# Minimizing the cost of a curved corner trapezoidal canal section

Ehab M. Fattouh and Neveen Y. Saad

## ABSTRACT

Designing a curved corner trapezoidal channel section with a minimal cost, which is the study's objective function, encompasses minimizing the channel lining and excavation costs. The discharge, as the prime constraint, and the permissible velocities, as subsidiary constraints, were considered to solve the problem. Mathematical optimization was used to obtain the optimum canal dimensions. The results were represented in chart form to facilitate easy design of the optimal channel dimensions with minimum cost. To demonstrate the practicability of the proposed method, a design example has been included. A comparison between the parameters and the cost of the proposed section with the conventional trapezoidal section revealed that the proposed section is more economic, and more suitable from a maintenance point of view. At last, sensitivity analysis was derived to show the effect of changing the canal dimensions on the cost.

**Key words** | canal cross section, cost, excavation, lining, optimal dimensions

**Ehab M. Fattouh**
**Neveen Y. Saad** (corresponding author)
Faculty of Engineering,
Ain Shams University,
Cairo,
Egypt
E-mail: *neveen_yousif@eng.asu.eg.edu*

## HIGHLIGHTS

- The proposed section is the standard section shape existing in some countries.
- As the canals need a primary amount of finance, their design must be implemented on an optimization basis.
- Comparing the parameters and cost of the proposed section to the most economic trapezoidal section resulting from another study illustrated that the proposed section is more economic and preferable for easier maintenance.

## INTRODUCTION

Open channels are considered as water transmitters for many purposes, such as irrigation, waterpower systems, domestic and industrial uses. They need a primary amount of finance depending on their cross sections and lengths. So, their design must be implemented on an optimization basis. Therefore, it is essential to obtain the optimum canal cross section dimensions with minimal construction cost, which involves the earthworks and lining costs. Canal linings are essential for using the water resources and land efficiently. Lining controls seepage loss and this minimizes the waterlogging in the contiguous areas, and conserves water for further expansion of the irrigation network. Also, lining helps to reduce the canal cross-section area due to the increase in the permissible velocity.

Many studies have been implemented to save water in irrigation systems and achieve the optimum design of the channel cross-sections. Different cross section types were involved, including traditional sections such as trapezoidal ones (Bhattachariya & Satish (2007) and Jain *et al.* (2004)), or non traditional ones such as parabolic (Aksoy & Altan-Sakarya (2006) and Chahar (2005)), curvilinear bottomed channel (Chahar 2007), elliptical sides with horizontal bottoms (Easa 2016) and semi-elliptical channels (Motlagh *et al.* 2018).

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Generally, the objective function is to minimize the total canal construction cost, 'excavation, lining, and land acquisition', which is subject to a constraint represented by Manning's equation. So, the most economical section optimization model involves nonlinear constraints and objective function (Han *et al.* 2018).

An optimization algorithm is a process that is carried out iteratively by comparing various solutions until an optimum solution is achieved. The classical optimization techniques, such as Lagrange's method of undetermined multipliers, are analytical methods that use differential calculus to find the optimum solution. The great advantage of Lagrange's method of undetermined multipliers is that it allows the optimization to be solved without explicit parameterization in terms of the constraints.

Evolutionary algorithms such as genetic algorithm (GA) and linear genetic programming (LGP) are very popular. GA solves optimization problems relying on natural operators such as selection, mutation and crossover. The main advantage of GA is that it is applicable to optimization problems whenever the objective function is non differentiable, highly nonlinear, stochastic, or discontinuous. LGP is a specified subset of genetic programming in which computer programs in a population are outlined as a series of instructions from machine language or imperative programming language. The drawbacks of these algorithms are poor constraint management ability, limited problem size, and problem-specific parameter tuning (Venter 2010).

Different techniques were used by researchers to develop models that minimize the cost of the section. Froehlich (1994) determined the best trapezoidal section dimensions using 'Lagrange's method of undetermined multipliers'. Bhattacharjya & Satish (2007) considered the safety of the canal slope to design a stabilized trapezoidal canal by implementing a hybrid optimization technique. Tofiq & Guven (2015) analyzed the canal width to depth ratio '$b/d$' effects on the trapezoidal lined canal cost by applying linear genetic programming techniques. Jain *et al.* (2004) studied the optimum section for minimizing the composite channels' total construction cost by using the genetic algorithm. Chen *et al.* (2019) presented a group of general exact solutions of the optimum hydraulic horizontal-bottomed power-law sections based on 'Gauss Hyper geometric mathematics' and 'Lagrange multiplier'. Das (2007) minimized the cost for a cross section with parabolic sides by using the Lagrange Multiplier technique. Chahar (2007) implemented Lagrange's undetermined multiplier method to obtain optimum parameters corresponding to the minimal area section and seepage loss section for channel sections with curvilinear bottom. Easa (2009, 2016) and Easa & Vatankhah (2014) minimized the construction cost for elliptical sections with horizontal bottom and two-segment parabolic sides by using 'Premium Solver Software'. Also, some researchers examined the effectiveness of different algorithms in designing optimum canal cross sections of different geometries (Kentli & Mercan (2014) and Turan & Yurdusev (2016)). Maintenance is costly for some recent shapes like the power- law section and elliptical shape (Hussein 2008).

