth, free setting spacing, low working head, and low power consumption. It is more suitable for deep-rooted plants and can keep the soil around the root system moist.

The volume of moist soil is an important basis for optimizing the spacing of irrigation devices, horizontal arrangement, and choosing a suitable irrigation system, and it is also one of the most important factors in the design of irrigation systems. Studies have shown that the wetting depth in the soil profile of the surface drip irrigation
and the wetting radius of the soil surface are the main components of the soil wetting pattern (Dabral et al. 2012; Al-Ogaidi et al. 2016a), and that the main components of the soil wetting pattern of subsurface drip irrigation are the wetting depth, wetting radius, and upward wetting distance in the soil profile (Yao et al. 2011). Vertical moistube irrigation is a buried irrigation technology, and the main components of the soil wetting pattern are similar to those of underground drip irrigation (Fan et al. 2018a). During irrigation, the size of the components of the wetting pattern must be determined according to the cross section of the wetting pattern and the volume of the wet soil (Fernandez-Galvez & Simmonds 2006). Domestic and foreign scholars have developed some analytical models (Philip 1984; Chu 1994; Cook et al. 2003; Moncef & Khemaies. 2016), HYDRUS-2D models (Brandt et al. 1997; Šimůnek et al. 2008), and empirical models (Malek & Peters 2011; Al-Ogaidi et al. 2016; Fan et al. 2018b) to quantify the wet soil volume. Researchers compared and evaluated the above three types of models (Kandelous & Šimůnek 2010; Subbaiah 2013). The results show that analytical and HYDRUS-2D models are usually employed to solve control flow equations under specific initial and boundary conditions. Analytical models are based on point source assumptions and special forms of homogeneous soil physical properties. The HYDRUS-2D model can realize a wide range of boundary conditions, including irregular boundaries and soil heterogeneity, improving the popularity of its analysis in the design of moistube irrigation systems. Kandelous & Šimůnek (2010) believe that HYDRUS-2D can estimate soil moisture distribution under drip irrigation conditions more accurately than analysis and empirical models. The model has been successfully applied to simulate the distribution of soil moisture under different irrigation methods. Although each empirical model is relatively simple in form, it is obtained by analyzing laboratory and field data using regression and dimensional analysis methods. The empirical formula is only applicable to specific irrigation techniques, such as the developed point source or line source drip irrigation wetted pattern size, and is not applicable to vertical moistube irrigation (Malek & Peters 2011). Therefore, the development of an empirical model that can predict the size of the soil wetting pattern of vertical moistube irrigation would provide a convenient and practical method for quantifying the size of the wetting pattern and solving the problem that the wetting pattern is difficult to observe.

The combination of numerical simulation and experimental verification is a common method for studying the laws of soil water movement. Numerical simulation can simulate the process of soil water movement under different soil characteristics and different technical elements (Saito et al. 2006; Han et al. 2015; Šimůnek et al. 2016). Experiments can verify the laws summarized by numerical simulation and improve the reliability of research results. Kanda et al. (2020) used the HYDRUS-2D model to numerically simulate the soil moisture distribution of two soil textures (ST; loamy sand and sandy clay loam) under horizontal moistube irrigation, and verified them with experiments, with the results showing that the simulated values are very consistent with the experimental observations. Fan et al. (2018c) verified the reliability of the HYDRUS-2D simulation results of horizontal moistube irrigation by using indoor soil box experimental data. On this basis, the dynamic changes of soil wetting pattern under different ST and loams with different initial water content ($\theta_0$), pressure head ($H$), and depth of burial ($D$) were studied. The results show that the movement distance of the soil wet front conforms to the rule of vertical downward > horizontal direction > vertical upward, the contours of soil moisture content are all approximately concentric circles, and that the texture of the soil has a significant effect on the characteristics of the wet body. Zhang et al. (2016) explored the infiltration of water in different soil textures under the condition of horizontal moistube irrigation based on laboratory experiments, with the results showing that the cumulative infiltration amount was negatively correlated with the soil clay content, that the wetting front was an approximate circle centered on the moistube, and that the relationship between the distance and time of the wetting front was approximately a power function. Fan et al. (2018a) determined the specific flow calculation formula of the vertical moistube through indoor experiments, and verified the accuracy of HYDRUS-2D simulation. On this basis, using HYDRUS-2D simulation to study the influence of ST, $\theta_0$, $H$, moistube length ($L$) and $D$ on the characteristics of vertical moistube irrigation wetting pattern shows that the contour of the wetting pattern and the water content was approximately an ellipsoid around the moistube. ST has a
significant impact on the wetting pattern, and with the increase of soil clay content, the volume of wet soil decreases. The $\theta_0$, $H$, and $L$ have a greater impact on the distance of the wetting front and the volume of the wet soil, and $D$ affects the position of the wetting pattern.

