

Improved fuzzy chance-constrained optimization of booster strategy for water distribution system under uncertainty

Yumin Wang and Guangcan Zhu

ABSTRACT

To sustain water quality in a water distribution system (WDS), disinfectant generally chlorine is boosted to the WDS. However, the concentration of chlorine should be limited to acceptable levels. The upper boundary of the range is set for preventing the occurrence of a disinfectant by-product, which is harmful to human health. The lower boundary of the range is set for controlling the growth of microorganisms as well as reducing the injection mass. As such, an optimization model was applied to solve the water quality issue in a WDS. However, in a WDS, chlorine decays and varies with time and space, affected by pipe material, temperature, pH value, and chlorine injection, etc. Therefore, in this paper, an improved fuzzy chance-constrained optimization model was proposed to optimize chlorine injection and location to maintain chlorine in a WDS distributed uniformly. The proposed model was applied to two WDSs to analyze the effect of reliability level and preference parameters on chlorine injection and location. The results indicated that the injection mass increased with the increase in preference parameter. The results also indicated that more booster stations can lower the injection mass, and two booster stations were suitable for WDS in Case 2. The method proposed can be applied to decide how and where to inject chlorine in a WDS under uncertainty, and can help managers determine whether an optimistic or pessimistic attitude should be taken under various reliability levels.

Key words | booster chlorination, improved fuzzy chance-constrained optimization, water distribution system

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HIGHLIGHTS

- Fuzzy chance-constrained programming model proposed for chlorine injection determination.
- m_α -measure method incorporated into fuzzy chance-constrained programming model with consideration of optimism and pessimism.
- Effects of reliability level and preference parameter on injection mass are analyzed.
- Help determine the location of booster stations.
- Help managers determine what attitude should be taken.

INTRODUCTION

The water distribution system (WDS) is one of the most important urban facilities, and should be designed to satisfy

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the customers' demand for water pressure, flow, and water quality. To meet with the water quality standard, chlorine, as a disinfectant, is applied widely to maintain water quality at acceptable levels for the WDS. The chlorine residue in WDS should be kept between maximum and minimum limits. The aim of setting a maximum limit is for preventing

the formation of disinfectant by-product, and unpleasant taste and odor (Köker & Altan-Sakarya 2015). Similarly, the minimum limit is for controlling the growth of microorganism. As such, the lower chlorine concentration is preferred while satisfying the minimum limit. To supply chlorine at the far ends of a WDS, high concentrations of chlorine should be injected, which leads to the chlorine concentration at the nodes near the source become excessive. To maintain water quality and reduce the injection cost, various operations of booster disinfection have been investigated by numerous researchers through optimization methods, including single-objective and multi-objectives optimizing models (Vasan & Simonovic 2010; Xu & Qin 2014; Marques *et al.* 2015; Xin *et al.* 2019). The results indicated that the total disinfectant mass can be reduced by booster chlorination and the boosters' locations can affect the disinfection efficiency significantly (Boccelli *et al.* 1998). The overall cost of booster placement, construction, and operation was minimized by setting the required chlorination dose of boosters for delivering water at acceptable residual chlorine and trihalomethanes concentrations (Ohar & Ostfeld 2014).

Since chlorine concentration in the WDS is affected by various uncertain problems such as nodal demand, pipe roughness coefficient, pipe diameter, chlorine decay coefficient, etc., it is difficult to simulate accurately the chlorine decay process in a WDS through water quality models. The conventional optimization models under uncertainty become complex and difficult to solve. As such, the chance-constrained programming (CCP) model was introduced to minimize the cost of WDS with consideration of uncertainty in nodal demands, pipe roughness coefficients, etc. (Babayan *et al.* 2005). The CCP model requires all the constraints be satisfied in a proportion of cases under a given reliability level, which is effective in reflecting the probability distribution in the right-hand sides of the constraints when the randomness occurs only at the right-hand side vector (Zhao *et al.* 2016). However, it cannot deal with the ambiguity in the constraints. Fuzzy programming can deal with real-world problems with vague information expressed as fuzzy sets, which is effective in reflecting ambiguity and vagueness in the constraints (Guo & Huang 2008). Therefore, a fuzzy chance-constrained programming (FCCP) model is

suitable for solving the scheduling of booster disinfection under the condition of fuzzy chlorine concentrations, which has been proved to be an effective approach to tackle ambiguity (Li *et al.* 2015).

In this paper, firstly, an m_λ -measure fuzzy chance-constrained optimization model is proposed to optimize the booster disinfection based on uncertainty. The objection function is to minimize the disinfection while maintaining the chlorine concentration with the specified limits simultaneously. Since chlorine concentration is uncertain due to the effects of initial chlorine injection, temperature, pH value, etc., the m_λ -measure approach is applied to set the constraint of chlorine concentration for both maximum and minimum limits in the fuzzy chance constraint optimization model. Secondly, the improved fuzzy chance-constrained method was applied to two cases. Thirdly, the effect of reliability level and preference parameters on the injection mass was analyzed and a conclusion was drawn.

FUZZY CHANCE CONSTRAINED PROGRAMMING MODEL

Booster optimization model

The optimization model for a typical daily operation of booster is formulated by Equation (1) as follows:

$$\text{Minf} = \sum_{j=1}^{n_t} \sum_{i=1}^{n_b+n_s} x_i^j \quad (1a)$$

$$C_L \leq C_{ir,tr} = \sum_{j=1}^{n_t} \sum_{i=1}^{n_b+n_s} \beta_{ir,tr}^{i,j} x_i^j \leq C_U \quad (1b)$$

$$x_i^j \geq 0 \quad (1c)$$

where n_t, n_b , and n_s are dosage schedule time interval, number of possible booster locations, and source locations, respectively, x_i^j is injection mass at booster or source location (M), C_U and C_L are the acceptable maximum and minimum of chlorine concentration (M/L³), $\beta_{ir,tr}^{i,j}$ is the response coefficient of chlorine concentration at consumer node ir at monitoring time tr to the injection rate at booster

or source location i at time j , defined as $\beta_{ir,tr}^{ij} = \partial C_{ir,tr} / \partial x_i^j$, $[(M/L^3)/(M)]$.

In WDS, hydraulic constraints are found by a hydraulic solver EPANET. For each junction, the mass conservation regulation should be satisfied, which is shown in Equation (1d) as follows:

$$\sum Q_{in} - \sum Q_{out} = q \quad (1d)$$

where Q_{in} and Q_{out} are the input and output flow of the node, q is the external inflow or demand at the node.

For each loop in WDS, the conservation of energy constraint is expressed by Equation (1e) as follows:

$$\sum_L \Delta H = 0, \quad \forall L \in NL \quad (1e)$$

where ΔH is the head loss of pipes in the loop L , and NL is the number of loops in WDS. The head loss for each pipe is calculated by Hazen-Williams equation.

In the booster optimization model, the objection function is to minimize the total chlorine mass injected to the WDS, which is obtained by summarizing the decision variable of injection rate x_i^j from location i and at time j for all boosters and source locations of $n_b + n_s$ and all time periods of n_t . The constraints include the chlorine concentration limits and non-negative limit for injections. The effect of individual injection on the response nodes can be expressed as linear function of the injections according to superposition principle (Lansley *et al.* 2007). As such, the response matrix of chlorine concentration at node ir and at monitoring time tr to the chlorine injection mass of x_i^j at location i and at time j can be expressed as $\beta_{ir,tr}^{ij} x_i^j$. The chlorine concentration at all consumer nodes at all monitoring times can be obtained by summarizing the response to each booster location and injection period, which is expressed by Equation (1b). As such, the optimization model can be treated as a linear programming model, which is expressed by Equation (2) as follows:

$$\text{Min}f = \sum_{j=1}^{n_t} X^T \quad (2a)$$

Subject to:

$$C_L \leq C = BX^T \leq C_U \quad (2b)$$

$$X^T \geq 0 \quad (2c)$$

where X^T is the transpose of the booster injection mass matrix, and B is all-in-one response coefficient matrix, which is obtained by adding an amount of chlorine at only one booster node to the WDS, and record the response of each consumer node for each monitoring time interval, which is expressed by Equation (3) as follows:

$$\beta_{ir,tr}^{ij} = \frac{C_{ir,tr}}{x_i^j} \quad (3)$$

By repeating the process for each booster station, the response coefficient matrix B is formed.