The trapezoidal shape is the most common cross section used for water transmission. One practical implication of constructing curved section canals is the strength of rigid lining. A curved section is stronger than a section with a linear base and sides. Cracks at the base of the side slopes in a trapezoidal lining are the common cause of its failure, where the geometric discontinuity produce intense stresses from soil movement, soil pressure, or external loads (Chahar & Basu 2009).

Froehlich (2008) evaluated the cross-sectional dimensions of the most hydraulically efficient lined channels, based on an analysis of the trapezoidal shape with rounded bottom vertices used in India. He used the 'Lagrange multiplier technique' to find optimal section dimensions, in which the only constraint considered is that of uniform flow. Muzaffar *et al.* (2012) applied the 'Lagrange multiplier technique' to minimize the lining costs of a trapezoidal canal with rounded corners. They found that a trapezoidal section with rounded corners is more economic than the conventional trapezoidal one, but their study did not consider the excavation cost. Neither Froehlich (2008) nor Muzaffar *et al.* (2012) took into consideration the presence of the free board to simplify the analysis. Channels without freeboards are impractical, as they are constructed in practice with freeboards to provide a factor of safety against raising the water level more than anticipated.

Despite the trapezoidal canal section with curved corners being the standard section shape existing in some countries, the only previous study that considered minimizing the cost of the lined trapezoidal canal section with curved corners was carried out by Muzaffar *et al.* (2012). However, this study did not take into consideration the excavation cost or the free board, which makes it impractical for use.

In the current study, the 'Lagrange undetermined multiplier technique' is implemented to achieve an optimum design of a curved corner trapezoidal section corresponding to the minimal construction cost. This cost includes the lining and excavation costs per canal unit length. As the freeboard is part of the practical canal cross section, it was integrated into the objective function. This study presents a solution to the cost minimization problem when the material cost of the base is different from that of the canal's sides and curved parts. The nonlinear optimization model is executed by the 'Solver' Module of MS-Excel software.

## THE OPTIMIZATION MODEL STRUCTURE

The canal design optimization process involves two main factors, which are the objective function and the constraints. The objective function is to get an optimal canal section (canal bottom width, depth, side slope, and corner radius for trapezoidal channels) with minimum costs (excavation, canal lining). Figure 1 represents the geometric properties of the trapezoidal curved corners section. The constraints are the restrictions that should be applied in the optimization operation to solve the objective function.

The following assumptions are considered:

- The flow is uniform and steady.
- Manning's and the continuity equations are considered as the calculations' base.
- Concrete lining is supposed to be used.
- Manning's roughness coefficient 'n' is 0.0167.
- The channel cross-section has a trapezoidal shape with curved corners.

Four values of the side slope are assumed, which are 0.5, 1, 1.5 and 2. Discharge is assumed to be within the range 6< $Q \leq 180$ m$^3$/s. This range was undertaken according to the common discharge values of channel discharges in Egypt.

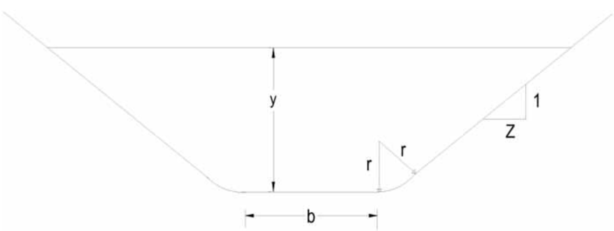


**Figure 1** | Definition sketch for the curved corner trapezoidal canal section.

### Geometrical parameters

The radius of a curved corner trapezoidal section r is a function of the water depth y;

$$r = Iy \tag{1}$$

where, I is a constant and its value should be less than 1.0

The flow area 'A' and wetted perimeter 'P' of this section 'Figure 1' could be expressed as follows (Chahar & Basu 2009):

$$A = b\,y + 2I\,y^2\beta_1 + y^2Z - 2I^2y^2\beta_1 + I^2y^2\beta_2 \tag{2}$$

$$P = b + 2I\,y\,\beta_2 + 2y\sqrt{1+Z^2} - 2I\,y\,\beta_1 \tag{3}$$

where, Z = 'channel side slope', b = 'channel bottom width', y = 'channel water depth', and

$$\beta_1 = \sqrt{1+Z^2} - Z, \qquad \beta_2 = \tan^{-1}\left(\frac{1}{Z}\right) * \frac{\pi}{180} \tag{4}$$

It could be noted that in the case that I = 0, the section will be converted to a normal trapezoidal section.