In previous research (Fan et al. 2018a), the authors of this article conducted a qualitative study on the soil characteristic parameters and irrigation technical elements affecting the soil wetting pattern of vertical moistube irrigation. Based on the previous qualitative research on the influencing factors of the vertical moistube irrigation, this study increases the simulation volume and uses the HYDRUS-2D software to obtain the wetting pattern size data with different factors; the laws of the simulation data are analyzed and summarized to construct a simplified empirical model for predicting the size of the soil wetting pattern; and experimental data is used to verify the reliability of the model in order to provide a scientific basis for the design, operation, and management of the moistube irrigation engineering.

### MATERIALS AND METHODS

#### Laboratory experiments

The silt loam in Qilihe District, Lanzhou, and the sandy loam and sandy clay loam in Minqin County, Wuwei, Gansu Province, are used for vertical moistube irrigation experiments, with the sampling locations shown in Figure 1. The saturated water content $\theta_s$ and saturated hydraulic conductivity $K_s$ of the experimental soil are measured by the ring knife method and the constant head method, respectively. The experimental soil characteristic parameters and irrigation technical parameters are shown in Table 1.

The experiment device consists of five parts: a height adjustable stand, a mariotte bottle, a hydraulic hose, a moistube, and a soil box (Figure 2). The soil box is made of 10 mm thick plexiglass, and the size is 60 cm (length) $\times$ 60 cm (width) $\times$ 100 cm (height). There are multiple ventilation holes (diameter 2 mm) at the bottom of the soil box to prevent air resistance in the soil. In the test, the mariotte

![Figure 1](image-url)  Study area and the soil sampling locations in the study area situated in Gansu, China.

<p>| Characteristic parameters and irrigation technical parameters of experimental soils |
|---------------------------------|---------------|---------------|----------------|---------|---------|---------|</p>
<table>
<thead>
<tr>
<th>ST</th>
<th>$\gamma$ (g·cm$^{-3}$)</th>
<th>$K_s$ (cm·min$^{-1}$)</th>
<th>$\theta_0$ (cm$^3$·cm$^{-3}$)</th>
<th>$\theta_s$ (cm$^3$·cm$^{-3}$)</th>
<th>$L$ (cm)</th>
<th>$D$ (cm)</th>
<th>$H$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt loam</td>
<td>1.33</td>
<td>0.0143</td>
<td>0.147</td>
<td>0.450</td>
<td>20</td>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>1.56</td>
<td>0.0390</td>
<td>0.096</td>
<td>0.410</td>
<td>20</td>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>1.45</td>
<td>0.0505</td>
<td>0.158</td>
<td>0.390</td>
<td>20</td>
<td>30</td>
<td>150</td>
</tr>
</tbody>
</table>

Downloaded from http://iwaponline.com/ws/article-pdf/21/1/331/840169/ws021010331.pdf by guest
bottle provides a constant head, and the contour of the wetted body at different times is drawn with a marker.

### Mathematical modeling

#### Basic equations

Vertical moistube irrigation is a spatial three-dimensional infiltration process under the condition of insufficient water supply, and the basic equation to soil water movement is the Richards equation (Richards 1953):

\[
\frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r K(h) \frac{\partial h}{\partial r} \right] + \frac{\partial}{\partial z} \left[ K(h) \frac{\partial h}{\partial z} \right] - \frac{\partial K(h)}{\partial z} \tag{1}
\]

where \( r \) is the radial coordinate (cm); \( z \) is the vertical coordinate (cm), which specifies that \( z \) is positive downward; \( \theta \) is the soil moisture content (cm³/cm³); \( h \) is the pressure head (cm); \( t \) is the infiltration time (min); and \( K(h) \) is the soil unsaturated hydraulic conductivity (cm/min).

The relationship among \( \theta, h, \) and \( K(h) \) in Equation (1) is fitted by the van Genuchten–Mualem (VG–M) model (van Genuchten 1980), which is:

\[
\theta = \theta_r + \frac{\theta_s - \theta_r}{\left(1 + |a h|^{\frac{1}{m}}\right)^{m}} \tag{2}
\]

\[
K(h) = K_s S_e^m \left[1 - \left(1 - S_e^{1/m}\right)^{m/2}\right] \tag{3}
\]

where \( S_e \) is the relative saturation of the soil, and \( S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \); \( \theta_r \) is the residual soil moisture content (cm³/cm³); \( a \) is an empirical parameter, which is inversely proportional to the intake air value (cm¹); \( n \) and \( m \) are empirical constants affecting the shape of the soil moisture characteristic curve, with \( n > 1 \) and \( m = 1 - 1/n \); \( l \) is the empirical coefficient, usually 0.5.