Fuzzy chance-constrained programming model

As for parameters with multiple uncertainties, the distribution function cannot be expressed by fuzzy or stochastic distribution, but combinations of fuzzy and probability distributions, which is incapable to be dealt with by the aforementioned CCP method. As such, fuzzy chance-constrained programming (FCCP) was proposed to deal with the constraints containing fuzzy and probability distributions at the same time.

A general FCCP is expressed by Equations (4a)–(4d) as follows (Li *et al.* 2015):

$$\text{Min}f = \sum_{j=1}^{n_t} X^T \quad (4a)$$

Subject to

$$\text{Cr}(BX^T \leq \tilde{C}_U) \geq \zeta_U \quad (4b)$$

$$\text{Cr}(BX^T \geq \tilde{C}_L) \geq \zeta_L \quad (4c)$$

$$X^T \geq 0 \quad (4d)$$

where B is the All-In-One coefficients matrix in the left-hand side of constraints, \tilde{C}_U and \tilde{C}_L are the fuzzy sets for

acceptable upper limits (ULs) and lower limits (LLs) for chlorine concentration in the right-hand side of constraints, Equation (4a) is the objective function in the optimal framework, and Equations (4b) and (4c) are predetermined confidence levels ζ_U and ζ_L for constraints ($BX^T \leq \tilde{C}_U$) and ($BX^T \geq \tilde{C}_L$), respectively.

In fuzzy set theory, the chance of a fuzzy event was commonly reflected by possibility and necessity measures, which are the fundamental concepts of fuzzy mathematical programming (Zhang *et al.* 2018). For a fuzzy variable \tilde{b} with triangular distribution, with the lower bound b_1 , the most likely value b_2 , and the upper bound b_3 , the membership function $\mu(x)$ of fuzzy variable \tilde{x} can be expressed by Equation (5) as follows:

$$\mu(x) = \begin{cases} \frac{x - b_1}{b_2 - b_1}, & b_1 \leq x < b_2 \\ \frac{x - b_3}{b_2 - b_3}, & b_2 \leq x < b_3 \\ 0, & \text{others} \end{cases} \quad (5)$$

Suppose a be an arbitrary subset of \mathfrak{X} , then the possibility measure of a fuzzy event, characterized by $a \leq \tilde{b}$ is defined by Equation (6) as follows:

$$\text{Pos}(a \leq \tilde{b}) = \sup \{\mu(x) | x \in \mathfrak{X}, a \leq x = \sup_{a \leq x} \mu(x)\} \quad (6a)$$

Based on Equations (5) and (6a), the possibility of fuzzy event $a \leq \tilde{b}$ can be calculated by Equation (6b) as follows:

$$\text{Pos}(a \leq \tilde{b}) = \begin{cases} 1, & a \leq b_2 \\ \frac{a - b_3}{b_2 - b_3}, & b_2 \leq a < b_3 \\ 0, & a > b_3 \end{cases} \quad (6b)$$

Similarly, the necessary of fuzzy event $a \leq \tilde{b}$ represents the impossibility of the opposite event, i.e., proposition 'a is less than or equal to \tilde{b} ' is true, and can be defined by Equation (7a) as follows:

$$\begin{aligned} \text{Nec}(a \leq \tilde{b}) &= \inf \{1 - \mu(x) | x \in \mathfrak{X}, a \leq x\} \\ &= 1 - \text{Pos}(a > \tilde{b}) \end{aligned} \quad (7a)$$

Similarly, the necessity can be calculated by Equation (7b) as follows:

$$\text{Nec}(a \leq \tilde{b}) = \begin{cases} 1, & a \leq b_1 \\ \frac{a - b_2}{b_1 - b_2}, & b_1 \leq a \leq b_2 \\ 0, & a > b_2 \end{cases} \quad (7b)$$

An integrated credibility measure was proposed as average of the possibility and necessity measures, which is expressed by Equation (8a) as follows (Li *et al.* 2015):

$$\text{Cr}(a \leq \tilde{b}) = \frac{1}{2} (\text{Pos}\{a \leq \tilde{b}\} + \text{Nec}\{a \leq \tilde{b}\}) \quad (8a)$$

Based on the credibility definition and the rule of fuzzy operations, we have (Li *et al.* 2013)

$$\text{Cr}(a \leq \tilde{b}) = \begin{cases} 1, & a \leq b_1 \\ \frac{2b_2 - b_1 - a}{2(b_2 - b_1)}, & b_1 \leq a \leq b_2 \\ \frac{b_3 - a}{2(b_3 - b_2)}, & b_2 \leq a \leq b_3 \\ 0, & a \geq b_3 \end{cases} \quad (8b)$$

The fuzzy set for fuzzy variable \tilde{b} and possibility, necessity, and credibility measure for fuzzy event $a \leq \tilde{b}$ are shown in Figure 1.

Similar to the definition of the possibility, necessary and credibility measure of fuzzy event $a \leq \tilde{b}$, the possibility, necessary, and credibility measure of fuzzy event $a \geq \tilde{b}$ can be defined by Equations (9a)–(9c) as follows:

$$\text{Pos}(a \geq \tilde{b}) = \begin{cases} 0, & a \leq b_2 \\ \frac{a - b_2}{b_3 - b_2}, & b_2 \leq a < b_3 \\ 1, & a \geq b_3 \end{cases} \quad (9a)$$

$$\text{Nec}(a \geq \tilde{b}) = \begin{cases} 0, & a \leq b_1 \\ \frac{a - b_1}{b_2 - b_1}, & b_1 \leq a \leq b_2 \\ 1, & a > b_2 \end{cases} \quad (9b)$$

$$\text{Cr}(a \geq \tilde{b}) = \begin{cases} 0, & a \leq b_1 \\ \frac{a - b_1}{2(b_2 - b_1)}, & b_1 \leq a \leq b_2 \\ \frac{a + b_3 - 2b_2}{2(b_3 - b_2)}, & b_2 \leq a \leq b_3 \\ 1, & a \geq b_3 \end{cases} \quad (9c)$$

