Experimental analysis of scour under circular pier

Ashish Malik, S. K. Singh and Mohit Kumar

ABSTRACT

For economical design, scour around the bridge piers is required to be controlled. In the present study, an attempt has been made to minimize scour depth by placing a triangular prism on the downstream side of a circular pier (35 mm dia) with one of its noses facing the direction of flow and other facing opposite to the direction of flow. Three different bed samples collected from Ghaggar, Patialki-Rao and the Kotla super-passage have been placed in a rectangular flume. Discharge values were varied from 0.0015 to 0.0186 m^3/sec. Results are compared for observed scour-depth for upstream (U/S) and downstream (D/S) piers with and without protection. Arrangement with a triangular prism of 2.5 times the diameter of the circular pier in the upstream direction of the flow is very effective in reducing scour depth. Further, it is possible to reduce the scour depth with an average efficiency of 65% for Ghaggar, 56% for Patialaki-Rao and 64% for the Kotla super-passage with respect to the circular pier. The comparison of observed values of scour-depth with computed values of Lacey's scour-depth was underestimated with a maximum of ±70%. Hence, a new site-specific relationship between scour depth, discharge intensity and silt factor has been proposed. Validation of the new proposed relationship with observed data is in a good agreement ±20%.

Key words | circular pier, flow altering device, open channel flow, scouring, triangular prism

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HIGHLIGHTS

- Scour at the location of bridge piers founded on erodible beds is a major problem and concern.
- Foundation depth increases with depth of scour and hence the cost of the bridge.
- Sometimes the foundations of bridge piers are eroded/scoured due to more scouring by the river than was anticipated at the design stage. In that case, scour depth can be minimized in the existing bridges with the external device as attaching a triangular prism.

INTRODUCTION

Rivers are natural flowing systems. Construction of a bridge pier across a river obstructs the flow of the river. The flowing water in the river erodes the sediment around bridge piers and all bridges and structures associated with waterways are potentially at risk of failure from hydraulic scour. Shirole & Holt (1991) stated that 60% of the 1,000-plus bridge failures occurring in the USA could be attributed to undermining of pier foundations following scouring or erosion of the channel bed. To protect bridges from scouring action, study needs to be carried out so that scour can be minimised and henceforth also the percentage of failures due to undermining.

'Man, who overlooked water under bridge will find bridge under water' (Neill 1973). This citation highlights the capability of water to erode the river bed and expose the foundation of bridge to scour. Scour is a natural phenomenon caused by the erosive action of flowing water on the bed and banks of alluvial channels. For economical design, problem

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scour around bridge piers is required to be controlled. A primary reason for concern about the stability of bridge foundations is the occurrence of scour around the pier. An experimental study has been under taken by Hamidifar & Omid (2017) to analyze the physics of scour holes for a sand clay mix bed that lies downstream of an apron. Sharafati *et al.* (2020) have used stochastic modelling to enhance the accuracy of scour prediction, whereas Pandey *et al.* (2018) experimentally investigated the temporal scour variation around bridge piers under clear water. Singh *et al.* (2020b) performed an experimental study to analyze the clear water scour under short contractions for sand beds. Illustration of the complex relation of flow and sediment with time and monitoring of the key consequences of bridge pier failure due to scour have been studied by Link *et al.* (2020).