The field water holding capacity is obtained using the prediction model established by Rab et al. (2011), and the specific expression is:

\[
FC = 0.0805 + 1.68PWP - 1.62PWP^2 \tag{4}
\]

where \( FC \) is the field water holding capacity (cm³/cm³), and \( PWP \) is the withering coefficient (cm³/cm³).

Naglic et al. (2014) showed that the withering coefficient of different soil textures can be expressed by the residual moisture content in the parameters of the VG–M model, namely:

\[
PWP = \theta_r \tag{5}
\]

Using Equations (4) and (5), we obtain:

\[
FC = 0.0805 + 1.68\theta_r - 1.62\theta_r^2 \tag{6}
\]

#### Initial and boundary conditions

Figure 3 shows the initial and boundary conditions considered when simulating different modeling scenarios in this study. Considering the axial symmetry of the vertical moistube, the area shown in Figure 3 is selected as the simulation calculation domain. The level of soil moisture content determines the rate of water movement in the soil. Therefore, in all simulation scenarios, the soil moisture content is set at a given initial moisture content.
The accurate setting of boundary conditions is critical to the simulation results. During the irrigation process, the surface soil is a dry soil layer, the amount of evaporation is very small, and therefore the influence of rainfall and evaporation is not considered at the upper boundary. When the simulation calculation domain is selected, the lower boundary is not affected by irrigation, and the left and right vertical boundaries are non-flux boundaries. During the irrigation process, the specific discharge of moistube ($Q$) at each node is basically constant (Niu et al. 2017; Fan et al. 2018a), and the Equation (7) established by Fan et al. (2018a) is used for the calculation:

$$q = K(H + M + a\gamma B + b)$$

where $M$ is the distance between the computing node and the water inlet (cm); $\gamma$ is the soil bulk density (g/cm$^3$); $B$ is the buried depth of the calculation node (cm); $a$ and $b$ are the fitting parameters.

In summary, the initial conditions can be expressed as:

$$\theta(r, z, t) = \theta_0(r, z), \ 0 \leq r \leq 30 \text{ cm}, \ 0 \leq z \leq 100 \text{ cm}, \ t = 0$$

where $\theta_0(r, z)$ is the initial soil moisture content (cm$^3$/cm$^3$).

The boundary conditions can be expressed as:

$$\begin{align*}
-K(\frac{\partial h}{\partial r}) + K(h) &= 0, \ 0 \leq r \leq 30 \text{ cm}, \ z = 0 \text{ or } z = 100 \text{ cm}, \ t \geq 0 \\
-K(\frac{\partial h}{\partial r}) &= 0, \ r = 0, \ 0 \leq z \leq D \text{ or } (D + L) \leq z \leq 100 \text{ cm}, \ t \geq 0 \\
-K(\frac{\partial h}{\partial r}) &= 0, \ r = 30 \text{ cm}, \ 0 \leq z \leq 100 \text{ cm}, \ t \geq 0 \\
-K(\frac{\partial h}{\partial r}) &= \frac{Q}{\pi d}, \ r = 0, \ D \leq z \leq (D + L), \ t \geq 0
\end{align*}$$

where $d$ is the diameter of moistube (cm).

**Simulation scheme**

In order to construct an empirical model of the size of the soil wetting pattern of vertical moistube irrigation, seven STs are selected. For each ST, the settings are as follows: three initial water content (60%FC, 70%FC, and 80%FC), three lengths of moistube (10, 15, and 20 cm), three buried depths of moistube (20, 30, and 40 cm) and three pressure heads (100, 150, and 200 cm), a total of 63 groups. During the simulation, the irrigation time is set to 192 h. The field water holding capacity of the soil is calculated using Equation (6), and $Q$ is obtained using Equation (7). The parameters of VG-M model of ST were taken from Carsel and Parrish’s data (Carsel & Parrish 1988), and the soil bulk density $\gamma$ was taken from Pachepsky and Park’s data (Pachepsky & Park 2015).