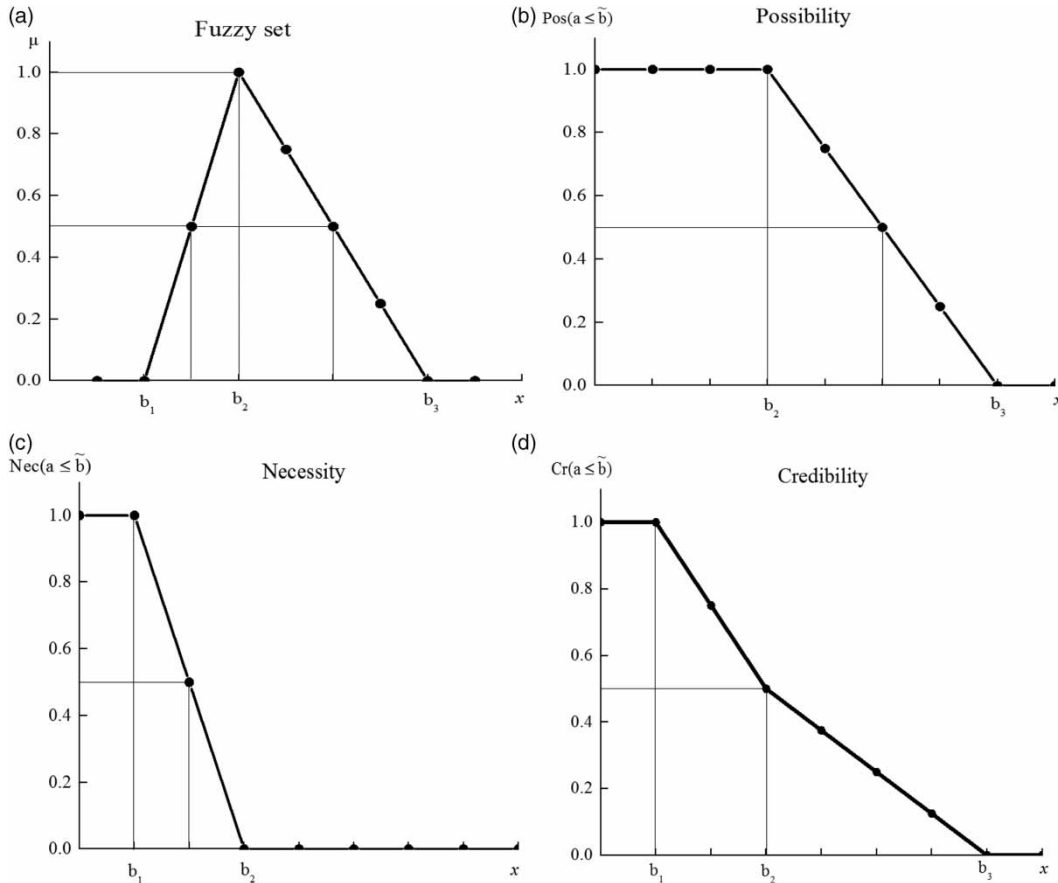


Figure 1 | The fuzzy set, possibility, necessity, and credibility measure for fuzzy event $a \leq \tilde{b}$.

The fuzzy set for fuzzy variable \tilde{b} and possibility, necessity, and credibility measure for fuzzy event $a \geq \tilde{b}$ are shown in Figure 2.

For fuzzy ULs and LLs with triangular distribution, the lower bound, the most likely value, and the upper bound are defined as C_{U1} , C_{U2} , and C_{U3} for variable \tilde{C}_U , and C_{L1} , C_{L2} , and C_{L3} for \tilde{C}_L , respectively.

For the constraint expressed by Equation (4b) as $\text{Cr}(BX^T \leq \tilde{C}_U) \geq \zeta_U$, by substituting BX^T by S , Equation (4b) can be transformed into Equation (10a) as follows:

$$\text{Cr}(S \leq \tilde{C}_U) \geq \zeta_U \quad (10a)$$

Let $\mu_{\tilde{C}_U} = \text{Cr}(S \leq \tilde{C}_U)$ donate the credibility of variables S for \tilde{C}_U . Since the confident level should be greater than 0.5 to make the constraints meaningful, the Equation (10a) can

be substituted by Equation (10b) as follows:

$$1 \geq \mu_{\tilde{C}_U} \geq \zeta_U \geq 0.5 \quad (10b)$$

Then we have

$$\text{Cr}(S \leq \tilde{C}_U) = \frac{2C_{U2} - C_{U1} - S}{2(C_{U2} - C_{U1})} \geq \zeta_U \quad (11)$$

which can be transformed into a deterministic constraint expressed by Equation (12a) as follows:

$$S \leq C_{U2} + (1 - 2\zeta_U)(C_{U2} - C_{U1}) \quad (12a)$$

Similarly, for the constraint expressed by Equation (4c) as $\text{Cr}(BX^T \geq \tilde{C}_L) \geq \zeta_L$, we can obtain the other

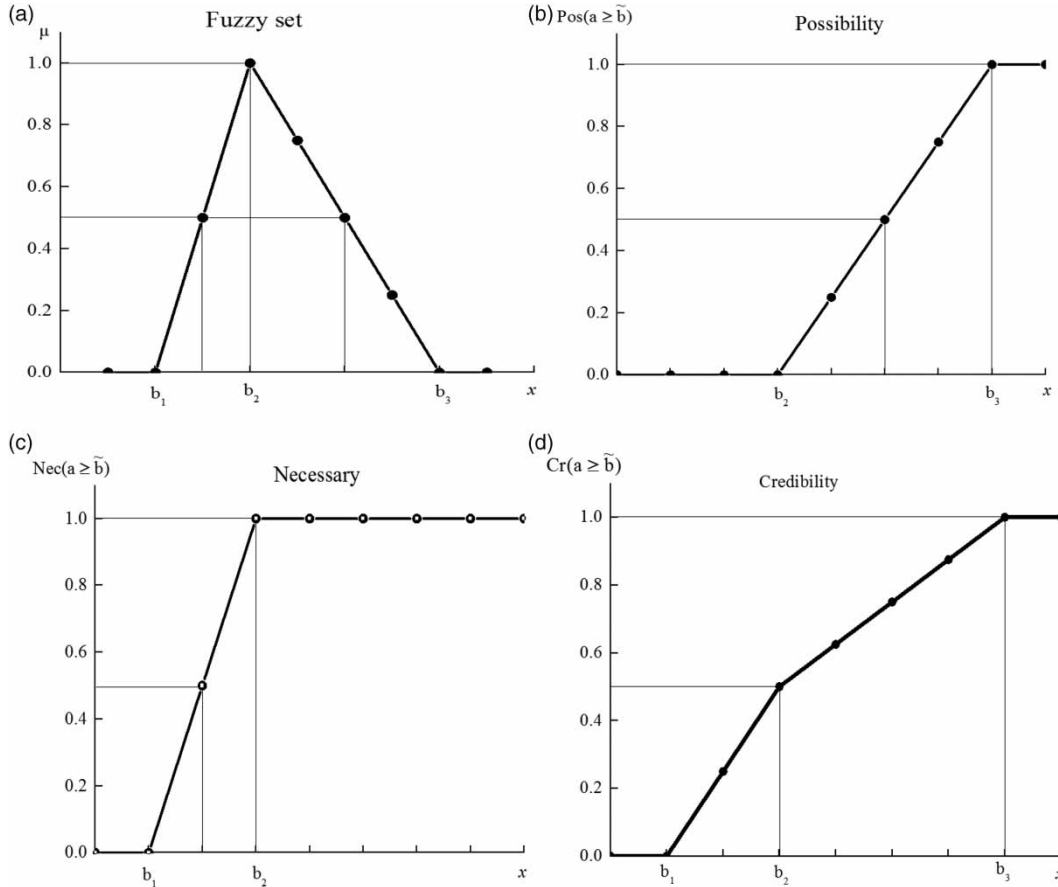


Figure 2 | The fuzzy set, possibility, necessity, and credibility measure for fuzzy event $a \geq \tilde{b}$.

deterministic constraint expressed by Equation (12b) as follows:

$$S \geq 2\zeta_L C_{L2} - 2\zeta_L C_{L1} + C_{L1} \tag{12b}$$

As such, in the FCCP, the objective function and constraints are linear, which can be solved by ‘Solver’ add-on in Microsoft Excel. By applying fuzzy chance-constrained optimization formulations, the total booster injection to the WDS can be obtained.

m_λ -measure fuzzy chance-constrained programming (MFCCP)

Generally speaking, the optimistic managers prefer to possibility measure, while pessimism ones prefer to necessary measure. To balance optimism and pessimism, a m_λ -measure is introduced, and expressed by Equations

(13a) and (13b) as follows:

$$m_\lambda(a \leq \tilde{b}) = \lambda \text{Pos}(a \leq \tilde{b}) + (1 - \lambda) \text{Nec}(a \leq \tilde{b}) \tag{13a}$$

$$m_\lambda(a \geq \tilde{b}) = \lambda \text{Pos}(a \geq \tilde{b}) + (1 - \lambda) \text{Nec}(a \geq \tilde{b}) \tag{13b}$$

where λ is parameter given by managers with consideration of tradeoff between possibility and necessity ($\lambda \in [0, 1]$). Therefore, m_λ -measure includes the above-mentioned possibility measure ($\lambda = 1$), necessity measure ($\lambda = 0$), and credibility measure ($\lambda = 0.5$), which can be expressed by Equations (14a) and (14b) as follows:

$$m_\lambda(a \leq \tilde{b}) = \begin{cases} 1, & a \leq b_1 \\ \frac{(1 - \lambda)a + \lambda b_1 - b_2}{b_1 - b_2}, & b_1 \leq a \leq b_2 \\ \frac{\lambda(a - b_3)}{b_2 - b_3}, & b_2 \leq a \leq b_3 \\ 0, & a > b_3 \end{cases} \tag{14a}$$

$$m_\lambda(a \geq \tilde{b}) = \begin{cases} 0, & a \leq b_1 \\ \frac{(1-\lambda)(a-b_1)}{b_2-b_1}, & b_1 \leq a \leq b_2 \\ 1 + \frac{\lambda(a-b_3)}{b_3-b_2}, & b_2 \leq a \leq b_3 \\ 1, & a > b_3 \end{cases} \quad (14b)$$

As such, m_λ -measure is self-dual, and reflects the fuzzy information comprehensively. The relationship m_λ -measure of fuzzy event $a \leq \tilde{b}$ and $a \geq \tilde{b}$ with the triangular distribution of \tilde{b} are shown in Figure 3.