Numerous researchers studied the effect of parameters that influence scour around a bridge pier for cohesive soils (Najafzadeh & Barani 2014; Devi & Barbhuiya 2017; Mahalder 2018). Estimation of the equilibrium scour depth is needed for economical and secure design of the infrastructural components of bridges Briaud *et al.* (1999). While underestimation of the scour depth leads to the design of too shallow a bridge foundation, on the other hand overestimation leads to uneconomic design (Ting *et al.* 2001). Khwairakpam & Mazumdar (2009) determined local scour around hydraulic structures. Singh *et al.* (2020a) reviewed the remedial techniques for local scour around bridge piers. A handy number of scour studies have been carried out with the aim of predicting equilibrium scour depth (Kothyari *et al.* 1992; Kothyari 2007; Jueyi *et al.* 2010; Masjedi *et al.* 2010; Fathi *et al.* 2011; Farooq & Ghumman 2019; Liang *et al.* 2020). Use of flow diverging devices to protect the bridge pier from scour has been studied by Zahedani *et al.* (2018). Memar *et al.* (2020) found a reduction in scour depth at couple of piers in a tandem arrangement due to the influence of collars. A number of studies has been carried out with a view to determining the equilibrium scour depth for clear water scour under steady flow condition. Pandey *et al.* (2017) evaluated the efficiency of the prevailing equations for scour around circular bridge piers. Numerous researchers have also presented equations to calculate equilibrium scour depth in the case of piers (Laursen 1958; Shen *et al.* 1969; Neill 1973; Breusers *et al.* 1977; Raudkivi & Ettema 1983; Yanmaz & Altinbilek 1991; Melville & Chiew 1999). Based on the studies of Laursen & Toch (1956), Breusers *et al.* (1977) established that the scour depths increase when discharge increases. The coarser sediment additionally produced a lesser scouring effect at the piers. Pandey *et al.* (2020) predicted the scour depth under a circular pier for a non-uniform gravel bed and validated the proposed expression.

Flow altering devices are used so that the shear stresses on the riverbed, in the vicinity of the pier, are reduced by altering the flow pattern around a pier, which in turn reduces the scour depth at the pier. The different flow altering devices, namely scour reduction using slots, scour reduction using collars, scour reduction using sacrificial piles, Delta-wing passive devices, and stone gabions with geo-textile filters have been used in the past to alter the flow and reduce scour (Beg & Beg 2013). The majority of the studies are devoted to determining the maximum depth of the scour around the bridge elements. A slot may be choked by floating debris and its construction is difficult as well. Sacrificial piles might become ineffective when the flow advancing on the piers changes its direction. Stone gabions and geotextile filters are likely to be clogged with the passage of time.

Scour at the location of bridge piers founded on erodible beds is a major problem and concern. Foundation depth increases with depth of scour and hence the cost of bridge. Sometimes, the foundation of a bridge pier gets eroded/scoured due to more scouring by river than was anticipated at the design stage. In that case, scour depth can be minimized in existing bridges with an external device, such as by attaching a triangular prism. A very few studies on rectangular slots are available in the literature (Kumar *et al.* 1999) and research on reducing scour depth under circular piers with a triangular prism arrangement is very scanty. The triangular prism may be an effective arrangement to protect a circular pier from the scouring action of a river.

In the current study, the problem of scour around circular bridge piers has been addressed in order to meet the research gap discussed above. A triangular prism model has been used to understand the variation in scour depth. Laboratory experiments have been performed on the different prism arrangements at various values of discharge. The experimental results of temporal change in scour depths of a circular pier without and with a triangular prism (facing towards U/S and D/S) direction of flow have been compared. The effectiveness of triangular prism models with respect to simple circular piers has also been found. The experimental results of data measured for circular piers was compared with the equation proposed by Lacey for scour depth and was found to be unsatisfactory. Hence, a new relationship is proposed and then validated with observed data.

SITE DESCRIPTION

Three river sites were selected to take the bed sample for the experimental run; the sites are in the vicinity of Chandigarh, India. The first site is Ghaggar river, which is an intermittent stream in India, streaming amid the storms. It starts in the town of Dagshai in the Shivalik Hills of Himachal Pradesh and moves through Punjab and Haryana and weakens in Rajasthan, southwest of Sirsa, Haryana. This site was selected because Ghaggar river is known for its flashy floods in monsoons. Many times, the water of these floods reaches Fort Abbas in Pakistan. The sample was collected from a place with the co-ordinates 30°40′53″N-76°52′32″E.

The second site is Patiala Ki Rao, a seasonal stream that starts in Shivalik Range, Punjab, enters Chandigarh then onto Mohali and later converges into the Ghaggar River. The stream keeps running crosswise over Fateh Burj at Chappar Chiri. It is one of the seasonal rivulets amongst others in Chandigarh, which incorporates Sukhna Choe in the East and N Choe in the West. The sample was collected from a place with co-ordinates 30°47′01.6″N - 76°47′41.4″E.

