Optimal operation of multi-reservoir systems: comparative study of three robust metaheuristic algorithms

Saeid Akbarifard, Mohammad Reza Sharifi, Kourosh Qaderi and Mohamad Reza Madadi

ABSTRACT

In this study, the capability of the recently introduced moth swarm algorithm (MSA) was compared with two robust metaheuristic algorithms: the harmony search (HS) algorithm and the imperialist competitive algorithm (ICA). First, the performance of these algorithms was assessed by seven benchmark functions having 2–30 dimensions. Next, they were compared for optimization of the complex problem of four-reservoir and 10-reservoir systems operation. Furthermore, the results of these algorithms were compared with nine other metaheuristic algorithms. Sensitivity analysis was performed to determine the appropriate values of the algorithms’ parameters. The statistical indices coefficient of determination (R²), root mean square error (RMSE), mean absolute error (MAE), mean square error (MSE), normalized MSE (NMSE), mean absolute percentage error (MAPE), and Willmott’s index of agreement (d) were used to compare the algorithms’ performance. The results showed that MSA was the superior algorithm for solving all benchmark functions in terms of obtaining the optimal value and saving CPU usage. ICA and HS were ranked next. When the dimensions of the problem were increased, the performance of ICA and HS dropped but MSA has still performed extremely well. In addition, the minimum CPU usage and the best solutions for the optimal operation of the four-reservoir system were obtained by MSA, with values of 269.7 seconds and 308.83, which are very close to the global optimum solution. Corresponding values for ICA were 486.73 seconds and 306.47 and for HS were 638.61 seconds and 264.61, which ranked them next. Similar results were observed for the 10-reservoir system; the CPU time and optimal value obtained by MSA were 725.2 seconds and 1,195.58 while for ICA they were 1,421.62 seconds and 1,136.22 and for HS they were 1,963.41 seconds and 1,060.76. The R² and RMSE values achieved by MSA were 0.951 and 0.528 for the four-reservoir system and 0.985 and 0.521 for the 10-reservoir system, which demonstrated the outstanding performance of this algorithm in the optimal operation of multi-reservoir systems. In a general comparison, it was concluded that among the 12 algorithms investigated, MSA was the best, and it is recommended as a robust and promising tool in the optimal operation of multi-reservoir systems.

Key words | evolutionary algorithms, reservoir operation, soft computing, test functions, water resources

HIGHLIGHTS

- This is the first application of robust MSA in the optimization of water resources.
- The capability of 11 metaheuristic algorithms in optimization of multi-reservoir systems was compared.
- MSA was the superior model in terms of CPU time and obtaining the optimal solution.

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INTRODUCTION

The optimal operation of reservoirs is one the most important issues in water resources management, especially for multi-reservoir systems. Many control variables determine the operation strategies for scheduling a sequence of releases to meet a large number of demands for different users. Optimal reservoir operation defines the optimum policies to retain or release water from a reservoir at different periods of the year according to the inflows and demands.

To develop a comprehensive and optimal operation policy for a reservoir, the sources of uncertainties should be considered. The uncertainties may be due to different factors such as inaccurate predictive models, data scarcity, measurement and observation errors, the measurement site being unrepresentative, or problems in aggregating or disaggregating data. For a multi-reservoir system, the uncertainties may be caused by the stochastic nature of the system inputs (rainfall, reservoir inflow, evaporation, leakage, release, etc.), the nonlinearity of functions, the multiple constraints, the large number of decision variables, and other spatial and temporal variations of system components. Consideration of these uncertainties can help in the design of efficient operational policies and to develop robust predictive models.