**Analysis method**

Studies have shown that the maximum size of soil wetting patterns in different directions must be considered when describing soil wetting patterns (Kilic 2018). The maximum sizes of the soil wetting pattern in the vertical upward, horizontal, and vertical downward directions are located at the highest point (point A), the middle point (point B), and the lowest point (point C) of the moistube, respectively (as shown in Figure 3). Therefore, the three characteristic values – vertical upward, horizontal direction, and vertical downward – are selected to describe the soil wetting pattern of vertical moistube irrigation. Many scholars have found that the migration process of point source and line source infiltration can be described by power function with high accuracy.
The parameters that the empirical model has good predictive performance. If the comparison result shows that MAE, root mean square error (RMSE), and Nash–Sutcliffe efficiency coefficient (NSE) are used to evaluate the performance of the equation. The mean absolute error (MAE), root mean square error (RMSE), and Nash–Sutcliffe efficiency coefficient (NSE) are used to evaluate the performance of the empirical model. If the comparison result shows that MAE and RMSE are closer to 0, and NSE is closer to 1, it indicates that the empirical model has good predictive performance. The parameters $R^2$, RMSE, MAE, and NSE are calculated using the following equations (Moriasi et al. 2007):

$$R^2 = \frac{\left(\sum_{i=1}^{N} O_i C_i - \sum_{i=1}^{N} C_i \sum_{i=1}^{N} O_i\right)^2}{\left(\sum_{i=1}^{N} (C_i)^2 - \left(\sum_{i=1}^{N} C_i\right)^2\right) \left(\sum_{i=1}^{N} (O_i)^2 - \left(\sum_{i=1}^{N} O_i\right)^2\right)}$$

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |O_i - C_i|$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (O_i - C_i)^2}$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N} (O_i - C_i)^2}{\sum_{i=1}^{N} (O_i - O_m)^2}$$

where $O_i$ is the $i$th measured value; $C_i$ is the $i$th calculated value; $O_m$ is the average value of the measured value; and $N$ is the total number of data.

**RESULTS AND DISCUSSION**

Parameter determination

Wetting pattern size of vertical upward

Based on the HYDRUS-2D simulation results, using Equation (10), the fitting parameters $a_i$ and $b_i$ in different influencing factor combinations are obtained through Origin 9.0 fitting. The fitting results are shown in Table 2.

It can be seen from Table 2 that using Equation (10) to fit the relationship between the vertical upward wetting pattern size of vertical moistube irrigation and the irrigation time, $R^2$ is 0.985. It shows that Equation (10) demonstrates the relationship between the vertical upward wetting pattern size of vertical moistube irrigation and irrigation time.

Fitting parameter $b_i$

Table 2 shows that $ST$, $\theta_0$, $H$, $L$, and $D$ have little influence on the values of fitting parameter $b_i$, which fluctuate around the average value 0.476, and the fluctuation range is small. To simplify the calculation, the average value of the fitting parameter $b_i$ can be taken. Therefore, Equation (10) can be further expressed as:

$$Z_A = a_1 \theta_0^{0.476}$$

(17)

Fitting parameter $a_i$

A further analysis of Table 2 finds that the value of the fitting parameter $a_i$ varies in a small range. However, previous studies have found that the size of the soil wetting pattern of vertical moistube irrigation is mainly affected by $ST$, $\theta_0$, $H$, $L$, and $D$, while $H$, $L$, and $D$ affect $Q$, which in turn affects the size of the wetting pattern (Fan et al. 2018a).

Therefore, taking the parameter $a_i$ as the average value alone does not meet the actual situation, nor can it meet the requirements for model universality. It is worth
considering that \( Q \) at different buried depths is different, and due to the influence of gravity, the discharge flow of the adjacent points above and below the moistube will have a superposition effect. Therefore, \( Q \) alone cannot fully reflect the water permeability of the moistube, and the product of the average specific flow rate \( (q) \) of the moistube and \( L \) can be used to characterize the water permeability of the moistube. The \( \theta_0 \) has a certain influence on the size of the wetting pattern; the influence mechanism is that during the irrigation process, the irrigation water moves around in the soil pores, and gradually fills the soil pores until the soil is fully saturated. Therefore, the final determination of the size