The third site is a super-passage site near Kotla power house, situated between Kiratpur Sahib and Anandpur Sahib in Punjab, India, on Satluj canal with co-ordinates 31°12′41.8″N - 76°33′05.0″E, as shown in Figure 1.

MATERIALS

Three soil samples were used in the present investigation from three different sites, namely Ghaggar, Patiala ki-Rao

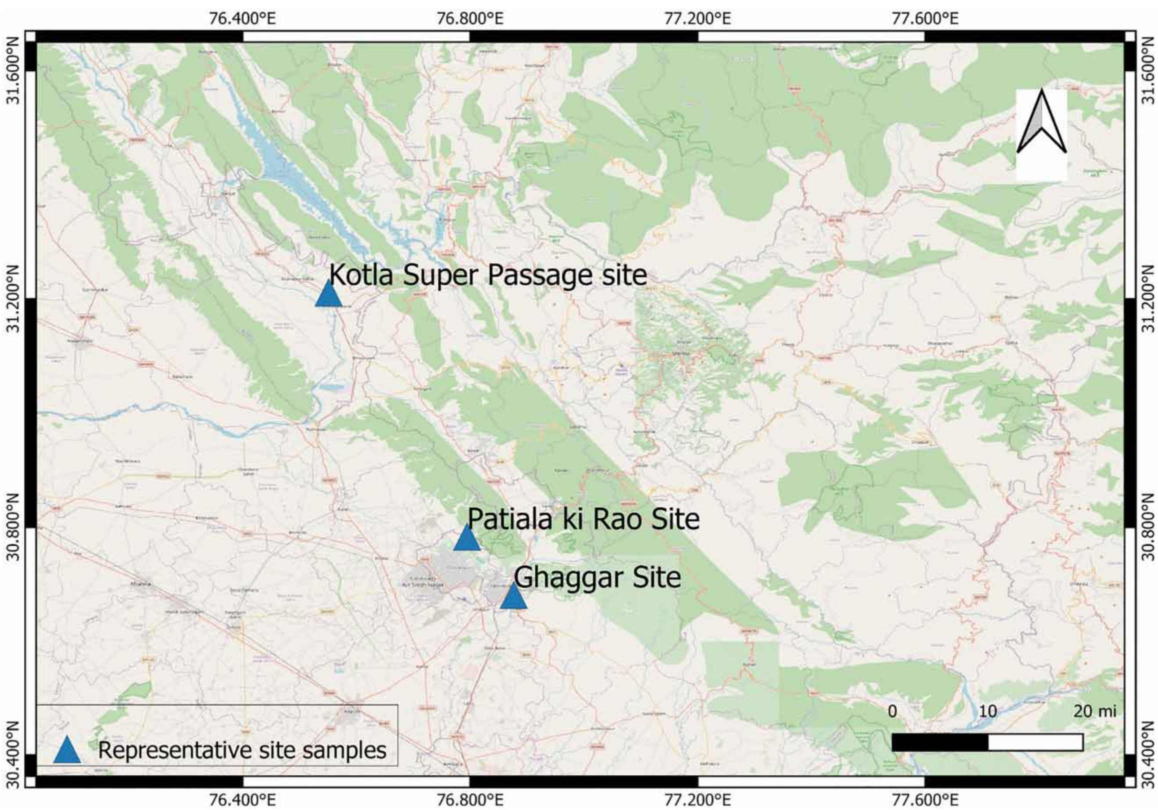

Figure 1 | Representative bed samples of flume from various sites.

and Kotla super-passage. To find the type of soil, grain size analysis was carried out in the laboratory. Figure 1 represents the sites from where the samples were obtained and Figure 2 represents the grain size analysis of the three soil samples. The values of d_{10}, d_{30} and d_{60} were obtained from Figure 2 and then the coefficient of uniformity (C_u) and coefficient of curvature (C_c) were found, as shown in Table 1. Hence, Ghaggar soil was found to be uniform soil (poorly graded) and Kotla super-passage and Patiala-ki-Rao samples were found to be non-uniform soil (well graded).