### Objective function

The objective function for the total canal cost comprises the excavation and the lining cost. The area of the free board above the water surface was included in the total construction area. Both the excavation and the lining costs are expressed below (Ghazaw 2011; Chahar & Basu 2009).

**Excavation cost ($C_e$)**

Excavation cost can be written as:

$$C_e = C_v[b\,D + 2\,I\,\beta_1 D^2 + Z\,D^2 - 2I^2\beta_1 D^2 + I^2\beta_2 D^2] \tag{5}$$

where $C_v$ = excavation cost per unit volume of the channel, D = excavation depth = y + f and f = free board of the channel.

**Lining cost ($C_{lin}$)**

The lining cost for the sides, curves and base are assumed to be different, because the performance of work on the curved portion and sides needs a special type of works and labors, let:

$C_b$, $C_c$, and $C_s$ = lining cost per unit area for the base, the curved portion and the sides, respectively.

So, the cost of canal lining per unit canal length is given as follows:

$$C_{lin} = C_b b + C_c * 2I\beta_2 D + C_S[2\sqrt{1+Z^2}\ D - 2I\,\beta_1 D] \tag{6}$$

**Total cost of the canal (C)**

The total cost of a curved corner trapezoidal canal, C, is the sum of the excavation cost and the lining cost, which may be written as:

$$C = C_e + C_{lin} \tag{7}$$

$$C = C_v\left[b\ D + 2\ I\ \beta_1 D^2 + Z\ D^2 - 2I^2\beta_1 D^2 + I^2\beta_2 D^2\right] + C_b b + C_C * 2I\beta_2 D + C_S\left[2\sqrt{1+Z^2}\ D - 2\ I\ \beta_1 D\right] \tag{8}$$

## Problem constraints

The constraints that were applied in the optimization process are:

**Flow constraint**

For problem optimization in the current study, the discharge, Q, should be the same value supposed by Manning's Equation (9) (Chow 1973).

$$Q = \frac{\aleph}{n}\frac{A^{5/3}}{P^{2/3}} S^{1/2} \tag{9}$$

where, Q = the flow rate ($m^3$/s), A = the flow area ($m^2$), $P$ = the wetted perimeter (m), $n$ = Manning's roughness coefficient, S = the longitudinal channel bed slope (dimensionless), and $\aleph$ = measuring system constant, 1.0 in Standard International units and 1.486 in English systems.

**Velocity constraint**

The mean velocity 'V = Q/A' should be compared with the minimum and maximum velocity limits, as: $V_{min} < V < V_{max}$

The minimum permissible velocity '$V_{min}$' is the minimum velocity that does not launch sedimentation or lead to vegetation growth, as those reduce the channels' capacities. Generally, a velocity range between 0.6 and 0.9 m/s could prevent sedimentation, and a velocity of 0.7 m/s is usually adequate to prevent vegetation growth (Chow 1973). On the other hand, the maximum allowable velocity '$V_{max}$' is the velocity that does not lead to erosion of the channel bed or sides. Velocity limits for different channel lining materials were given by Sharma & Chawla (1975).

## Optimization procedure

To obtain the minimal cost of the channel, the total cost per unit length C, demonstrated by Equation (8), and the discharge equation, Equation (9), the flow constraint, should be minimized. The discharge equation could be written as:

$$\varnothing\ (y, b, z) = A^{\alpha}P^{\beta} - S.F = A^{5/3} * P^{-2/3} - S.F = 0 \tag{10}$$

where, A = flow area, $P$ = wetted perimeter, $\alpha$ and $\beta$ = constants equal to (5/3) and (−2/3) consequently, and $S.F = \frac{Q\ \ n}{\aleph\ \sqrt{S}}$ is the section factor of the channel.

The Lagrange multiplier is a useful method for optimization to derive a general differential equation for the most economic section, as given by Han & Easa (2018). Applying Lagrange's method of undetermined multipliers with $\lambda$ as the undetermined multipliers, the next relations are obtained:

$$\frac{\partial C}{\partial y} + \lambda\frac{\partial \varnothing}{\partial y} = 0 \tag{11}$$

$$\frac{\partial C}{\partial Z} + \lambda\frac{\partial \varnothing}{\partial Z} = 0 \tag{12}$$

$$\frac{\partial C}{\partial b} + \lambda\frac{\partial \varnothing}{\partial b} = 0 \tag{13}$$

Substituting Equation (11) into Equation (13) ⇒

$$\frac{\partial C}{\partial b} + \frac{-\partial C/\partial y}{\partial \varnothing/\partial y} * \frac{\partial \varnothing}{\partial b} = 0 \tag{14}$$

which leads to:

$$\frac{\partial C}{\partial b} * \frac{\partial \varnothing}{\partial y} = \frac{\partial C}{\partial y} * \frac{\partial \varnothing}{\partial b} \tag{15}$$