<table>
<thead>
<tr>
<th>( H ) (cm)</th>
<th>( L ) (cm)</th>
<th>( D ) (cm)</th>
<th>( \theta_0 ) (cm³/cm³)</th>
<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>15</td>
<td>20</td>
<td>80%FC</td>
<td>0.124</td>
<td>0.542</td>
<td>0.993</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>60%FC</td>
<td>0.151</td>
<td>0.509</td>
<td>0.992</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>70%FC</td>
<td>0.183</td>
<td>0.489</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.210</td>
<td>0.462</td>
<td>0.991</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.241</td>
<td>0.465</td>
<td>0.991</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.160</td>
<td>0.468</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.250</td>
<td>0.434</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.219</td>
<td>0.443</td>
<td>0.990</td>
</tr>
<tr>
<td>150</td>
<td>15</td>
<td>20</td>
<td>80%FC</td>
<td>0.257</td>
<td>0.424</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>60%FC</td>
<td>0.218</td>
<td>0.438</td>
<td>0.990</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>70%FC</td>
<td>0.221</td>
<td>0.436</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.229</td>
<td>0.433</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>15</td>
<td>30</td>
<td>80%FC</td>
<td>0.230</td>
<td>0.432</td>
<td>0.989</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.290</td>
<td>0.415</td>
<td>0.989</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.168</td>
<td>0.452</td>
<td>0.990</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.234</td>
<td>0.430</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.212</td>
<td>0.456</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>15</td>
<td>20</td>
<td>80%FC</td>
<td>0.163</td>
<td>0.494</td>
<td>0.992</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>60%FC</td>
<td>0.162</td>
<td>0.486</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>70%FC</td>
<td>0.161</td>
<td>0.487</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.167</td>
<td>0.485</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>15</td>
<td>30</td>
<td>80%FC</td>
<td>0.172</td>
<td>0.481</td>
<td>0.991</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.209</td>
<td>0.470</td>
<td>0.991</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.131</td>
<td>0.495</td>
<td>0.992</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.183</td>
<td>0.472</td>
<td>0.991</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.164</td>
<td>0.478</td>
<td>0.991</td>
</tr>
</tbody>
</table>
of the wetting pattern is the amount of water required to fill the pores of the soil to achieve approximate saturation. It can be expressed by the difference between $\theta_s$ and $\theta_0$, that is, $\theta_s - \theta_0$. In addition, the influence of $ST$ on the size of the wetting pattern can be characterized by $K_s$. Therefore, the influencing factors of the simplified vertical moistube irrigation soil wetting pattern size mainly include $K_s$, $qL$, and $\theta_s - \theta_0$.

Based on this, the function expression of fitting parameter $a_1$ is established by multiple regression analysis as follows:

$$a_1 = \lambda_1(\theta_s - \theta_0)c^2K_s^d(qL)^e_1$$

(18)

where $\lambda_1$, $c_1$, $d_1$, and $e_1$ are all empirical parameters.

**Wetting pattern size of horizontal**

Table 3 shows the simulation results based on HYDRUS-2D. Using Equation (11), the values of fitting parameters $a_2$ and $b_2$ under different influencing factor combinations and $R^2$ are obtained by fitting with Origin 9.0. It can be seen from Table 3 that using Equation (11) to fit the relationship between the horizontal wetting pattern size of vertical moistube irrigation and the irrigation time gives $R^2 \geq 0.985$, indicating that Equation (11) demonstrates the relationship between the horizontal wetting pattern size of vertical moistube irrigation and the irrigation time.

**Fitting parameter $b_2$**

A comparative analysis of Table 3 finds that $ST$, $\theta_0$, $H$, $L$, and $D$ also have little effect on the value of the fitting parameter $b_2$, all fluctuating around 0.406 on the average. To simplify the calculation, the average value of the fitting parameter $b_2$ is taken. Therefore, Equation (11) can be further expressed as:

$$R_B = a_2t^{0.406}$$

(19)

**Fitting parameter $a_2$**

A further analysis of Table 3 finds that the fitting parameter $a_2$ is affected by $ST$, $\theta_0$, $H$, $L$, and $D$, and that the value fluctuates greatly. The size of the horizontal wetting pattern is a component of the size of the soil wetting pattern of the vertical moistube irrigation, and the analysis of the fitting parameter $a_1$ shows that the factors affecting the size of the soil wetting pattern of the vertical moistube irrigation have been simplified to $K_s$, $qL$, and $(\theta_s - \theta_0)$. Therefore, the functional expression of the fitting parameter $a_2$ can be established by multiple regression analysis, which is:

$$a_2 = \lambda_2(\theta_s - \theta_0)c^2K_s^d(qL)^e_2$$

(20)

where $\lambda_2$, $c_2$, $d_2$, and $e_2$ are all empirical parameters.

**Wetting pattern size of vertical downward**

Using the HYDRUS-2D simulation results and Equation (12), the fitting parameters $a_3$ and $b_3$ under different combinations of influencing factors were obtained by Origin 9.0. The values of parameters $a_3$ and $b_3$ and $R^2$ are shown in Table 4. It can be seen from Table 4 that Equation (12) is used to fit the relationship between wetting pattern dimension of vertical downward of vertical moistube irrigation and irrigation time, and $R^2 \geq 0.989$, indicating that Equation (12) demonstrates the relationship between wetting pattern dimension of vertical downward of vertical moistube irrigation and irrigation time.