EXPERIMENTAL SETUP

The experimental setup consists of a rectangular hydraulic flume, gauges for the measurement of depth at various sections, a discharge measuring tank, different pier arrangements, a 36 HP motor and sump. The diameter (D) of the pier was taken one-tenth of width of flume (i.e. 35 mm) so that the effect of streamlines on pier could be neglected.

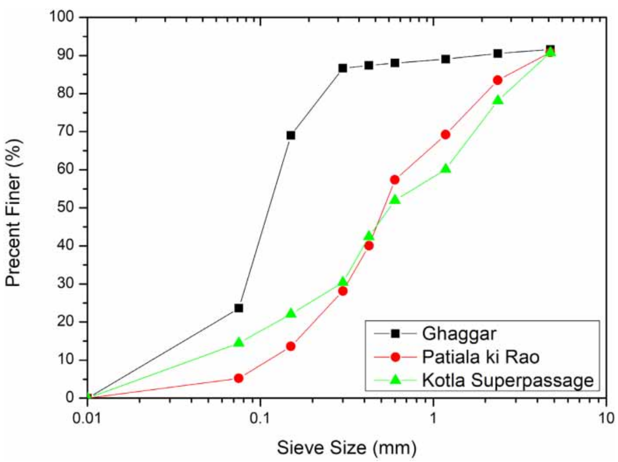

Figure 2 | Grain size distribution curves of soil sample.

Table 1 | Soil classification

Soil sample	d_{10} (mm)	d_{30} (mm)	d_{60} (mm)	C_u	C_c
Ghaggar	0.025	0.09	0.104	4.16	3.11
Patiala ki Rao	0.12	0.31	0.7	5.83	1.14
Kotla	0.04	0.3	1.12	28	2

d_{10}, d_{30} and d_{60} = grain size of bed material corresponding to finer than 10, 30 and 60% respectively.
C_u = coefficient of uniformity and C_c = coefficient of curvature.

The pier material was a hollow steel pipe filled with concrete. The prism selected has its nose at 2.5D from the outer face of pier with a nose angle of 19°12′. The triangular prism was welded with the piers, and both pier and prism had adequate strength to sustain wear and tear caused due to flow. The schematic view of the flume and discharge measuring tank in the Water Resources laboratory of Punjab Engineering College (Deemed to be University), Sector-12, Chandigarh, is shown in Figure 3.

METHODOLOGY

The soil properties were studied, and respective silt factors were calculated for the three soil samples collected from the three-river site in the vicinity of Chandigarh (India). The soil samples were then placed one by one on the hydraulic flume with adequate compaction. Simultaneously, the pier arrangements were set up. Three pier arrangements were selected and placed in the hydraulic flume at 1.22 m, 3.05 m and 4.87 m from the inlet, respectively. The first pier, which was 1.22 m away from the inlet, was a simple circular pier. The second pier, which was 3.05 m from the inlet, was a circular pier with a triangular prism facing upstream attached to it; and, thirdly, the last pier, which was 4.87 m from the inlet, was also a circular pier attached to a triangular prism with the prism facing downstream. Discharge was measured in the discharge measuring tank with the help of a ball gate valve. The readings of scour depths were measured during the experimental run at 15-minute intervals using an inverted pointer gauge. Each soil sample was laid 4–5 times in the flume and readings of scour depth were obtained for different discharge values using an inverted pointer gauge that could measure precisely up to 0.5 mm. Figure 4(a) shows the arrangements of pier and triangular prism, Figure 4(b) shows the circular pier before the experimental run and Figure 4(c) shows the final scour in a circular pier after the experiment.