Substituting Equation (12) into Equation (13) ⇒

$$\frac{\partial C}{\partial b} + \frac{-\partial C/\partial Z}{\partial \varnothing/\partial Z} * \frac{\partial \varnothing}{\partial b} = 0 \tag{16}$$

which leads to:

$$\frac{\partial C}{\partial b} * \frac{\partial \varnothing}{\partial Z} = \frac{\partial C}{\partial Z} * \frac{\partial \varnothing}{\partial b} \tag{17}$$

From Equation (10) $\varnothing = A^{\alpha} P^{\beta} - S.F$, therefore:

$$\frac{\partial \varnothing}{\partial y} = A^{\propto} \beta P^{\beta-1} \frac{\partial P}{\partial y} + P^{\beta} \alpha A^{\alpha-1} \frac{\partial A}{\partial y} = P^{\beta} A^{\propto} \frac{\beta}{P} \frac{\partial P}{\partial y} + P^{\beta} A^{\alpha} \frac{\alpha}{A} \frac{\partial A}{\partial y} \tag{18}$$

$$\frac{\partial \varnothing}{\partial y} = P^{\beta} A^{\propto} \left( \frac{\alpha}{A} \frac{\partial A}{\partial y} + \frac{\beta}{P} \frac{\partial P}{\partial y} \right) \tag{19}$$

Similarly

$$\frac{\partial \varnothing}{\partial b} = P^{\beta} A^{\propto} \left( \frac{\alpha}{A} \frac{\partial A}{\partial b} + \frac{\beta}{P} \frac{\partial P}{\partial b} \right) \tag{20}$$

$$\frac{\partial \varnothing}{\partial Z} = P^{\beta} A^{\propto} \left( \frac{\alpha}{A} \frac{\partial A}{\partial Z} + \frac{\beta}{P} \frac{\partial P}{\partial Z} \right) \tag{21}$$

**Curved corner radius (r = I y)**

From Equation (2) $A = b\,y + 2\,I\,y^2\beta_1 + y^2 Z - 2\,I^2 y^2 \beta_1 + I^2 y^2 \beta_2$

$$\frac{dA}{dy} = b + 4I\,y\,\beta_1 + 2\,y\,Z - 4I^2 y\,\beta_1 + 2I^2 y\,\beta_2, \quad \frac{dA}{db} = y \tag{22}$$

From Equation (3)

$$P = b + 2\,I\,y\,\beta_2 + 2\,y\sqrt{1+Z^2} - 2I\,y\,\beta_1$$

$$\frac{dP}{dy} = 2I\beta_2 + 2\sqrt{1+Z^2} - 2I\beta_1, \quad \frac{dP}{db} = 1 \tag{23}$$

From Equation (8) $C = C_v[b\,D + 2\,I\,C = C_v[b\,D + 2\,I\,\beta_1 D^2 + Z\,D^2 - 2I^2\beta_1 D^2 + I^2\beta_2 D^2] + C_b b +$

$C_c * 2I\beta_2 D + C_S[2\sqrt{1+Z^2}D - 2\,I\beta_1 D]$

$$\frac{\partial C}{\partial b} = C_v D + C_b \tag{24}$$

$$\frac{\partial C}{\partial y} = C_v[b + 4I\,\beta_1 D + 2\,Z\,D - 4I^2\beta_1 D + 2I^2\beta_2 D] + C_c * 2I\beta_2 + C_S[2\sqrt{1+Z^2} - 2I\beta_1] \tag{25}$$

Substituting Equations (19) and (20) in Equation (15)

$$\frac{\partial C}{\partial b} * P^{\beta} A^{\propto} \left[ \frac{\propto}{A} \frac{\partial A}{\partial y} + \frac{\beta}{P} \frac{\partial P}{\partial y} \right] = \frac{\partial C}{\partial y} * P^{\beta} A^{\propto} \left[ \frac{\propto}{A} \frac{\partial A}{\partial b} + \frac{\beta}{P} \frac{\partial P}{\partial b} \right] \tag{26}$$

Substituting Equations (22)–(25) in Equation (26) leads to;

$$[C_v D + C_b] * \left[ \frac{(5/3) * (b + 4\,I\,y\,\beta 1 + 2\,y\,Z - 4\,I^2\,y\,\beta 1 + 2\,I^2\,y\,\beta 2)}{b\,\,y + 2\,I\,y^2\beta 1 + y^2\,Z - 2\,I^2 y^2 \beta 1 + I^2 y^2 \beta 2} - \frac{(2/3) * \left(2\,I\,\beta 2 + 2\sqrt{1+Z^2} - 2\,I\,\beta 1\right)}{b + 2\,I\,y\,\beta 2 + 2\,y\sqrt{1+Z^2} - 2\,I\,y\,\beta 1} \right] =$$

$$\left[C_v[b + 4\,I\beta_1 D + 2\,Z\,D - 4\,I^2\beta_1 D + 2\,I^2\beta_2 D] + C_c * 2I\beta_2 + C_S[2\sqrt{1+Z^2} - 2\,I\beta_1]\right] * \left[ \frac{(5/3)\,y}{b\,\,y + 2\,I\,y^2\beta 1 + y^2\,Z - 2\,I^2 y^2 \beta 1 + I^2 y^2 \beta 2} - \frac{(2/3) * (1)}{b + 2\,I\,y\,\beta 2 + 2\,y\sqrt{1+Z^2} - 2\,I\,y\,\beta 1} \right] \tag{27}$$