Over the past few years, several evolutionary algorithms have been developed and applied to solving reservoir optimization problems (Table 1). They are considered to be very effective alternatives for solving complex optimization problems with either single or multiple objectives. These algorithms offer an expanded capability to systematically select the optimal solutions given the objectives and constraints (Labadie 2004). Garousi-Nejad et al. (2016) applied the firefly algorithm (FA) to optimal operation of reservoirs used for irrigation supply and hydropower production. The results demonstrated the superior performance of FA compared to genetic algorithms (GA) in terms of the convergence rate and obtaining optimal value. Qaderi et al. (2018) used a water cycle algorithm (WCA) to derive operating policy for a multi-reservoir system. They reported a high performance of WCA compared to other well-known algorithms. Ehteram et al. (2017) used the GA-krill hybrid for the optimization of multi-reservoir systems operation and showed that it outperformed the traditional nonlinear programming models. Ehteram et al. (2018) successfully used the spider monkey algorithm (SMA) to optimize a multi-reservoir system with the aim of decreasing irrigation deficiencies. Asadieh & Afshar (2019) used the charged system search (CSS) algorithm to optimize water-supply and hydropower reservoir operation. The results demonstrated the robustness and superiority of the CSS algorithm in solving long-term reservoir operation problems, compared to alternative methods. Feng et al. (2019) proposed the k-means clustering method and extreme learning machine based on particle swarm optimization (PSO) for the operation rule derivation for two hydropower reservoirs in China. They reported the satisfactory performance of proposed method in real-world cases. Mohammadi et al. (2019) reported the high performance of the hybrid whale-genetic algorithm in the optimal operation of multi-reservoir benchmark systems. Using the long-term data of Hongjiadu reservoir in China, Niu et al. (2019) evaluated the capability of four methods: multiple linear regression (MLR), artificial neural network (ANN), extreme learning machine (ELM), and support vector machine (SVM) in deriving the operation rule of the hydropower reservoir. They reported that the three artificial intelligence algorithms (ANN, SVM, and ELM) showed better performances than the conventional MLR and scheduling graph method. Ehteram et al. (2019) proposed the crow algorithm (CA) for optimizing hydropower generation in multi-reservoir systems. They documented the high potential of the proposed CA for achieving optimal solutions to complex optimization problems associated with dam and reservoir operations. Zhou et al. (2019) identified efficient operating rules for hydropower reservoirs using the system dynamics approach. The Three Gorges Reservoir in central China was used as a case study. The results showed that the system dynamics simulation is an efficient way to simulate a complicated reservoir system using feedback and causal loops. Dehghani et al. (2019) applied the grey wolf optimization (GWO) method coupled with an adaptive neuro-fuzzy inference system (ANFIS) to forecast hydropower generation. The results indicated that the hybrid GWO-ANFIS model was
Table 1 | Literature review of the application of well-known evolutionary algorithms in the operation of reservoirs