As such, a general MFCCP is expressed by Equations (15a)–(15d) as follows:

$$\text{Min } f = \sum_{j=1}^{m_i} X^T \quad (15a)$$

Subject to

$$m_\lambda(BX^T \leq \tilde{C}_U) \geq \zeta_U \quad (15b)$$

$$m_\lambda(BX^T \geq \tilde{C}_L) \geq \zeta_L \quad (15c)$$

$$X^T \geq 0 \quad (15d)$$

Substitute BX^T by S , the Equations (15b) and (15c) can be transformed into Equations (16a) and (16b) as follows:

$$m_\lambda(S \leq \tilde{C}_U) \geq \zeta_U \quad (16a)$$

$$m_\lambda(S \geq \tilde{C}_L) \geq \zeta_L \quad (16b)$$

Let $\mu_{\tilde{C}_U} = m_\lambda(S \leq \tilde{C}_U)$ represent the m_λ -measure of variables S for \tilde{C}_U . In addition, b_1, b_2 , and b_3 in Figure 3(a) refer to C_{U1}, C_{U2} , and C_{U3} , respectively. According to Figure 3(a), when $S \leq C_{U1}$, the fuzzy event is completely invalid due to $\mu_{\tilde{C}_U} = 0$, and when $S > C_{U3}$, the fuzzy event is completely satisfied since $\mu_{\tilde{C}_U} = 1$. In case of $C_{U1} \leq S \leq C_{U3}$, $\mu_{\tilde{C}_U}$ is a monotonically decreasing function between 0 and 1, which means that only a single solution S exist in $[C_{U1}, C_{U3}]$ with a given confident level ζ_U . Since the confident level should be greater than 0.5 to make the constraints meaningful, the Equation (16a) can be substituted by Equation (17a) as follows:

$$1 \geq \mu_{\tilde{C}_U} \geq \zeta_U \geq 0.5 \quad (17a)$$

The cases can be divided into three scenarios as (1) $0 \leq \lambda \leq 0.5$ and $\lambda \leq \zeta_U$ (2) $0.5 < \lambda \leq 1$ and $\lambda \leq \zeta_U$ (3) $\lambda > \zeta_U$. The three scenarios can be summarized into two cases:

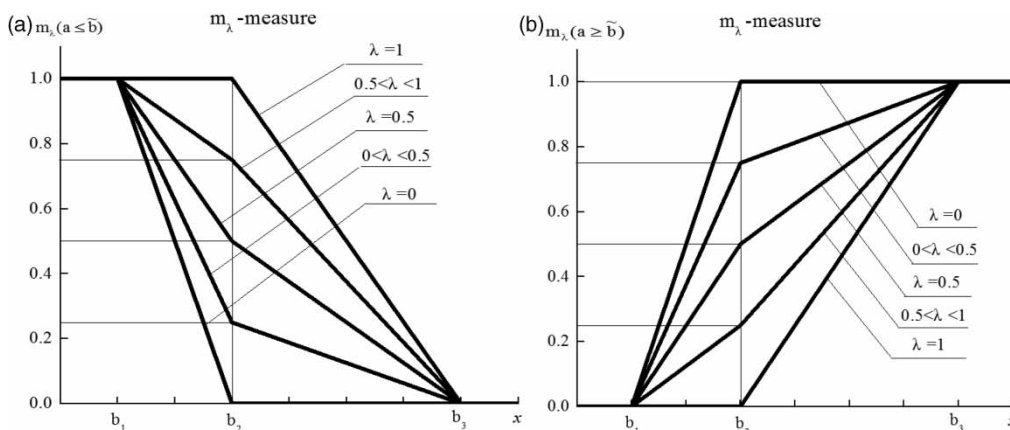


Figure 3 | m_λ -measure of fuzzy event (a) $a \leq \tilde{b}$ and (b) $a \geq \tilde{b}$.

(1) $\lambda \leq \xi_U$, then we can get Equation (17b) expressed as follows:

$$\frac{(1-\lambda)S + \lambda C_{U1} - C_{U2}}{C_{U1} - C_{U2}} \geq \xi_U \Rightarrow S \leq \frac{(1-\xi_U)C_{U2} + (\xi_U - \lambda)C_{U1}}{1-\lambda} \quad (17b)$$

(2) $\lambda > \xi_U$, then we can get Equation (17c) expressed as follows:

$$\frac{\lambda S - \lambda C_{U3}}{C_{U2} - C_{U3}} \geq \xi_U \Rightarrow S \leq \frac{(\lambda - \xi_U)C_{U3} + \lambda C_{U2}}{\lambda} \quad (17c)$$

Similarly, let $\mu_{\tilde{C}_L} = m_\lambda(S \geq \tilde{C}_L)$ represent the m_λ -measure of variables S for \tilde{C}_L . In Figure 3(b), b_1 , b_2 , and b_3 refer to C_{L1} , C_{L2} , and C_{L3} , respectively. According to Figure 3(b), when $S \leq C_{L1}$, the fuzzy event is completely invalid due to $\mu_{\tilde{C}_L} = 0$, and when $S > C_{L3}$, the fuzzy event is completely satisfied since $\mu_{\tilde{C}_L} = 1$. In case of $C_{L1} \leq S \leq C_{L3}$, $\mu_{\tilde{C}_L}$ is a monotonically increasing function between 0 and 1, which means that only a single solution S exist in $[C_{L1}, C_{L3}]$ with a given confident level ζ_L . Since the confident level should be greater than 0.5 to make the constraints meaningful, the Equation (16b) can be substituted by Equation (18a) as follows:

$$1 \geq \mu_{\tilde{C}_L} \geq \zeta_L \geq 0.5 \quad (18a)$$

The cases can also be summarized into two cases:

(1) $\lambda \leq \xi_L$, then we can get Equation (18b) expressed as follows:

$$1 + \frac{\lambda(S - C_{L3})}{C_{L3} - C_{L2}} \geq \zeta_L \Rightarrow S \geq C_{L3} + \frac{(\zeta_L - 1)(C_{U3} - C_{U2})}{\lambda} \quad (18b)$$

(2) $\lambda > \xi_L$, then we can get Equation (18c) expressed as follows:

$$\frac{(1-\lambda)(S - C_{L1})}{C_{L2} - C_{L1}} \geq \zeta_L \Rightarrow S \geq C_{L1} + \frac{\zeta_L(C_{U2} - C_{U1})}{1-\lambda} \quad (18c)$$

As such, the constraints Equations (15b) and (15c) can be solved by substituting BX^T with S . The MFCCP model can deal with fuzzy uncertainty in the right-hand side constraints.

As such, by combining Equations (15a), (17b), (17c), and (15d), fuzzy chance constrain optimization model for upper limit (UL) can be obtained with constant LL of 0.2 mg/L, and by combining Equations (15a), (18b), (18c), and (15d), fuzzy chance constrain optimization model for LL can be obtained with upper limit of 4 mg/L as well. In addition, by combining Equations (15a), (17b), (18b), and (15d), fuzzy chance-constrained optimization model under both limits (BL) can be solved. Similarly, by combining Equations (15a), (17c), (18c), and (15d), fuzzy chance-constrained optimization model under BL condition can also be solved. The scheme of the model is shown in Figure 4. The process for applying MFCCP model to solve the optimization of booster station can be summarized as follows:

- (1) Formulate the MFCCP model (Equations (15a)–(15d));
- (2) Incorporate fuzzy parameters into the related uncertain constraints;
- (3) Incorporate various preference parameter and reliability level to construct scenarios for upper boundary, lower boundary, and both upper and lower boundaries, respectively, and generate the optimal solutions;
- (4) Analyze the optimal solutions and give the optimal number and injection mass of booster stations.