RESULTS AND DISCUSSION

The readings of scour depth observed after two hours in every experimental run have been tabulated in Table 2,

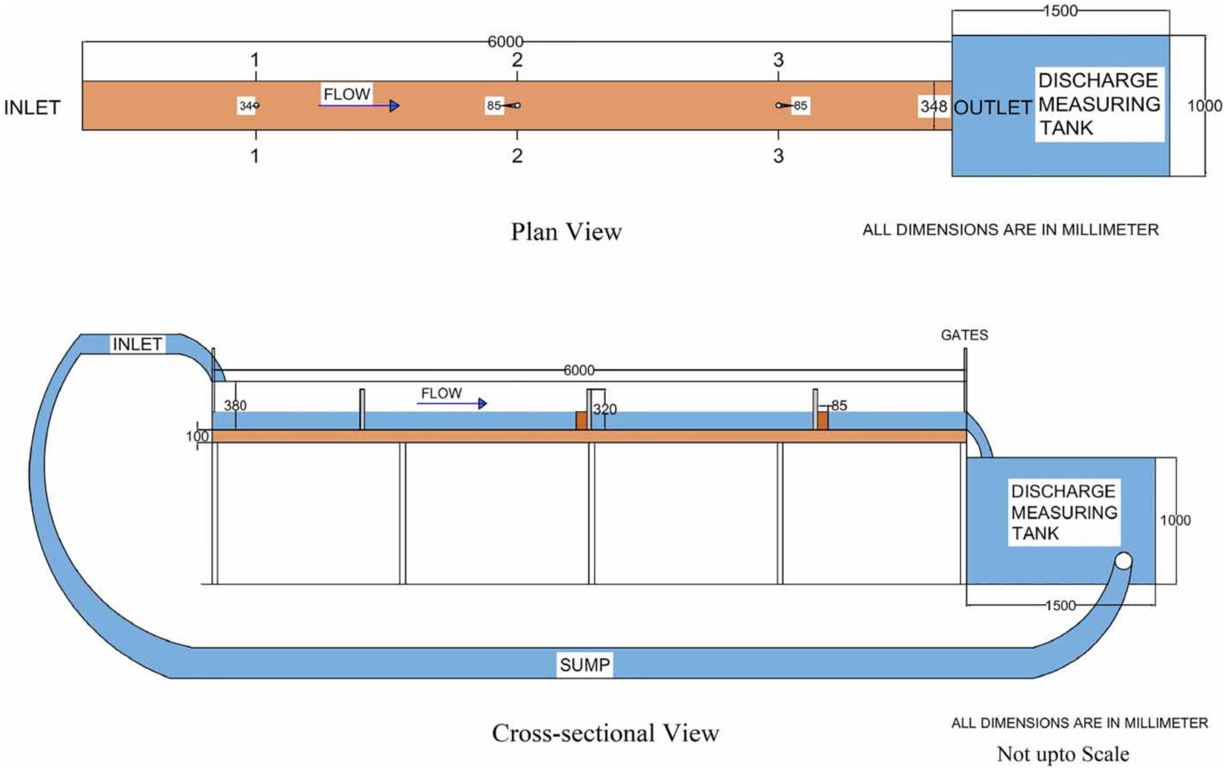

Figure 3 | Schematic view of experimental setup.

Figure 4 | (a) Triangular nose on a circular pier, (b) Simple circular pier, (c) Scour action of water on circular pier.

were compared with Lacey's scour depth and plotted against each other, as shown in Figure 5(a). The observed readings of scour depth under the circular pier were much less as compared to Lacey's scour depth, as shown in Figure 5. It was observed that Lacey's values were overestimated and a calibration is required. After calibration, a new