<table>
<thead>
<tr>
<th>Evolutionary algorithm</th>
<th>References</th>
<th>Problem definition</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genetic algorithm (GA)</td>
<td>Sharif &amp; Wardlaw (2000)</td>
<td>Optimization of a multi-reservoir system in Indonesia by GA and discrete differential dynamic programming (DP)</td>
<td>✓ The GA results are very close to the optimum, it does not need trial state trajectories to initiate the search and it does not need discretization of state variables, in contrast to DP.</td>
</tr>
<tr>
<td></td>
<td>Chang et al. (2005)</td>
<td>Using GA to derive operating rule curves for the Shih-Men reservoir in Taiwan</td>
<td>GA provides an effective way for searching the rule curves.</td>
</tr>
<tr>
<td></td>
<td>Garousi-Nejad et al. (2016)</td>
<td>Optimal operation of reservoirs for irrigation supply and hydropower generation using GA and FA</td>
<td>FA is superior in terms of convergence and variance of the results.</td>
</tr>
<tr>
<td>Genetic programming (GP)</td>
<td>Fallah-Mehdipour et al. (2015a, 2015b)</td>
<td>Extraction of optimal operation rules in an aquifer-dam system with developed version of GP in comparison with GA</td>
<td>Developed GP is more flexible and effective in determining optimal rule curves for a conjunctive aquifer-dam system.</td>
</tr>
<tr>
<td></td>
<td>Fallah-Mehdipour et al. (2022)</td>
<td>Application of GP for real-time operation of reservoir</td>
<td>The GP-based rule is effective in determining optimal rule curves.</td>
</tr>
<tr>
<td></td>
<td>Ashofteh et al. (2015a)</td>
<td>Using multi-objective GP for evaluation of the climatic change impacts on multi-objective reservoir operation</td>
<td>Reservoir-operating rules that take into account changing climate would lead to improvements in reservoir performance in the order of 29–32% relative to operating rules based on baseline climatic conditions.</td>
</tr>
<tr>
<td></td>
<td>Fallah-Mehdipour et al. (2015a, 2015b)</td>
<td>Developing operational decision rules of multi-purpose reservoirs by GP, GA and linear, integer, nonlinear, and global optimization (LINGO)</td>
<td>The objective function value is significantly enhanced by GP.</td>
</tr>
<tr>
<td></td>
<td>Ashofteh et al. (2015b)</td>
<td></td>
<td>The GP-based operational rule is effective in determining optimal rule curves for reservoirs.</td>
</tr>
<tr>
<td>Ant colony optimization (ACO)</td>
<td>Jalali et al. (2006)</td>
<td>Optimal operation of Dez reservoir with ACO</td>
<td>ACO is quite sensitive to setup parameters, and provides better and more comparable results with known global optimum results.</td>
</tr>
<tr>
<td></td>
<td>Moeini &amp; Afshar (2015)</td>
<td>Optimal operation of multi-reservoir systems by ACO and constrained ACO</td>
<td>Constrained ACO was better than conventional ACO.</td>
</tr>
<tr>
<td></td>
<td>Mohammed et al. (2018)</td>
<td>Optimization of Darbandikhan reservoir operation using a developed version of ACO</td>
<td>Developed ACO showed a high performance in exploring the optimum solutions for the operation of the Darbandikhan reservoir.</td>
</tr>
<tr>
<td>Particle swarm optimization (PSO)</td>
<td>Afshar (2015)</td>
<td>Using PSOs for optimal operation of multi-reservoir systems</td>
<td>PSOs are very effective in locating optimal solutions and very efficient in terms of the convergence rate.</td>
</tr>
<tr>
<td></td>
<td>Ghimire &amp; Reddy (2015)</td>
<td>An elitist-mutated PSO (EMPSO) is applied for weekly operation policies of the Upper Seti Hydro-Power Reservoir for wet, dry and normal water years</td>
<td>The EMPSO can generate 3% more hydropower than the planned hydropower production with a sustainability index of 0.75.</td>
</tr>
<tr>
<td></td>
<td>Afshar (2012)</td>
<td>Using constrained PSOs (CPSO) for optimization of large reservoir operation compared to GA and conventional PSO</td>
<td>CPSOs were superior to conventional PSO and GA in locating near-optimal solutions and convergence characteristics.</td>
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<td></td>
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<td>CPSOs were more insensitive to the swarm size and initial swarm.</td>
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</tr>
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<tbody>
<tr>
<td><strong>Al-Aqeeli &amp; Agha (2020)</strong></td>
<td></td>
<td>Optimality operation of Mosul and Badush reservoirs system for hydropower production using PSO</td>
<td>✓ PSO has high performance in real-time operation of single- and multi-reservoir systems.</td>
</tr>
<tr>
<td><strong>Harmony search (HS)</strong></td>
<td>Bashiri-Atrabi et al. (2015)</td>
<td>Application of HS for optimization of the Narmab reservoir operation for flood management</td>
<td>✓ HS has high convergence rate, it can be effectively used for operation of reservoirs for flood management.</td>
</tr>
<tr>
<td></td>
<td>Kougias &amp; Theodossiou (2013)</td>