CASE STUDY

Case 1

The proposed methodology was applied for a small WDS, shown in Figure 5. The WDS has 10 nodes connected by 12 pipes with a reservoir at a water level of 243.8 m. The pump has a shutoff head value of 101.3 m, a maximum flow rate of 189.3 L/s. The tank is cylindrical with a diameter of 15.4 m. The water is delivered to a storage elevated tank at node 10 (at a ground level of 259.1 m) and to eight consumers located at nodes 1–8. The base demand at various nodes varies from 6.5 to 13 L/s, and demand multipliers ranges from 0.4 to 1.6. The initial residual chlorines at nodes and reservoir are assumed to be 0.5 mg/L and 1.0 mg/L, respectively. The roughness coefficients of pipes are assumed to be 100. During the water quality simulation

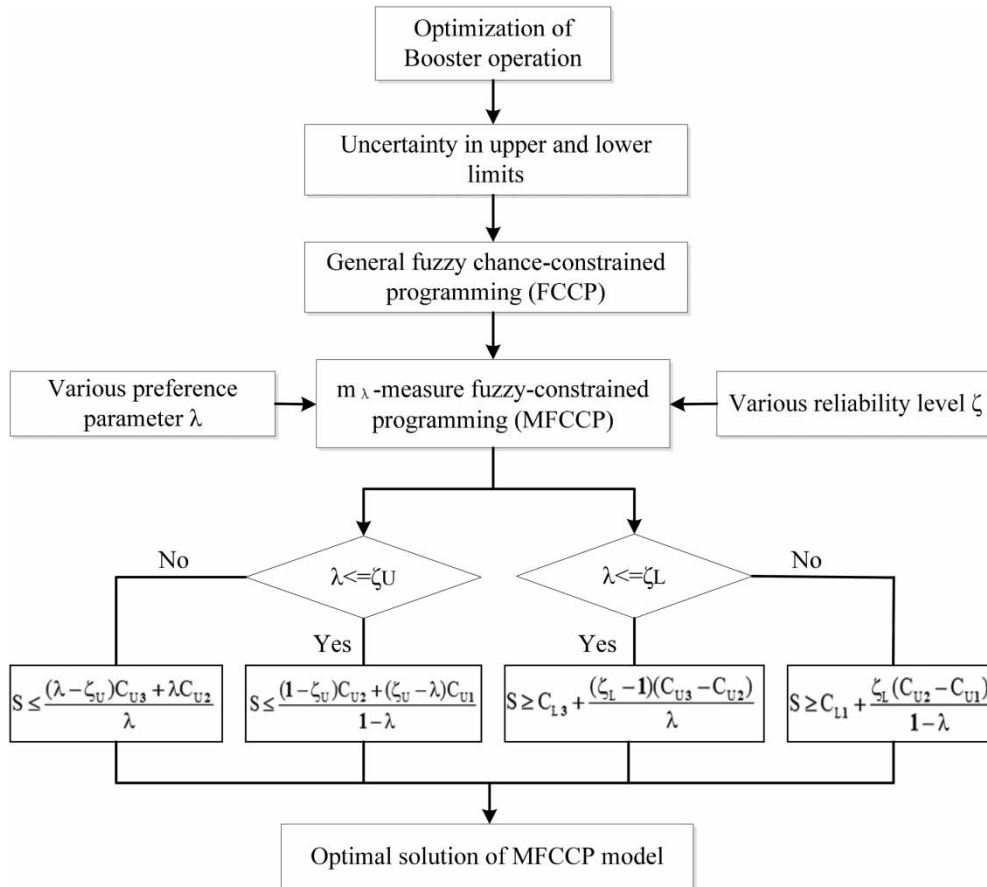


Figure 4 | The general framework of this study.

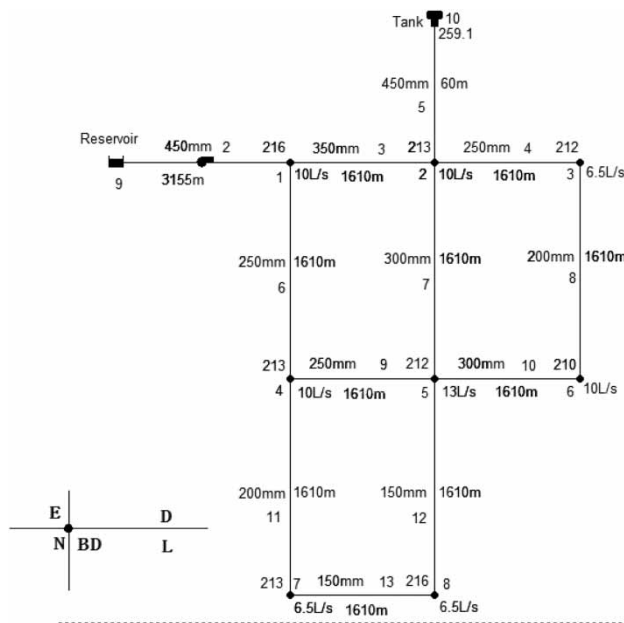


Figure 5 | Pipe-net layout of Example 1.

progress, the chlorine decay coefficient k_0 was set to be $-1.0/\text{day}$. The lower boundary, the most likely value, and the upper boundary for fuzzy upper limit are taken as 3 mg/L, 4 mg/L, and 5 mg/L, respectively. The lower boundary, the most likely value, and the upper boundary for fuzzy LL are taken as 0.1 mg/L, 0.2 mg/L, and 0.3 mg/L, respectively.

Case 2

In this paper, the Brushy Plain water distribution network system was applied, shown in Figure 6. The WDS is composed of one source node with a pump station, 34 consumer nodes, one storage tank, and 40 pipes. The physical properties such as lengths, diameters, and roughness coefficients of pipes and operational properties such as pump and demand multipliers are same as defined in

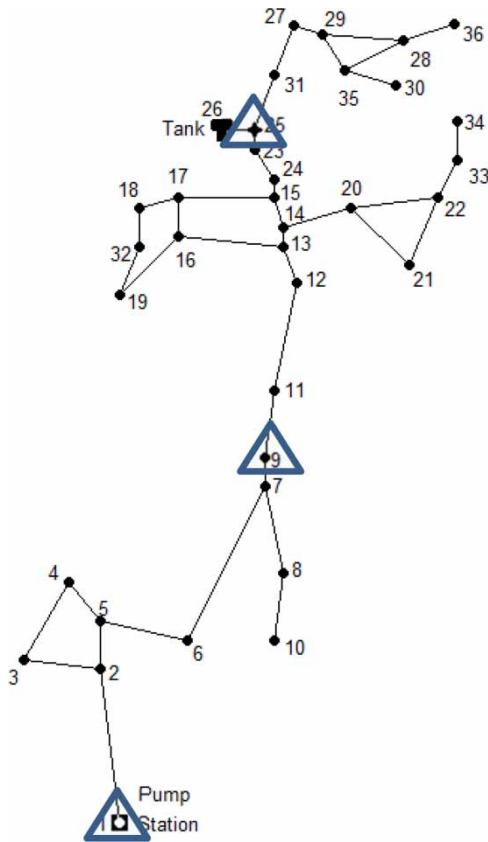


Figure 6 | Pipe-net layout of Example 2.