Table 2 | Comparison b/w Lacey's scour depth and observed data

Material	Q(cumec)	b(m)	q(cumec/m)	Silt factor f	Lacey's scour depth (m)	Observed circular pier (m)
Ghaggar	0.00167	0.34	0.00491	0.622	0.0457	0.031
	0.00371	0.34	0.01091	0.622	0.0778	0.032
	0.00934	0.34	0.27471	0.622	0.1439	0.046
	0.01338	0.34	0.03935	0.622	0.1830	0.053
Patialaki Rao	0.00152	0.34	0.00447	1.244	0.0341	0.034
	0.00375	0.34	0.01103	1.244	0.0622	0.045
	0.00866	0.34	0.02547	1.244	0.1087	0.055
	0.01585	0.34	0.04662	1.244	0.1626	0.062
	0.01866	0.34	0.05488	1.244	0.1813	0.065
Kotla super-passage	0.00154	0.34	0.00453	1.305	0.0338	0.03
	0.00391	0.34	0.0115	1.305	0.0629	0.036
	0.00685	0.34	0.02015	1.305	0.0915	0.045
	0.0114	0.34	0.03353	1.305	0.1285	0.05
	0.01606	0.34	0.04724	1.305	0.1614	0.056

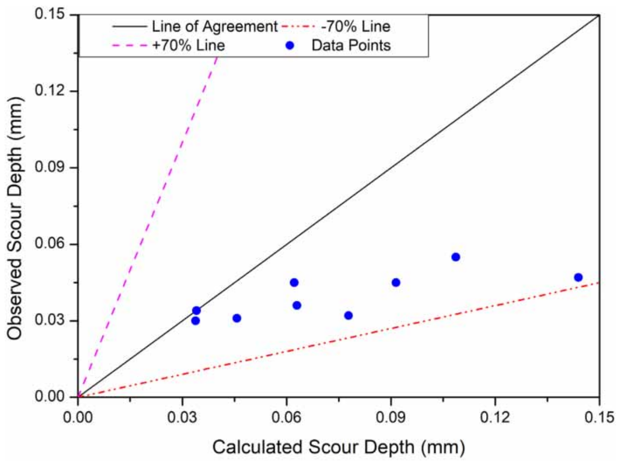

Figure 5 | Plot between observed scour depth v/s calculated scour depth.

relationship was proposed between scour depth and (q^2/f) value, as shown in Figure 5(b).

Lacey's scour depth formula is given by Equation (1)

$$\text{Scour depth} = 1.35\left(\frac{q^2}{f}\right)^{1/3} \tag{1}$$

q – Discharge per unit width
f – Silt factor

The results showed up to 70% variation between Lacey's scour depth and observed data, as shown in Figure 5. Therefore, a new relationship that was proposed is given by Equation (2) with a moderate R^2 value of 0.58, as depicted in Figure 6.

$$\text{New scour depth proposed} = 0.1036\left(\frac{q^2}{f}\right)^{0.1091} \tag{2}$$

Validation of model

A relationship between scour depth and (q^2/f) was proposed, as given in Equation (2). The relationship was obtained with three values of scour after two hours from each soil sample. This relationship was required to be validated. The values left were used in validating the relationship, the model was validated by putting the values

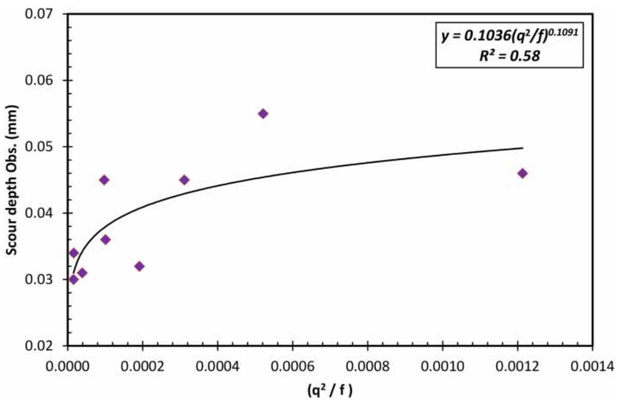

Figure 6 | Proposed relationship b/w scour depth and (q^2/f).