<td>Application of HS for optimum operation of a four-reservoir system over 24 hours</td>
<td>✓ HS has high potential for the optimization of multi-reservoir systems.</td>
</tr>
<tr>
<td></td>
<td>Mirbeyk et al. (2020)</td>
<td>Using HS for optimal operation of Dez reservoir</td>
<td>✓ HS has the ability to solve real large reservoir problems.</td>
</tr>
<tr>
<td><strong>Water cycle algorithm (WCA)</strong></td>
<td>Bozorg Haddad et al. (2015)</td>
<td>Comparison of WCA and GA for optimal operation of Karon-4 reservoir in Iran</td>
<td>✓ The results demonstrate the high efficiency and reliability of WCA in solving reservoir operation problems.</td>
</tr>
<tr>
<td><strong>Honey-bee mating optimization (HBMO)</strong></td>
<td>Bozorg Haddad et al. (2011)</td>
<td>HBMO was compared with linear programming (LP), DP, differential DP, discrete differential DP and GA in the optimal operation of multi-reservoir systems</td>
<td>✓ The high efficiency and rapid convergence rate of HBMO compared to other algorithms make it a robust tool for the optimal operation of reservoirs.</td>
</tr>
<tr>
<td></td>
<td>Soghari &amp; Mocini (2020)</td>
<td>Performance of HBMO for optimization of Dez hydropower reservoir operation was compared with artificial bee colony (ABC) algorithm, GA, improved particle swarm optimization (IPSO) algorithm, ACO and GSA</td>
<td>✓ Using ABC gave the best results with low computational costs.</td>
</tr>
<tr>
<td><strong>Imperialist competitive algorithm (ICA)</strong></td>
<td>Afshar et al. (2015)</td>
<td>Optimizing water supply and hydropower reservoir operation rule curves by ICA for Dez reservoir in Iran</td>
<td>✓ ICA converged to near-optimal solutions efficiently in the case study. It performed quite well in reservoir operation optimization.</td>
</tr>
<tr>
<td></td>
<td>Hosseini-Moghari et al. (2015)</td>
<td>Optimal operation policy of the Karun4 reservoir using ICA, GA, cuckoo optimization algorithm (COA), and nonlinear programing (NLP).</td>
<td>✓ Both COA and ICA showed high performance in extraction of optimal operation policies of Karun4.</td>
</tr>
<tr>
<td></td>
<td>Rahimi et al. (2020)</td>
<td>Optimization of hydropower energy and flood control for a real multi-objective multi-reservoir system by ICA</td>
<td>✓ ICA has a fast convergence rate and a high adaptability to the problem constraints.</td>
</tr>
<tr>
<td><strong>Spider monkey algorithm (SMA)</strong></td>
<td>Ehteram et al. (2018)</td>
<td>Investigating the capability of SMA compared to well-known optimization algorithms in the optimization of the Golestan and Voshmgir dam operations</td>
<td>✓ The SMA, with its high convergence rate, is suggested as an appropriate tool for optimizing the operation policy of cascade reservoirs.</td>
</tr>
<tr>
<td><strong>Bat algorithm (BA)</strong></td>
<td>Ahmadianfar et al. (2016)</td>
<td>An improved version of BA was used to optimize hydropower generation through two multi-reservoir benchmark problems</td>
<td>✓ The improved BA indicated a high performance in hydropower optimization.</td>
</tr>
</tbody>
</table>
capable of predicting hydropower generation satisfactorily. Soghrati & Moeini (2020) proposed an improved artificial bee colony (ABC) algorithm to solve the single-reservoir operation optimization problem. They documented the capability of proposed algorithms to solve large reservoir operation optimization problems. Moeini & Babaei (2020) applied a hybrid of the constrained version of the improved particle swarm optimization (CIPSO) algorithm with a support vector machine (SVM) called the hybrid SVM-CIPSO method for the optimal operation of reservoirs for uncertain water inflow conditions. They reported the acceptable accuracy of this model in predicting the optimal water release for future conditions. Feng et al. (2020) proposed adaptive mutation sine cosine algorithm (ASCA) to optimize multiple hydropower reservoir operation. In this algorithm, they used the elite mutation strategy to increase individual diversity, the simplex dynamic search strategy to improve solution accuracy, and the neighborhood search strategy improve the convergence rate. The simulations of 25 test functions and a real-world hydropower system in China indicated the superiority of ASCA over several existing methods. Al-Aqeeli & Agha (2020) successfully employed PSO for the optimal operation of a multi-reservoir system (Mosul and Badush reservoirs in Iraq) for hydroelectric generation. Azizipour et al. (2020) employed the hybrid cellular automata-simulated annealing approach for optimal hydropower operation of multi-reservoir systems. The case study was a three-reservoir system in the USA. The results indicated that the proposed method was much more efficient than existing algorithms. Bozorg-Haddad et al. (2020) applied the flower pollination algorithm (FPA) to optimize single- and multi-reservoir systems. They reported the superiority of FPA over PSO and nonlinear programming method (NLP) in finding the optimal solutions.