**Fitting parameter $b_3$**

A comparative analysis of Table 4 shows that $ST$, $\theta_0$, $H$, $L$, and $D$ also have a small effect on the value of the fitting parameter $b_3$, all fluctuating around 0.428 on average. In order to simplify the calculation, the average value of the fitting parameter is taken as $b_3$. Equation (12) can be expressed as:

$$Z_C = a_3t^{0.428}$$

(21)

**Fitting parameter $a_3$**

A further analysis of Table 4 finds that the value of the fitting parameter $a_3$ fluctuates greatly, with $ST$, $\theta_0$, $H$, $L$, and $D$ all affecting it. The vertical downward wetting pattern size and the vertical upward and horizontal wetting pattern size can completely describe the vertical moistube irrigation soil wetting pattern. According to the analysis of fitting parameters $a_1$, $a_2$, and $a_3$, the fitted values of $a_1$, $a_2$, and $a_3$ are shown in Table 4.
and $a_2$, the function expression of fitting parameter $a_3$ is established based on multiple regression analysis, which is:

$$a_3 = \lambda_3 (\theta_s - \theta_0)^{c_3} K_d^{d_3} (qL)^{e_3}$$  \hspace{1cm} (22)

where $\lambda_3$, $c_3$, $d_3$, and $e_3$ are all empirical parameters.

**Establishment of the empirical model**

Equations (18), (20), and (22) were substituted into Equations (17), (19), and (21), respectively. With 63 simulation data, the empirical model of soil wetting pattern dimensions under vertical moistube irrigation was obtained.
by Origin 9.0; namely:

\[ Z_A = 0.197(\theta_s - \theta_0)^{0.016} K_s^{-0.045}(qL)^{0.206} \theta^{0.476} \]  
\[ R_B = 0.694(\theta_s - \theta_0)^{-0.055} K_s^{0.008}(qL)^{0.267} \theta^{0.406} \]  
\[ Z_C = 0.629(\theta_s - \theta_0)^{-0.151} K_s^{0.110}(qL)^{0.228} \theta^{0.428} \]  

Evaluation of the empirical model

To evaluate the accuracy of the prediction model of the soil wetting pattern size of the vertical moistube irrigation, the indoor experiments is used to verify the model, with the calculated value of the prediction model compared with the measured value (Figure 4). Based on Equations (14)–(16),

Table 4 | Parameter values of \(a_3\) and \(b_3\), and coefficients of determination \(R^2\)