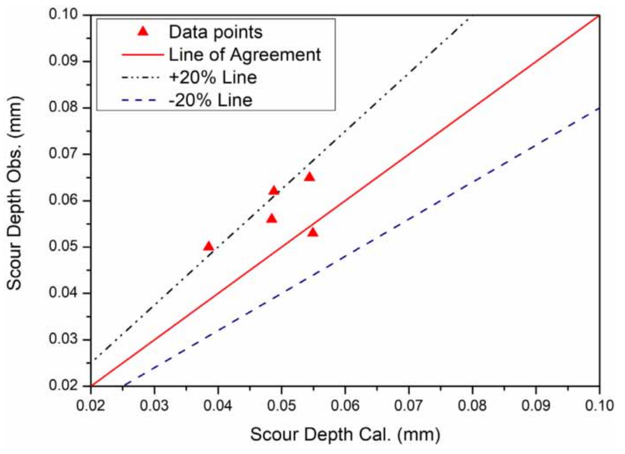

Figure 7 | Validation of proposed relationship.

of (q^2/f) in the proposed relation and comparing those values with observed values. The comparison brought satisfactory results with minor variation of ±20% from observed values, as shown in Figure 7.

Performance evaluation of models

After performing the experiments, it was evident that the scour was minimum in the case of a triangular prism facing in the upstream direction compared with the scour value of a simple circular pier without prism. The value of scour depth under a constant discharge was found to be maximum in the case of a simple circular pier followed by the prism facing the (D/S) direction and the prism facing the (U/S) direction of flow, as shown in Table 3. Efficiency of the model was found using Equation (3)

$$\text{Efficiency of prism (protected) model w.r.t. simple circular (unprotected) model} = \left(\frac{Unprotected - Protected}{Unprotected}\right) \times 100 \quad (3)$$

It was observed from Table 3 that scour depths obtained at a certain discharge are in the following sequence in all the experimental runs: simple circular pier > circular pier with prism at D/S end > circular pier with prism at U/S end.

CONCLUSIONS

This study is important in finding the scour depth around circular piers in the vicinity of Chandigarh city; scour depth can be minimized for the existing bridges with an external device by attaching a triangular prism. The study also helped to analyse the effect of discharge and different arrangements of the circular pier with triangular prism on the maximum scour depth. Experiments were performed in a rectangular hydraulic flume with three different soil samples from North India. For each soil sample, experiments were run for varying discharge values for three

Table 3 | Efficiencies of prism models with respect to a circular pier

Material	Q(cumec)	Circular (mm)	Prism facing U/S direction	Efficiency U/S prism(%)	Prism facing D/S direction	Efficiency D/S prism (%)
Ghaggar	0.00167	31	10	67.74%	21	32.26%
	0.00371	32	10	68.75%	23	28.13%
	0.00934	46	18	60.87%	37	19.57%
	0.01338	53	20	62.26%	40	24.53%
Patialaki-Rao	0.00152	34	16	52.94%	27	20.59%
	0.00375	45	17	62.22%	35	22.22%
	0.00866	55	24	56.36%	40	27.27%
	0.01585	62	28	54.84%	41	33.87%
	0.01866	65	30	53.85%	48	26.15%
Kotla super-passage	0.00154	37	14	62.16%	26	29.73%
	0.00391	36	14	61.11%	26	27.78%
	0.00685	45	16	64.44%	33	26.67%
	0.0114	50	17	66.00%	37	26.00%
	0.01606	56	18	67.86%	41	26.79%

different arrangements. The existing relationship for scour depth proposed by Lacey was not found to be satisfactory for the present study. Hence, a new relationship for these was derived and validated. The efficiency of the triangular prism in U/S direction of flow was found to be maximum and the amount of scour reduction is of the order of 61–68% for the Ghaggar soil sample, 54–62% for the Patialaki-Rao soil sample and 53–68% for the Kotla super-passage soil sample for various discharge values. The efficiency of the triangular prism in the D/S direction of flow is lower than that of the U/S case. The amount of scour reduction is of the order of 20–32% for the Ghaggar soil sample, 18–33% for the Patialaki-Rao soil sample and 13–27% for the Kotla super-passage soil sample for varying discharge values. The experimental data did not comply with the calculated by Lacey's formula. The comparison of the observed value of scour depth with computed values clearly shows that all the data points are underestimated by ±70%. The comparison brought unsatisfactory results and hence a site specific relationship was proposed and validated for the present study for the region around Chandigarh city from where the samples from the river bed were collected.

CONFLICT OF INTEREST

None.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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