Using Equations (10) and (27), the optimal canal cross section dimensions can be obtained. Once these values of b and y are calculated and assuming the discharge Q is known, the corresponding value of the mean velocity V is determined. This velocity should be less than the limiting velocity $V_{max}$ and greater than $V_{min}$. If V is more than $V_{max}$, then the section should be redesigned with revised surface roughness or bed slope (Chahar 2007).

## COMPUTER PROGRAM AND DESIGN CHARTS

Optimal values of the bed width 'b' and the water depth 'y' have been obtained from Equations (10) and (27), for different values of relative costs (the ratio between the average lining cost per meter length for the side slope and the round part to its corresponding value for the bed to the excavation cost per unit volume).

Numerical computations of the optimization models need many iterations (trial and error method) to achieve the true solution. So, the 'Solver' Module of MS-Excel was utilized to execute the optimization algorithm and to save time for the designers, Figure 2 shows a flow chart to represent the optimization model in detail.

The obtained values of b and y have been plotted to construct a set of design charts for different values of side slopes, Z (Figure 3(a)–3(d)). These charts help the designer to obtain the optimum dimensions of the channel for known values of Q, S, n, Z and the relative costs.

## DISCUSSION

Figure 3(a)–3(d) show the optimal bed width values against the optimal water depth values for different section factors S.F and the corresponding relative costs, considering the variation of Z as 0.5, 1.0,1.5 and 2.0. It is noticed that, for the same S.F, as the relative costs increase, the bed width increases, and the water depth decreases. So, the optimum section is shallower and wider for the bigger relative costs.