As can be seen, several evolutionary algorithms have been developed in recent years. Because of the many advantages of these algorithms, such as rapid rate of convergence, the capability of a large number of simulations to arrive at an optimum solution, avoiding local optima, the ability for multi-objective optimization, easy handling of nonlinearity and non-convexity of the problem domain, lack of restriction by the number of dimensions and computational requirements, they have become an attractive alternative to the classical methods for the optimization of engineering problems.

The main question that this paper seeks to answer is that, among the several evolutionary algorithms that have been recently introduced by different researchers in different engineering fields, which one is the most appropriate for the optimal operation of multi-reservoir systems? Accordingly, this paper investigates the capability of the recently introduced moth swarm algorithm (MSA) for the optimal operation of four- and 10-reservoir systems in comparison to the harmony search (HS) algorithm and the imperialist competitive algorithm (ICA). To the authors’ knowledge, this is the first application of MSA for the optimal operation of multi-reservoir systems. The extraordinary performance of these three algorithms, as outlined in Table 2, has made them the most successful algorithms among dozens of algorithms in solving and optimizing various complex engineering problems (Yang et al. 2017; Yoo et al. 2018; Gerist & Maheri 2019; Kim et al. 2019; Afsari et al. 2020; Khamari et al. 2020; Sayed et al. 2020). The results of these three algorithms will be also compared with nine other algorithms developed by previous researchers, including GA, honey-bee mating optimization (HBMO), gravity search algorithm (GSA), improved bat algorithm (IBA), FA, modified FA (MFA), krill, hybrid GA-krill, WCA.

MATERIALS AND METHODS

As previously stated, this study investigates the capability of three robust algorithms, MSA, HS and ICA, for the optimal operation of multi-reservoir systems. The overall methodology of this study is shown in Figure 1. More details of methodology are presented below.

Moth swarm algorithm (MSA)

MSA is a robust metaheuristic algorithm which originates from moth behaviour in nature. Moths hide from predators during the day, and at night they use celestial navigation to orient themselves in the dark and exploit food sources. In celestial navigation, the flying direction lies at a constant angle to the moonlight as a remote light source. In this algorithm, the best solution is indicated by the position of the light source, and the quality of this solution is determined by the intensity of the light source. Each moth swarm includes three groups: pathfinders, prospectors and
onlookers. Pathfinders can explore the best point over the optimization space with the first-in-last-out principle to lead the movement of the main swarm. Prospectors wander into a random path close to the light sources, which have been marked by the pathfinders. Onlookers move directly toward the best global solution (moonlight), which has been obtained by prospectors.

The optimization procedure in MSA is such that, at any iteration, each moth is incorporated into the optimization problem to find the highest intensity of its corresponding light source. The best fitness in the swarm is considered as the pathfinders’ positions, and guidance for the next updated iteration. Hence, the second and third best groups are the prospectors and onlookers, respectively. The optimization of the algorithm is performed through three phases of initialization, reconnaissance and transverse orientation. In initialization, at the beginning of the flight, the initial positions of moths are randomly generated. Next, the moths are grouped into the three types (pathfinders, prospectors, onlookers) based on their calculated fitness. To prevent a probable premature convergence in this phase as well as to improve the diversity of solutions, pathfinders update their positions by crossover operations and Levy mutation. During this procedure, the type of each moth changes dynamically. For instance, if a prospector finds a solution that is brighter than the existing one, it is promoted to become a pathfinder moth. During the optimization procedures, by decreasing the number of prospectors, the number of onlookers increases, which may lead to a global solution. In this phase, MSA forces the onlookers to search more effectively (by celestial navigation) and find the optimal solution. The governing equations and mathematical explanation of MSA was provided by Mohamed et al. (2018). Less than 3 years after development of this algorithm, its extraordinary performance has been reported by several researchers (Zhou et al. 2018; Shilaja & Arunprasath 2019; Duong et al. 2020; Kotb & El-Fergany 2020).

### Table 2 | Main advantages and limitations of the MSA, ICA and HS

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Advantages</th>
<th>Limitations</th>
</tr>
</thead>
</table>
| MSA       | - Ease of implementation and execution  
- Fast convergence rate  
- Low computational overheads *(Zhou et al. 2018)*  
- Ability to avoid local optima and premature convergence  
- Capability of a large number of simulations to arrive at the optimum  
- Lack of restriction by the number of dimensions  
- High exploitation and exploration capabilities  
- Ability to solve large-scale and complex problems  
- Easily handling of nonlinearity and non-convexity of problem domain  
- Low number of parameter tuning | - Reduction of population diversity over time |
| ICA       | - Ease of implementation and execution  
- Lack of restriction by the number of dimensions  
- Less dependency on initial solutions | - Possibility of premature convergence  
- Possibility of trapping in local optima in high-dimensional problems  
- Failure to achieve the exact optimal point |
| HS        | - Ease of implementation and run  
- Lack of restriction by the number of dimensions  
- Ability to tackle several complex problems | - Possibility of premature convergence  
- Possibility of trapping in local optima in high-dimensional problems  
- Failure to achieve the exact optimal point |