<table>
<thead>
<tr>
<th>(H) (cm)</th>
<th>(L) (cm)</th>
<th>(D) (cm)</th>
<th>(\theta_s) (cm³/cm³)</th>
<th>(a_3)</th>
<th>(b_3)</th>
<th>(R^2)</th>
<th>(a_3)</th>
<th>(b_3)</th>
<th>(R^2)</th>
<th>(a_3)</th>
<th>(b_3)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>15</td>
<td>20</td>
<td>80%FC</td>
<td>0.493</td>
<td>0.403</td>
<td>0.988</td>
<td>0.485</td>
<td>0.406</td>
<td>0.988</td>
<td>0.415</td>
<td>0.396</td>
<td>0.988</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>60%FC</td>
<td>0.421</td>
<td>0.415</td>
<td>0.989</td>
<td>0.418</td>
<td>0.413</td>
<td>0.988</td>
<td>0.427</td>
<td>0.418</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>70%FC</td>
<td>0.416</td>
<td>0.420</td>
<td>0.989</td>
<td>0.417</td>
<td>0.416</td>
<td>0.989</td>
<td>0.440</td>
<td>0.416</td>
<td>0.989</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.432</td>
<td>0.415</td>
<td>0.989</td>
<td>0.425</td>
<td>0.418</td>
<td>0.989</td>
<td>0.439</td>
<td>0.418</td>
<td>0.989</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.403</td>
<td>0.398</td>
<td>0.988</td>
<td>0.415</td>
<td>0.396</td>
<td>0.988</td>
<td>0.426</td>
<td>0.396</td>
<td>0.988</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.599</td>
<td>0.367</td>
<td>0.986</td>
<td>0.584</td>
<td>0.372</td>
<td>0.986</td>
<td>0.599</td>
<td>0.372</td>
<td>0.986</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.574</td>
<td>0.365</td>
<td>0.986</td>
<td>0.548</td>
<td>0.373</td>
<td>0.986</td>
<td>0.577</td>
<td>0.370</td>
<td>0.986</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.478</td>
<td>0.419</td>
<td>0.989</td>
<td>0.455</td>
<td>0.425</td>
<td>0.989</td>
<td>0.467</td>
<td>0.427</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.476</td>
<td>0.401</td>
<td>0.988</td>
<td>0.485</td>
<td>0.400</td>
<td>0.988</td>
<td>0.543</td>
<td>0.391</td>
<td>0.987</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.403</td>
<td>0.398</td>
<td>0.988</td>
<td>0.415</td>
<td>0.396</td>
<td>0.988</td>
<td>0.426</td>
<td>0.396</td>
<td>0.988</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.599</td>
<td>0.367</td>
<td>0.986</td>
<td>0.584</td>
<td>0.372</td>
<td>0.986</td>
<td>0.599</td>
<td>0.372</td>
<td>0.986</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.574</td>
<td>0.365</td>
<td>0.986</td>
<td>0.548</td>
<td>0.373</td>
<td>0.986</td>
<td>0.577</td>
<td>0.370</td>
<td>0.986</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.282</td>
<td>0.505</td>
<td>0.992</td>
<td>0.445</td>
<td>0.421</td>
<td>0.989</td>
<td>0.413</td>
<td>0.429</td>
<td>0.989</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.232</td>
<td>0.521</td>
<td>0.992</td>
<td>0.406</td>
<td>0.418</td>
<td>0.989</td>
<td>0.384</td>
<td>0.424</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.228</td>
<td>0.523</td>
<td>0.992</td>
<td>0.395</td>
<td>0.431</td>
<td>0.989</td>
<td>0.364</td>
<td>0.438</td>
<td>0.990</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.239</td>
<td>0.520</td>
<td>0.992</td>
<td>0.404</td>
<td>0.423</td>
<td>0.989</td>
<td>0.376</td>
<td>0.429</td>
<td>0.989</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>30</td>
<td>80%FC</td>
<td>0.244</td>
<td>0.535</td>
<td>0.993</td>
<td>0.420</td>
<td>0.437</td>
<td>0.990</td>
<td>0.401</td>
<td>0.442</td>
<td>0.990</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>30</td>
<td>80%FC</td>
<td>0.210</td>
<td>0.508</td>
<td>0.992</td>
<td>0.340</td>
<td>0.428</td>
<td>0.989</td>
<td>0.347</td>
<td>0.420</td>
<td>0.989</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.343</td>
<td>0.469</td>
<td>0.991</td>
<td>0.500</td>
<td>0.398</td>
<td>0.988</td>
<td>0.462</td>
<td>0.406</td>
<td>0.988</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>40</td>
<td>80%FC</td>
<td>0.242</td>
<td>0.501</td>
<td>0.992</td>
<td>0.484</td>
<td>0.396</td>
<td>0.988</td>
<td>0.471</td>
<td>0.396</td>
<td>0.988</td>
</tr>
</tbody>
</table>
a statistical analysis is carried out on the calculated and measured values of the model (Table 5).

Figure 4 shows that the measured values of soil wetting pattern of vertical moistube irrigation is consistent with the calculated values of the model, but the horizontal fit is the best. It can be seen from Table 5 that MAE of the statistical indicators of the built model is between 0.30 and 1.42 cm, RMSE is between 0.42 and 1.65 cm, and NSE is not less than 0.93. This shows that the model prediction effect is good, but there are still some errors. The reason may be that when the prediction model of soil wet pattern size for vertical moistube irrigation is established, in different combinations of \( K_s \), \( qL \), and \( (\theta_s - \theta_0) \), the power function exponent of the wetting pattern size and irrigation time varies little in

**Table 5** | Statistical analysis of calculated values of the model and measured values for wetting pattern dimensions

<table>
<thead>
<tr>
<th>ST</th>
<th>Experimental setup</th>
<th>MAE (cm)</th>
<th>RMSE (cm)</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam</td>
<td>( L = 20 \text{ cm}, \ D = 30 \text{ cm}, \ H = 150 \text{ cm} )</td>
<td>0.30</td>
<td>0.42</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>( L = 10 \text{ cm}, \ D = 40 \text{ cm}, \ H = 200 \text{ cm} )</td>
<td>1.19</td>
<td>1.33</td>
<td>0.96</td>
</tr>
<tr>
<td>Silt loam</td>
<td>( L = 20 \text{ cm}, \ D = 30 \text{ cm}, \ H = 150 \text{ cm} )</td>
<td>0.46</td>
<td>0.61</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>( L = 10 \text{ cm}, \ D = 40 \text{ cm}, \ H = 200 \text{ cm} )</td>
<td>1.42</td>
<td>1.65</td>
<td>0.93</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>( L = 20 \text{ cm}, \ D = 30 \text{ cm}, \ H = 150 \text{ cm} )</td>
<td>0.56</td>
<td>0.75</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>( L = 10 \text{ cm}, \ D = 40 \text{ cm}, \ H = 200 \text{ cm} )</td>
<td>0.94</td>
<td>1.19</td>
<td>0.97</td>
</tr>
</tbody>
</table>
the vertical upward, horizontal, and vertical downward
direction. In order to simplify the calculation, the average
values are taken, which affects the accuracy of the calcu-
lation results to a certain extent. In addition, only using
the difference between $K_s$ and $(\theta_s-\theta_0)$ to characterize
the size of different ST wetting patterns is also one of the
reasons for the partial error. It should be noted that in the
process of irrigation advancement, the wetting boundary
will reach the surface and the vertical upward Equation
(23) is no longer applicable, but the buried depth value
can be taken when the estimated value is greater than the
buried depth of the moistube. The arrival of the wetting
boundary on the surface has little effect on the horizontal
and vertical downward migration of wetting patterns, and
Equations (24) and (25) still apply. For soils with large
pores between soil particles, such as sand, the model
should be used with caution.