EPANET. Node 1 is the source node, and node 9 and node 25 are considered to be probable booster locations, which are in accordance with the other study on the same WDS (Boccelli *et al.* 1998; Köker & Altan-Sakarya 2015). The pump located at node 1 has a negative demand of $-4,400 \times 10^{-5} \text{ m}^3/\text{s}$ with a certain pump demand multiplier. The tank at node 26 is a completely mixed cylindrical tank with a diameter of 15.25 m with maximum and minimum water levels of 15.25 m and 21.35 m, respectively. The monitoring time interval is set to be 1 h for each consumer node. The response coefficient matrix B is obtained by setting the source type as mass booster type with time step of 1 h in a total of 24 h to be coincidence with the hydraulic cycle time of 24 h. By simulating hydraulic and water quality analysis in 960 h to make sure the system become stable and periodicity is obtained, the last 24 h analysis result was used. The global bulk and wall decay coefficients are set to be $k_b = 0.53/\text{day}$ and $k_w = 5.1 \text{ mm}/\text{day}$, respectively.

The lower boundary, the most likely value, and the upper boundary for fuzzy ULs and LLs are the same as for Case 1.

In the improved FCCP, the objective function and constraints are linear, which can be solved by ‘Solver’ add-on in Microsoft Excel. By applying improved fuzzy chance-constrained optimization formulations, the total booster injection to the WDS for triangular probability distribution is obtained.

RESULTS AND DISCUSSION

Application to Case 1

The confidence levels for ζ_U and ζ_L are taken as the same value between 0.5 and 1.0. The preference parameter λ is taken as a value between 0.1 and 0.9. By solving the optimization model for UL and LLs and both limits (BL) for upper limit ζ_U and lower limit ζ_L , the optimization solution of the decision variable and objective function can be obtained. The comparisons among ULs, LLs and BLs for reliability levels of 0.7, 0.8, and 0.9 are shown in Table 1, respectively. For the UL application, the optimization solution remains to be 11.27 kg/day with no relationship with reliability level ζ and preference parameter λ , which indicated that upper limits had no effect on the optimization solution. As for LL and BL applications, the injection mass is the same for the same reliability level ζ and preference parameter λ . The results indicated that the optimization results are only affected by lower concentration limit. The effect of various decision preference λ and confidence level ζ on the total injection mass is shown in Figure 7(a) and 7(b). For the same preference parameter λ , i.e., the manager’s decision attitude is unchangeable, the total injection mass increased with the reliability level $\zeta_U = \zeta_L$ for two cases of $\lambda \leq \zeta_U = \zeta_L$ and $\lambda > \zeta_U = \zeta_L$ as well, which can also be observed in Figure 7(a) and 7(b). The preference parameter λ reflects the manager’s decision, aspiration, preference and attitudes, such as optimism and pessimism, especially in case of uncertain input information. The increase of preference parameter λ , i.e., possibility rises steadily and leads to an expanded decision space in the right-side hand constraint. In case of $\zeta_U = \zeta_L = 1.0$, the chance-constrained optimization model is deterministic with no relationship

Table 1 | Comparison of total injection mass for various reliability level ζ and preference parameter λ

| $\zeta_U = \zeta_L$ | U (kg/day) | | | $\zeta_U = \zeta_L$ | λ | U (kg/day) | | | $\zeta_U = \zeta_L$ | λ | U (kg/day) | | | |
|---------------------|------------|--------------|--------------|---------------------|-----------|------------|--------------|--------------|---------------------|-----------|------------|-------|-------|-------|
| | UL | LL | BL | | | UL | LL | BL | | | UL | LL | BL | |
| 0.70 | 0.10 | 11.27 | 0.00 | 0.00 | 0.80 | 0.10 | 11.27 | 5.64 | 5.64 | 0.90 | 0.10 | 11.27 | 11.27 | 11.27 |
| | 0.20 | 11.27 | 2.82 | 2.82 | | 0.20 | 11.27 | 11.27 | 11.27 | | 0.20 | 11.27 | 14.09 | 14.09 |
| | 0.30 | 11.27 | 11.27 | 11.27 | | 0.30 | 11.27 | 13.15 | 13.15 | | 0.30 | 11.27 | 15.03 | 15.03 |
| | 0.40 | 11.27 | 12.68 | 12.68 | | 0.40 | 11.27 | 14.09 | 14.09 | | 0.40 | 11.27 | 15.50 | 15.50 |
| | 0.50 | 11.27 | 13.53 | 13.53 | | 0.50 | 11.27 | 14.66 | 14.66 | | 0.50 | 11.27 | 15.78 | 15.78 |
| | 0.60 | 11.27 | 14.09 | 14.09 | | 0.60 | 11.27 | 15.03 | 15.03 | | 0.60 | 11.27 | 15.97 | 15.97 |
| | 0.70 | 11.27 | 14.50 | 14.50 | | 0.70 | 11.27 | 15.30 | 15.30 | | 0.70 | 11.27 | 16.11 | 16.11 |
| | 0.80 | 11.27 | 25.37 | 25.37 | | 0.80 | 11.27 | 15.50 | 15.50 | | 0.80 | 11.27 | 16.21 | 16.21 |
| | 0.90 | 11.27 | 45.10 | 45.10 | | 0.90 | 11.27 | 50.74 | 50.74 | | 0.90 | 11.27 | 16.29 | 16.29 |

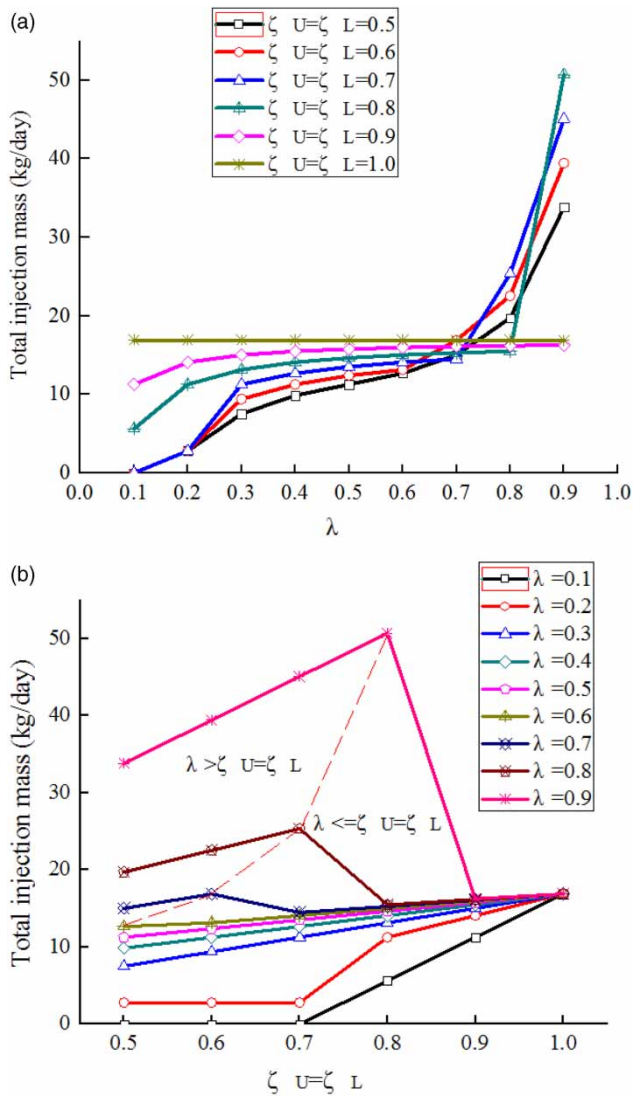


Figure 7 | (a) Relationship between total injection mass and preference parameter λ for Case 1. (b) Relationship between total injection mass and reliability level for Case 1.