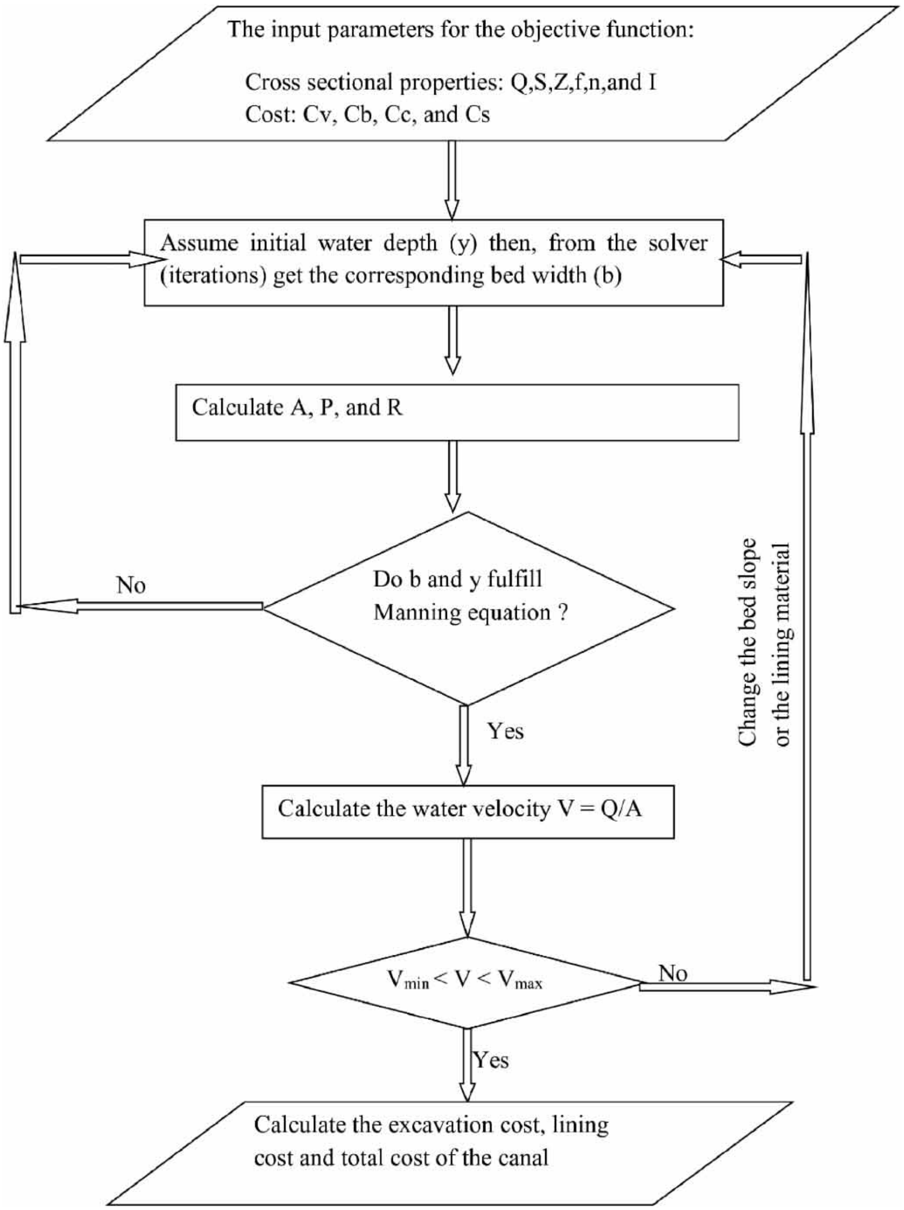


**Figure 2** | The optimization model flow chart.

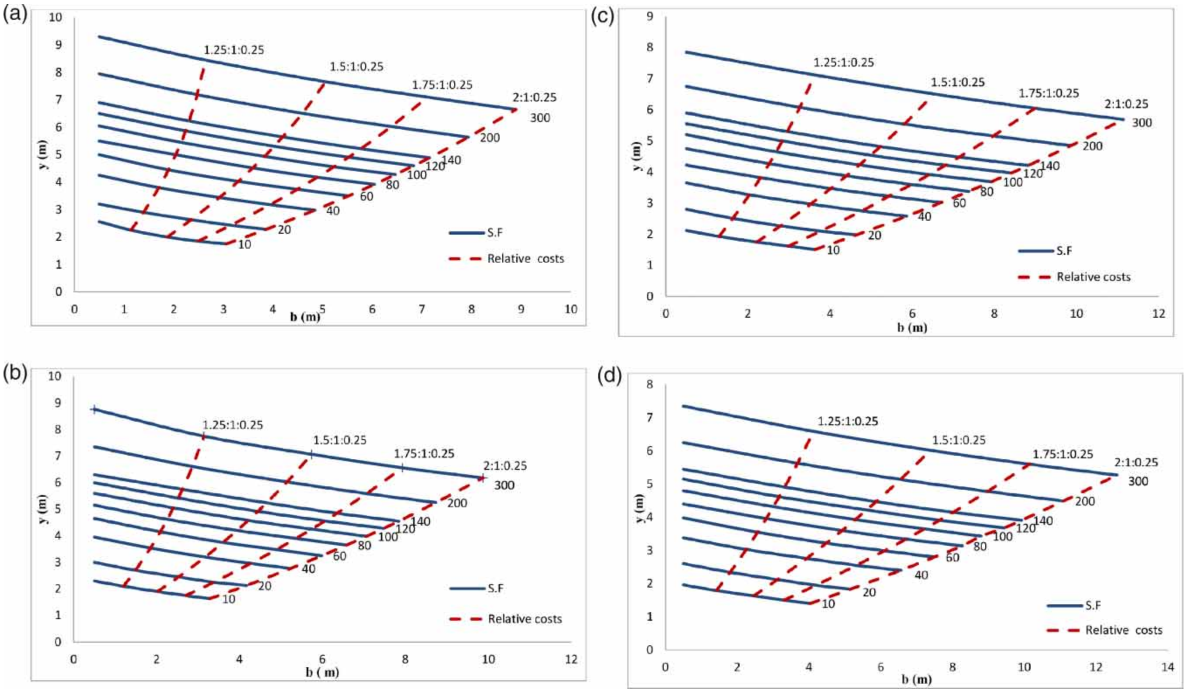


**Figure 3** | Cost minimization design solution (I = 0.7). (a) Z = 0.5. (b) Z = 1. (c) Z = 1.5. (d) Z = 2.

Also, for the same relative costs, as S.F increases, both the values of the optimum bed width and the water depth increases. But the increasing rate is high for small values of S.F and it diminishes for high S.F values.

Figure 4(a) represents the relation between the ratio of the lining cost/total cost (L/T) and the optimal bed width for a curved corner trapezoidal canal having a certain section factor (S.F), with respect to different side slopes Z. It can be noticed that, for the same Z, increasing the optimal bed width leads to an increase in the L/T. Also, L/T is least for the section with $Z = 0.5$. As for the same bed width, decreasing side slope Z leads to a decrease in the side length and consequently a decrease in the lining cost, which leads to decreasing L/T.

Figure 4(b) represents the relation between the L/T and the optimal water depth for a curved corner trapezoidal canal having a certain section factor (S.F), with respect to different side slopes Z. It can be noticed that increasing the optimal water depth leads to decrease in the L/T, this is due to the fact that a small increase in the water depth is accompanied by an increase in the side slope length by a small amount and a decrease in bed width by a tangible amount, therefore leading to a decrease in the lining cost.