CONCLUSIONS

On the basis of qualitative research on the influencing factors
of wetting pattern of vertical moistube irrigation, this study
adds the simulation quantity. The HYDRUS-2D software is
used to simulate the size data of the wetting pattern under
different factors. By analyzing and summarizing the laws of
simulation data, a simplified empirical model for predicting
the size of soil wetting patterns has been constructed, with
the reliability of the model verified with experimental data.
The following conclusions are obtained.

The power function is used to fit the relationship
between the size of the soil wetting pattern and the irrigation
time. The power function exponent varies slightly in the ver-
tical upward, horizontal, and vertical downward directions
of the moistube, with the average values taken as 0.476,
0.406, and 0.428, respectively. The power function coeffi-
cient conformed to the power function relationship with
$K_s$, $qL$, and $(\theta_s-\theta_0)$.

Based on this, an empirical model of the size of the ver-
tical moistube irrigation wetting pattern was established.
The model includes $t$, $K_s$, $qL$, and $(\theta_s-\theta_0)$. The MAE of the
statistical indicators of the built model is between 0.30 and
1.42 cm, RMSE is between 0.42 and 1.65 cm, and NSE is
not less than 0.93. The prediction effect of the model is
good, and it can provide a scientific basis for the design,
operation, and management of the moistube irrigation
engineering.

ACKNOWLEDGEMENTS

This study was supported by the National Natural Science
Foundation of China (No. 51409137 and 51969013), the
Natural Science Foundation of Gansu Province, China
(No. 18JR3RA144), and Hongliu Supporting Discipline of
Lanzhou University of Technology.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplemen-
tary Information.

REFERENCES

Al-Ogaidi, A. M. A., Wayayok, A., Rowsmon, M. K. & Abdullah,
A. F. 2016 Wetting patterns estimation under drip irrigation
systems using an enhanced empirical model. Agricultural

Brandt, A., Bresler, E., Diner, N., Ben-Asher, I., Heller, J. &
Goldberg, D. 1971 Infiltration from a trickle source: I.
mathematical models. Soil Science Society of America

Carsel, R. F. & Parrish, R. S. 1988 Developing joint probability
distributions of soil water retention characteristics. Water
Resources Research 24, 755–769.

Chu, S. T. 1994 Green-Ampt analysis of wetting patterns for
surface emitters. Journal of Irrigation and Drainage
Engineering 120, 414–421.

Cook, P. J., Thorburn, P. J., Fitch, P. & Bristow, K. L. 2005 Wetup:
a software tool to display approximate wetting patterns from

Dabra, P. P., Pandey, P. K., Ashish, P., Singh, K. P. & Sanjoy, S. M.
2012 Modeling of wetting pattern under trickle source in
sandy soil of Nirjuli Pradesh (India). Irrigation Science 30,
287–292.

Fan, Y. W., Huang, N., Zhang, J. & Zhao, T. 2018a Simulation of
soil wetting pattern of vertical moistube-irrigation. Water 10,
1–19.

Fan, Y. W., Shao, X. X., Wang, Y. & Gong, J. G. 2018b Empirical
model for predicting wetted soil dimensions under vertical
line source irrigation. Transactions of the Chinese Society for
Agricultural Machinery 49, 336–346.


Han, M., Zhao, C., Feng, G., Yan, Y. & Sheng, Y. 2015 Evaluating the effects of mulch and irrigation amount on soil water distribution and root zone water balance using HYDRUS-2D. Water 7, 2622–2640.


Moncef, H. & Khemaies, Z. 2016 An analytical approach to predict the moistened bulb volume beneath a surface point source. Agricultural Water Management 166, 123–129.


Philip, J. R. 1984 Travel times from buried and surface infiltration point sources. Water Resources Research 20, 990–994.


First received 7 July 2020; accepted in revised form 5 November 2020. Available online 18 November 2020