with preference parameter λ . For Case 1, the optimized injection mass is 16.91 kg/day. For the other cases of $\zeta_U = \zeta_L = 0.5, 0.6, 0.7, 0.8,$ and 0.9 , the total injection mass increased with the preference parameter λ for the same reliability level $\zeta_U = \zeta_L$. In Table 1, increasing the preference parameter λ from 0.10 to 0.90 results in the injection mass increases 45.10 kg/day, 45.10 kg/day, and 5.02 kg/day for reliability levels ζ of 0.70, 0.80, and 0.90, respectively. Since preference parameter λ reflects the manager's decision attitude of optimism and pessimism, the total injection mass based on optimistic attitude with higher λ is greater than the total injection mass based on the pessimistic attitude with lower λ for the same reliability level $\zeta_U = \zeta_L$. The reason is that in Equation (18b), the right-hand-side of the constraint increase with λ due to ζ_L is less than 1.0, which leads to the expanded LL and the increase of injection mass. Similarly, in Equation (18c), LL is expanded with the increase in λ , which also leads to the increase of injection mass. As such, the total injection mass increased with the preference parameter for $\lambda \leq \zeta_U = \zeta_L$ and $\lambda > \zeta_U = \zeta_L$ as well. However, the total injection mass has a significant increase from $\lambda \leq \zeta_U = \zeta_L$ to $\lambda > \zeta_U = \zeta_L$ for the same $\zeta_U = \zeta_L$, which can also be observed in Figure 7(a) and 7(b). Moreover, the increase is more significant for greater preference parameter λ . For preference parameter $\lambda = 0.6, 0.7, 0.8,$ and 0.9 , with the increase of reliability level $\zeta_U = \zeta_L$ from less than λ to greater than λ , the total injection mass had a significant decrease, which can be observed in Figure 7(b). The results indicated that if the manager takes a more optimistic attitude, for the reliability level less than the preference parameter expressed as $\lambda > \zeta_U = \zeta_L$, more

injection mass is needed. However, the effect of preference parameter λ on the total injection mass for $\lambda \leq \zeta_U = \zeta_L$ is not more significant than that for $\lambda > \zeta_U = \zeta_L$. The results indicated that for higher reliability level, there is not a significant increase from pessimistic attitude to optimistic attitude. However, for a lower reliability level, increase of injection mass from pessimistic attitude to optimistic attitude is significant.

Application to Case 2

Similar to Case 1, the effects of reliability level and preference parameter on the injection mass are shown in Figure 8(a) and 8(b). In case of $\zeta_U = \zeta_L = 1.0$, the chance-constrained optimization model is deterministic with no relationship with preference parameter λ . For Case 2, the optimized injection mass is 4.10 kg/day. The results indicated that the total injection mass increases with the preference parameter λ for $\lambda \leq \zeta_U = \zeta_L$ and $\lambda > \zeta_U = \zeta_L$ as well. The difference from Case 1 is that for preference parameter $\lambda = 0.9$, a feasible solution is not available for $\zeta_U = \zeta_L$ values of 0.7, and 0.8. Since for the same $\zeta_U = \zeta_L$, the LL increased with the preference parameter λ , however, the increase rate of LL is more significant for $\lambda > \zeta_U = \zeta_L$ than $\lambda \leq \zeta_U = \zeta_L$ according to Equations (18b) and (18c). As such, the range between upper and lower constraints is narrowed, which leads to the unavailable solutions for $\zeta_U = \zeta_L = 0.7$ and 0.8 for preference parameter $\lambda = 0.9$. In Case 1, the optimization results are only affected by lower concentration limit (LL). In Case 2, the available solution cannot be found for $\zeta_U = \zeta_L = 0.7$ and 0.8 for preference parameter $\lambda = 0.9$ under LL. However, under UL, when node 1 was considered as a booster station, the available solution cannot be found for $\zeta_U = \zeta_L = 0.5, 0.6, 0.7,$ and 0.8 for preference parameter $\lambda = 0.9$. The same condition can also be found when considering node 1 and node 9 as booster stations. The results indicated that upper limits can affect the available solution in Case 2. Similar to Case 1, with the increase of reliability level $\zeta_U = \zeta_L$ from less than λ to greater than λ , the total injection mass had a significant decrease except for the condition with unavailable solutions. Similarly, the decrease is more significant for greater preference parameter.

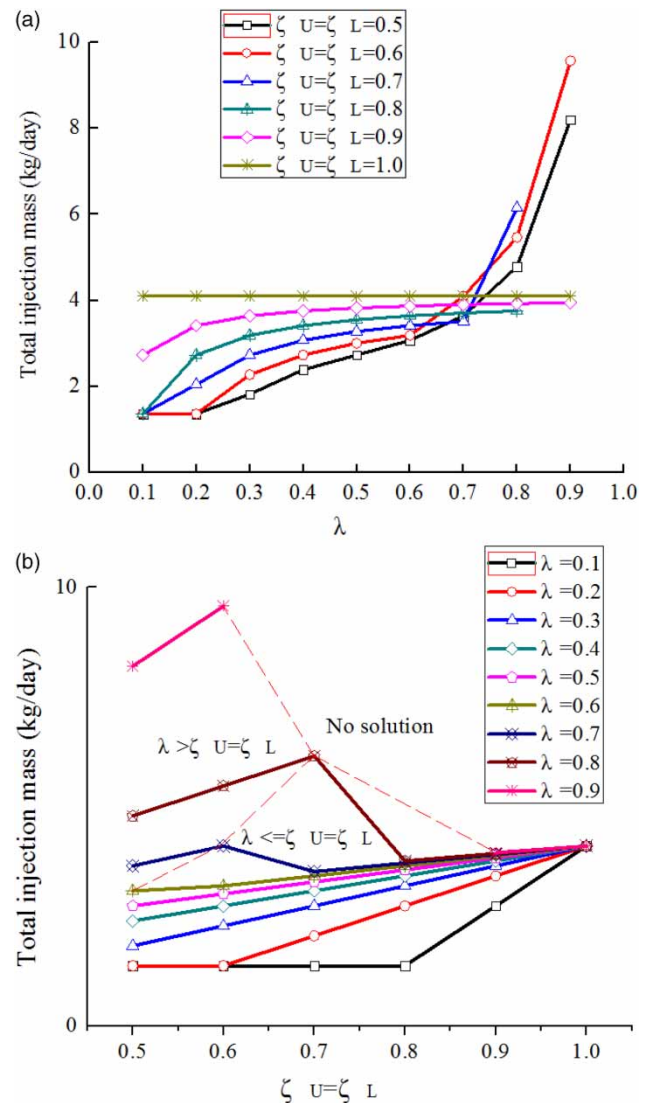


Figure 8 | (a) Relationship between total injection mass and preference parameter λ for Case 2. (b) Relationship between total injection mass and reliability level for Case 2.

The comparison of total injection mass for various booster stations under reliability level ζ between 0.5 and 0.9 and preference parameter λ of 0.1 and 0.9 is shown in Figure 9. The total injection mass for various booster chlorine injection stations with the same reliability level ζ of 0.5 and 0.9 and the same preference parameter λ of 0.5 and 0.9 is shown in Table 2. The same regulation can be observed that total injection mass increased with the preference parameter λ . For example, in case of only one booster station of node 1, preference parameter λ

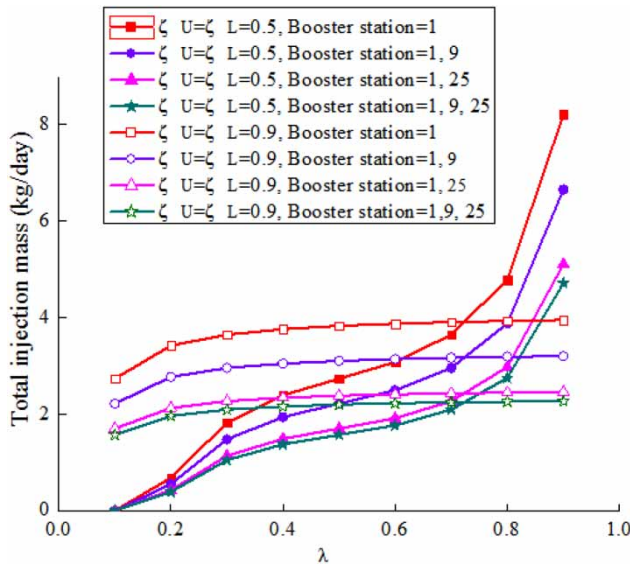


Figure 9 | Comparison of various scenarios for booster stations.

increasing from 0.5 to 0.9 gave rise to the total injection mass from 2.73 kg/day to 8.20 kg/day. With the increase of booster station number, the total injection mass decreased. In case of two booster stations, i.e., node 1 and node 9 or node 1 and node 25 are taken as booster stations, the total injection mass needed for node 1 and 9 is higher than that for node 1 and node 25 for the same reliability level ζ and preference parameter λ , which indicated that booster chlorine at the end of WDS can significantly improve the water quality. In Table 2, the decrease of total injection mass for two booster stations of node 1 and node 25 is almost two times the total injection mass for node 1 and node 9. The result can

also be found in other reports (Xin *et al.* 2019). Moreover, the number of booster station if increased from 2 to 3 cannot decrease the total injection mass significantly, and can almost be neglected. The same regulation can also be observed that for $\zeta_U = \zeta_L = 0.9$, preference parameter λ is always less than or equal to ζ , which leads to the increase rate of injection mass to preference parameter λ being relatively flat as observed in Figure 9. However, for $\zeta_U = \zeta_L = 0.5$, the increase rate of injection mass to preference parameter λ for $\lambda > \zeta_U = \zeta_L$ is greater than that for $\lambda \leq \zeta_U = \zeta_L$.