On the other hand, L/T is least for the section with side slope $Z = 2$. As for the same water depth, increasing the side slope Z leads to a decrease in the bed width and consequently a decrease in the lining cost, which leads to decreasing L/T.

## SENSITIVITY ANALYSIS

Sensitivity analysis was carried out in order to investigate the effect of changing both the bed width and the water depth above or below the optimum solution values on the total cost. It is obvious that the change in the total cost is more sensitive to the change in water depth than the change in the bed width, as shown in Figure 5. Also, it was noticed that the optimal is less sensitive to the increase in water depth and more sensitive to its decrease.

## DESIGN EXAMPLE

The following example presents the utility of the previous proposed design charts in Figure 3(a)–(d) to obtain the

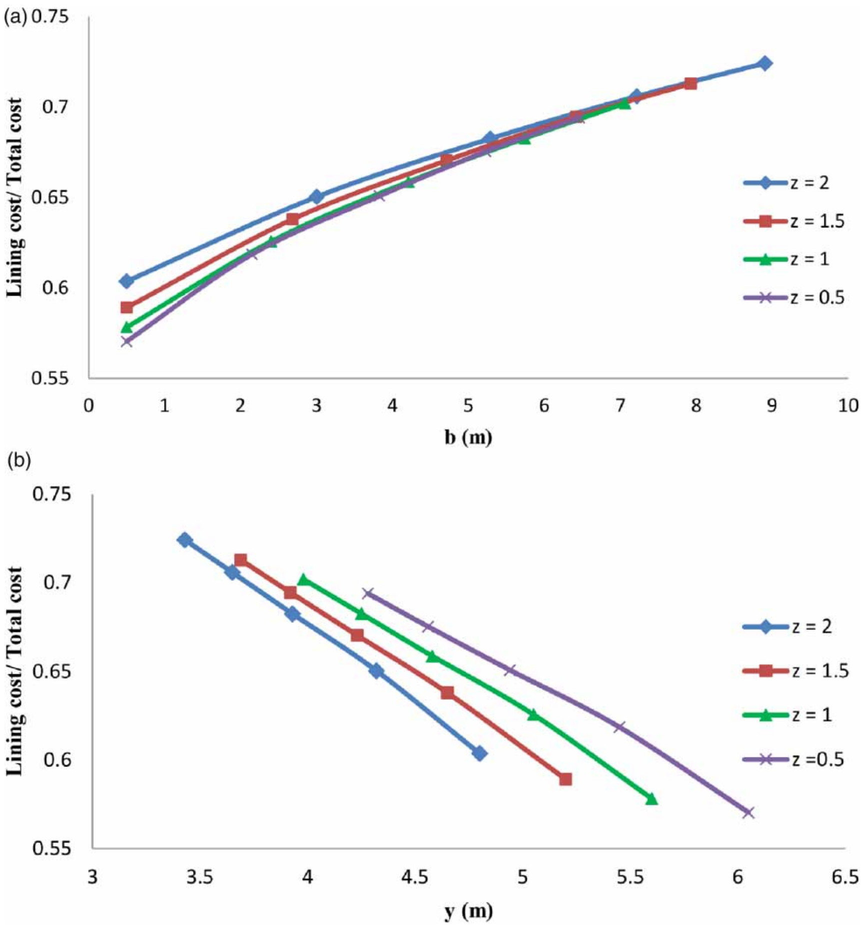


**Figure 4** | Relation between the lining cost/ total cost and (a) the optimal bed width, (b) the optimal water depth for different side slope z, S.F = 100.

cross section dimensions as described below. The required data are the discharge 'Q', the longitudinal bed slope 'S', side slope 'Z' and Manning's roughness coefficient 'n'. Also, the lining cost for the sides, the curves and the bed.

For a numerical example of designing a suitable curved corner trapezoidal channel dimensions, the following information is available as:

The flow rate $Q = 60\ m^3/s$, the longitudinal bed slope $S = 10$ cm/km, $Z = 1$ and Manning's roughness coefficient $n = 0.0167$ for concrete lining surface. The lining cost for the sides, the curves and the bed are 120,120 and 80 L.E/ meter length of the channel respectively.

So, the section factor of the channel is calculated as $S.F = \frac{Qn}{\sqrt{S}} = \frac{60*0.0167}{\sqrt{0.0001}} = 100$, the average lining cost for the curves and the sides is 120 L.E. The relative cost (the ratio between the average lining cost per meter length for the side slope and the round part to its corresponding value for the bed to the excavation cost per unit volume) = 120/80/20 = 1.5:1:0.25.

Using the design chart (3-b):

The bed width $b = 4.21$ m and the water depth $y = 4.58$ m.