**Harmony search (HS) algorithm**

HS is an optimization algorithm inspired by the musical improvisation process in music bands. It was first introduced by Geem et al. (2001), and in recent years, has been widely employed as an optimization tool in various engineering problems. Harmony in music is similar to the optimization solution vector and the musicians’ improvisations whereby the musicians’ search for harmony is analogous to local and global search schemes in the optimization procedure.
HS has many advantages (e.g. fewer mathematical requirements, no need for initial value setting of the decision variables, having stochastic random searches, and strong mechanism to generate new solution vectors) which made it a robust and prominent algorithm in the optimization of complex engineering problems. It has been successfully employed in the optimization of truss structural optimizations (Cheng et al. 2016; Saka et al. 2016), engineering management optimizations (Shen et al. 2017), economic load dispatch problem (Al-Betar et al. 2016); clustering applications (Abualigah et al. 2020; Talaei et al. 2020), electrical engineering (Shiva & Kumar 2020), medical sciences (Koti et al. 2020), civil and geotechnical engineering (Bekdas et al. 2020), optimal design of water distribution networks (Geem 2006; Geem & Cho 2011), optimal operation of multi-reservoir systems (Geem 2007), simulation of irrigation systems (Cisty 2008), optimization of groundwater management (Ayvaz 2009), identification of unknown groundwater pollution sources (Ayvaz 2010) and optimal operation of reservoir system with emphasis on flood forecasting (Bashiri-Atrabi et al. 2015).

**Imperialist competitive algorithm (ICA)**

ICA was proposed by Atashpaz-Gargari & Lucas (2007). It is based on a global heuristic search that applies imperialism and imperialistic competition process as a source of inspiration. ICA starts with an initial population. Some of the best individuals of this population, called countries, are picked up as the imperialist states and all the rest become...
the colonies of these imperialists. Due to the imperialists’ powers, which are inversely proportional to their cost, the colonies of the initial population are divided among them. Having distributed the colonies between the imperialists and established the initial empires, these colonies commence proceeding toward their relevant imperialist country. After the successful application of ICA in different engineering optimizations, it has been approved by several researchers as one of the most powerful optimization algorithms (Abd-Elazim & Ali 2019; Al Dossary & Nasrabadi 2016; Fathy & Rezk 2017; Mikaeil et al. 2018; Emami & Parsa 2020; Lei et al. 2020).

All the algorithms were coded in MATLAB 2016a using a PC with i7 CPU 1.8 GHz/16GB RAM/2TB HDD.

**Verification of the algorithms**

In order to evaluate the efficiency and validation of the developed models for the optimal operation of multi-reservoir systems, a set of standard benchmark functions was selected, as presented in Table 3.

The performance of MSA in solving these functions was compared with HS and ICA. The population size and the number of evaluations of benchmark functions in all the algorithms was identical and proportional to the dimensions of each function. As seen in Table 3, for benchmark functions with lower dimensions, the performance of all the algorithms was approximately similar, but for large dimension problems (Rosenbrock function with dimension of 10 and 30), MSA was the only algorithm which was capable of solving the problem. The performance of the other algorithms was so weak, the results of them dramatically diverged from the optimal value. Figure 2 shows the convergence rate of the algorithms in reaching the optimal value for standard benchmark functions. It indicates that for all benchmark functions, MSA reached its optimal value by the smallest number of iterations. The difference was more obvious in high dimension problems.

After successful verification of utilized algorithms by benchmark functions, their performance was investigated in the optimization of two multi-reservoir systems.

**Sensitivity analysis of model parameters**

Before using the algorithms to optimize the multi-reservoir system operation, a sensitivity analysis was performed to determine the best values for the algorithm parameters. The tuning parameters include number of iterations, number of variables, number of search agents and number of pathfinders (for MSA), number of iterations, number of variables, harmony memory size, harmony memory considering rate and pitch-adjusting rate (for HS), and number of iterations, number of variables, number of countries, revolution rate and assimilation coefficient (for ICA). The sensitivity of each model to all corresponding parameters was investigated and the best values for model parameters were identified by sensitivity analysis.