Since the total booster cost not only includes the booster chlorination injection cost (BCI), but also the booster chlorination capital cost (BCD) (Ostfeld & Salomons 2006), the economic comparison among various booster strategy in Case 2 was performed, shown in Table 2. The BCI is expressed by Equation (19) as follows:

$$BCI = \alpha \sum_{j=1}^{n_t} \sum_{i=1}^{n_b+n_s} x_i^j \tag{19}$$

where BCI refers to the booster chlorination operational injection cost ($\$/\text{day}^{-1}$), and α refers to the unit chlorine injection cost, which is assumed to be $\$2 \text{ kg}^{-1} \text{ Cl}$.

The BCD is expressed by Equation (20) as follows:

$$BCD = \sum_{i=1}^{n_b+n_s} \beta(x_i^{\max})^\gamma + \theta V_i \tag{20}$$

Table 2 | Effect of booster stations' number on total injection mass ($\zeta_U = \zeta_L = 0.5, 0.9, \lambda = 0.5, 0.9$).

| λ | Booster points | $\zeta_U = \zeta_L = 0.5$ | | | | | $\zeta_U = \zeta_L = 0.9$ | | | | |
|-----------|----------------|-------------------------------|-------------------|--------------|--------------|---------------------|-------------------------------|-------------------|--------------|--------------|---------------------|
| | | Total injection mass (kg/day) | Decrease (kg/day) | BCI (\$/day) | BCD (\$/day) | Total cost (\$/day) | Total injection mass (kg/day) | Decrease (kg/day) | BCI (\$/day) | BCD (\$/day) | Total cost (\$/day) |
| 0.5 | 1 | 2.73 | / | 5.46 | 6.45 | 11.91 | 3.83 | / | 7.66 | 6.74 | 14.40 |
| | 1, 9 | 2.22 | 0.51 | 4.44 | 11.15 | 15.59 | 3.10 | 0.73 | 6.20 | 11.65 | 17.85 |
| | 1, 25 | 1.70 | 1.03 | 3.40 | 11.06 | 14.46 | 2.38 | 1.45 | 4.76 | 11.55 | 16.31 |
| | 1, 9, 25 | 1.57 | 1.16 | 3.14 | 15.58 | 18.72 | 2.20 | 1.63 | 4.40 | 16.27 | 20.67 |
| 0.9 | 1 | 8.20 | / | 16.40 | 7.44 | 23.84 | 3.95 | / | 7.90 | 6.77 | 14.67 |
| | 1, 9 | 6.65 | 1.55 | 13.30 | 12.86 | 26.16 | 3.20 | 0.75 | 6.40 | 11.70 | 18.10 |
| | 1, 25 | 5.10 | 3.10 | 10.20 | 12.76 | 22.96 | 2.46 | 1.49 | 4.92 | 11.60 | 16.52 |
| | 1, 9, 25 | 4.71 | 3.49 | 9.42 | 17.97 | 27.39 | 2.27 | 1.68 | 4.54 | 16.34 | 20.88 |

where BCD is the booster chlorination capital cost ($\$ \text{ day}^{-1}$), x_i^{\max} is the maximum i^{th} booster chlorination injection rate (mg min^{-1}), V_i is the total i^{th} booster chlorination injection amount (mg), and β , γ , θ are empirical designed chlorination cost coefficients, which are assumed to be $\$2.21 (\text{mg min}^{-1} \text{ day})^{-\gamma}$, $\$0.13$, and $\$0 \text{ mg}^{-1}$, respectively (Ohar & Ostfeld 2010). Generally, BCI decreases when setting more booster stations, while BCD increases in case of more booster stations, which leads to the total cost including BCI and BCD increases with the number of booster stations. Under the condition of setting two booster stations, the total cost for setting node 1 and node 9 as booster stations is more expensive than setting node 1 and node 25 as booster stations, which indicated that setting booster stations far from the source node can decrease the injection mass significantly, which can also lower the total cost including BCI as well as BCD.

The residual chlorine at typical nodes of node 3, 10, 11, 19, 31, 34, and 36 under various booster stations for $\zeta_U = \zeta_L = 0.9$ and $\lambda = 0.1$ is shown in Table 3. When only one booster station at node 1 was set, injection mass of 2.73 kg/day can satisfy the upper and lower boundaries. Under conditions of two booster stations at nodes 1 and 9, and at nodes 1 and 25, the injection mass obtained are 2.22 kg/day, and 1.70 kg/day, respectively. When three booster stations at nodes 1, 9, and 25 were considered, the injection mass obtained was 1.57 kg/day. Under the four

scenarios, the residual chlorine concentration at nodes 3, 10, 11, 19, 31, 34, and 36 we kept between upper and LLs. As such, the total injection mass can be decreased by increasing the number of booster stations.

CONCLUSION

In this paper, the aim was to obtain satisfactory water quality while minimizing the cost for injection. The uncertainty of chlorine concentration is considered as chance constraint in the optimization model, which is applied to two cases. The results indicated that increasing the preference parameter λ results in the increase of total injection mass. However, the effect of reliability level ζ on total injection mass depends on the comparison of preference parameter λ and reliability level ζ . In case of $\lambda > \zeta_U = \zeta_L$ and $\lambda \leq \zeta_U = \zeta_L$, the total injection mass increased with ζ , respectively. When ζ increased from less than λ to greater than λ , the total injection mass had a significant decrease, even leading to unavailable feasible solutions. Moreover, the increase rate of total injection mass for $\lambda > \zeta_U = \zeta_L$ is more significant than for $\lambda < \zeta_U = \zeta_L$. The results indicated that the manager's optimistic or pessimistic attitude has no obvious effect in case of higher reliability level. While in case of lower reliability level, more injection mass is needed by the optimistic attitude than the pessimistic attitude.

Table 3 | Residual chlorine at typical nodes in Case 2

| Booster stations | Injection mass (kg/day) | Nodal chlorine concentration (mg/L) | | | | | | |
|------------------|-------------------------|-------------------------------------|---------|---------|---------|---------|---------|---------|
| | | Node 3 | Node 10 | Node 11 | Node 19 | Node 31 | Node 34 | Node 36 |
| 1 | 2.73 | 1.33 | 0.75 | 1.06 | 0.95 | 0.68 | 0.36 | 0.20 |
| 1 | 0.41 | | | | | | | |
| 9 | 1.81 | 0.20 | 0.37 | 1.00 | 0.91 | 0.68 | 0.36 | 0.20 |
| Total | 2.22 | | | | | | | |
| 1 | 1.03 | | | | | | | |
| 25 | 0.68 | 0.50 | 0.32 | 0.46 | 0.41 | 0.68 | 0.20 | 0.20 |
| Total | 1.70 | | | | | | | |
| 1 | 0.41 | | | | | | | |
| 9 | 0.49 | | | | | | | |
| 25 | 0.67 | 0.20 | 0.22 | 0.45 | 0.41 | 0.68 | 0.20 | 0.20 |
| Total | 1.57 | | | | | | | |

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CONFLICT OF INTEREST

None.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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