Then, the flow area A is calculated from Equation (2) as $51.97\ m^2$ and the flow velocity '$V = Q/A$' = 1.15 m/sec, which is less than the permissible velocity (2.5 m/s).

Substituting b, y, and z in Equation (8), the total cost is 3610 L.E, which comprises the lining cost (2378 L.E) and the excavation cost (1232 L.E).

## COMPARISON WITH A TRAPEZOIDAL SECTION

A comparison between the parameters and the cost of a trapezoidal section (Han *et al.* 2018) with a trapezoidal section

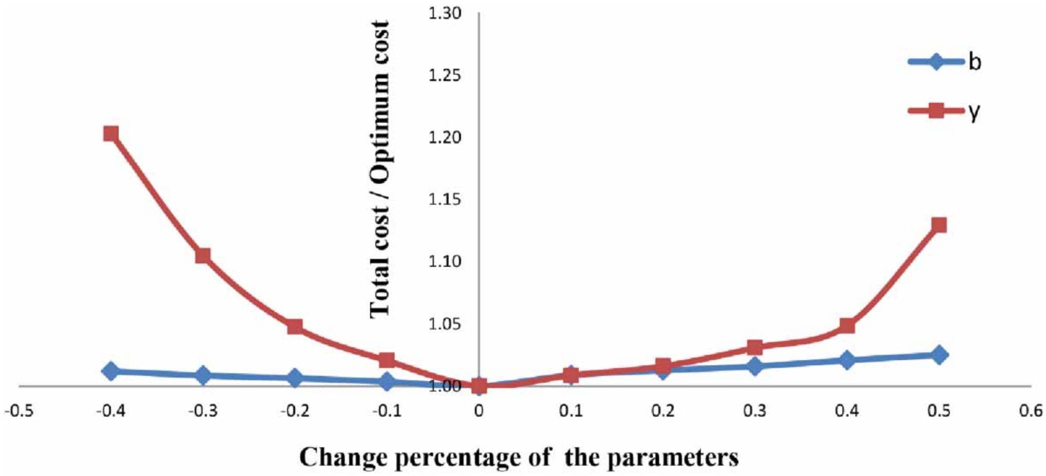


**Figure 5** | Sensitivity analysis of the optimal design.

**Table 1** | Comparison between the results of the proposed model and a trapezoidal one

| | Water depth y (m) | | Bed width b (m) | | Area A ($m^2$) | | The total cost L.E | |
|---|---|---|---|---|---|---|---|---|
| **Relative cost** | **The proposed model** | **Han *et al.* (2018)** | **The proposed model** | **Han *et al.* (2018)** | **The proposed model** | **Han *et al.* (2018)** | **The proposed model** | **Han *et al.* (2018)** |
| 1.25:1.0:0.25 | 4.27 | 4.69 | 2.52 | 2.67 | 45.66 | 45.57 | 3223 | 3225 |
| 1.50:1.0:0.25 | 3.88 | 4.69 | 4.4 | 2.69 | 45.89 | 45.57 | 3576 | 3653 |
| 1.75:1.0:0.25 | 3.6 | 4.68 | 5.98 | 2.7 | 46.36 | 45.57 | 3912 | 4081 |
| 2.00:1.0:0.25 | 3.38 | 4.68 | 7.37 | 2.71 | 46.79 | 45.57 | 4224 | 4509 |

that has curved corners is carried out. The input data for designing the channels are : $Q = 48\ m^3/s$, $S = 10\ cm/km$, $Z = 1.5$ and $n = 0.0167$. The results of the comparison for y, b, A, and the cost, for different values of the relative cost, are given in Table 1.

It could be noticed that the trapezoidal section with curved corners is more economic compared to the conventional trapezoidal one. Also, from a maintenance point of view, the trapezoidal section with curved corners is more suitable as it is shallower and has a wider bed.

## CONCLUSIONS

In the current research, the minimum total cost of the excavation and the lining for a curved corner trapezoidal canal sections considering the free board, which is the standard section shape existing in some countries, has been determined by applying Lagrange's method of undetermined multipliers. As the performance of work on the curved portion and the sides needs a special type of work and labor, the lining cost for the sides, curves, and the base is assumed to be unequal. The research results are demonstrated in a novel design chart form for different side slopes ($Z = 0.5$, 1.0, 1.5, 2), which simplifies the design of the optimal channel dimensions at a minimal cost. So, the proposed charts should be of interest to the hydraulics engineering community. A sensitivity analysis was carried out and it revealed that the optimal cost is less sensitive to the change in bed width while it is more sensitive to the change in water depth. Comparing the parameters and cost of the trapezoidal section with curved corners to the most economic trapezoidal section resulted from another study illustrated that the trapezoidal section with curved corners is more economic and preferable for easier maintenance.

It is worth noting that the analysis will be more complex if the seepage loss is integrated in the design process, which may be a subject of a future study.

## DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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