**Test cases: optimization of multi-reservoir systems**

The four-reservoir problem was first formulated and solved by Larson (1968) as a linear problem with a known global
optimum. This case was later used by a number of researchers for the assessment of different optimization algorithms (Wardlaw & Sharif 1999; Hosseini-Moghari et al. 2015; Ehteram et al. 2017). The four-reservoir system, as illustrated in Figure 3(a), consists of both series and parallel connections. The supplies from the system are used for hydropower generation and irrigation. Hydropower generation is possible from each reservoir. The outflow from reservoir four may be diverted for irrigation. Hydropower and irrigation benefits are quantified by linear functions of discharge. There are inflows to the first and second reservoirs only, and these are 2 and 3 units, respectively, in all time periods. The initial storage in all reservoirs is 5 units.

The 10-reservoir problem is more complicated, not only in terms of size, but also because of the many time-dependent constraints on storage. The layout of the 10-reservoir problem is shown in Figure 3(b). The system comprises reservoirs in series and in parallel, and a reservoir may receive supplies from one or more upstream reservoirs. Inflows are defined for each of the upstream reservoirs, and initial storage and target storage at the end of the operating period are specified for each reservoir. In addition, there are minimum operating storage amounts in each reservoir that must be satisfied, as well as constraints on minimum and maximum reservoir releases. Operation of the system is optimized over 12 operating periods to maximize hydropower production. More details of these problems can be found in Murray & Yakowitz (1979).

The objective function is the maximization of benefits from the systems over 12 two-hour operating periods,
defined as follows:

$$\text{Max } F = \sum_{k=1}^{K} \sum_{t=1}^{NT} b_k(t) \times R_k(t) + \sum_{t=1}^{NT} b_{k+1}(t) \times R_k(t)$$ (1)

where $K$ is the number of reservoir; $NT =$ total number of periods; $R_k(t) =$ releases in time period $t$ from reservoir $k$ ($k = 1, \ldots, K$); and $b_k(t)$ is the benefit functions for the $k_{th}$ reservoir. The function $F$, which should be maximized, is the sum of the returns due to power generated by the power plants and the return from the diversion of the irrigation project.

The fundamental constraints include the continuity constraints for each reservoir over each operating period $t$, defined as:

$$S_k(t + 1) = S_k(t) + I_k(t) - R_k(t) \quad k = 1, 2, \ldots, K$$ (2)

Constraints on reservoir storage:

$$S_k^{\text{min}}(t) \leq S_k(t) \leq S_k^{\text{max}}(t) \quad k = 1, 2, \ldots, K$$ (3)

And constraints on releases from the reservoirs:

$$R_k^{\text{min}}(t) \leq R_k(t) \leq R_k^{\text{max}}(t) \quad k = 1, 2, \ldots, K$$ (4)

Here $S_k(t) =$ storage at time $t$ in reservoir $k$ ($k = 1, \ldots, K$); $I_k(t) =$ inflows in time period $t$ to reservoir $k$; $S_k^{\text{min}}(t) =$ minimum storage in reservoir $k$; $S_k^{\text{max}}(t) =$ maximum storage in reservoir $k$; $R_k^{\text{min}}(t) =$ minimum release from reservoir $k$; $R_k^{\text{max}}(t) =$ maximum release from reservoir $k$.

Evaluation criteria

In order to evaluate the employed algorithms, the statistical indices of coefficient of determination ($R^2$), root mean square error (RMSE), mean absolute error (MAE), mean square error (MSE), normalized MSE (NMSE), mean absolute percentage error (MAPE), and Willmott’s index of agreement (d) were used as per Equations (5)–(11) (Willmott 1981).

$$R^2 = \left[1 - \frac{\sum (\text{Re}_{\text{opt}} - \text{Re}_t)(\text{Re}_t - \text{Re})}{\sum (\text{Re}_{\text{opt}} - \text{Re})^2 \times \sum (\text{Re}_t - \text{Re})^2}\right]^2$$ (5)

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum (\text{Re}_{\text{opt}} - \text{Re}_t)^2}$$ (6)

$$\text{MAE} = \frac{1}{n} \sum |\text{Re}_{\text{opt}} - \text{Re}_t|$$ (7)

$$\text{MSE} = \frac{\sum_{t=1}^{n} (\text{Re}_{\text{opt}} - \text{Re}_t)^2}{n}$$ (8)

$$\text{NMSE} = \frac{\sum_{t=1}^{n} (\text{Re}_{\text{opt}} - \text{Re}_t)^2}{\sum_{t=1}^{n} (\text{Re}_{\text{opt}} - \text{Re})^2}$$ (9)

$$\text{MAPE} = \frac{100}{n} \times \sum_{t=1}^{n} \left|\frac{\text{Re}_{\text{opt}} - \text{Re}_t}{\text{Re}_{\text{opt}}}ight|$$ (10)

$$d = 1 - \frac{\sum_{t=1}^{n} (\text{Re}_{\text{opt}} - \text{Re}_t)^2}{\sum_{t=1}^{n} (|\text{Re}_t - \text{Re}| + |\text{Re}_{\text{opt}} - \text{Re}_t - \text{Re}|)^2}$$ (11)

In the above equations, Re$_t$ is the releases in time period $t$ from the investigated algorithms, Re$_{\text{opt}}$ is the optimum release in time period $t$, Re$_{\text{opt}}$ is the mean of the optimum release, Re is the mean of the release, and $n$ is the number of total time periods. The variation domain of Willmott's index of agreement ranges from $-\infty$ to 1, so that 1 indicates perfect agreement between the optimum release and releases from the investigated algorithms.

A low value of RMSE and a high value of $R^2$ indicate acceptable accuracy of the algorithm and correlation between the data, and also imply its superiority over the other algorithms. Each of the MAE, MSE, NMSE, and
MAPE parameters shows the difference between the optimum release and releases from the investigated algorithms; the lower the values of these parameters, the more efficient the algorithm. MSE highlights the difference between the data, and its normalized form (NMSE) can be compared with other algorithms.

**RESULTS AND DISCUSSION**

**Results of sensitivity analysis**

As previously described, to ensure the reliability and validity of the models’ outputs, MSA, HS and ICA were put through 10 runs with different iterations. To achieve the appropriate number of iterations, a sensitivity analysis on the number of iterations was performed. Accordingly, each algorithm was run with 500, 1,000, 2,000 and 5,000 iterations. Table 4 shows the values of objective function for different numbers of iterations in the four-reservoir system.

Only MSA could reach the values very close to the optimal value after 1,000 iterations. Although the two other algorithms converged after 1,000 iterations, they have a large difference from the optimal value. For iterations over 1,000, while the execution time of the algorithms dramatically increased, the variations of the objective function were negligible. Therefore, the number of iterations in each algorithm was considered to be 1,000. The results of sensitivity analysis for the four-reservoir system are shown in Figures S1–S4 in the Supplementary Information. In addition, Figure 4 shows the variations of objective function by increasing the number of iterations. Similar results were obtained for the 10-reservoir system. Table 5 shows the final values of algorithm parameters for multi-reservoir systems, all of which were derived by sensitivity analysis.

**Convergence rate and models performance**

Figure 5 shows the convergence rate of applied algorithms to reach the optimum value in the operation of four- and 10-reservoir systems. It indicates the rapid convergence of MSA compared to HS and ICA. The results in Figure 5 are consistent with those in Figure 2, in which MSA is the best and HS is the worst algorithm for solving the large-scale problems. It can be seen from these that MSA reaches the optimum value in fewer iterations, but HS has premature convergence. Premature convergence means the solution is stuck within the local optimum.

Tables 6 and 7 show the values of objective function and the average CPU run time of each algorithm for 10 runs of the algorithms for the four-reservoir and 10-reservoir systems, respectively. It was found that MSA was able to produce the best solutions for both the four- and 10-reservoir systems. The results of 10 different runs of MSA generated an optimum solution close to 100% of the global optimum for both multi-reservoir systems. As can be seen in Table 6, the value of the objective function achieved by MSA was 16.03% and 3.86% better than that of HS and ICA, respectively. The calculation time was 638 seconds for HS, 486 seconds for ICA, and 269 seconds for MSA, indicating the outstanding speed of MSA execution. Thus, MSA ranks best in terms of the objective function production and saving calculation time. Also, according to Table 7, the
value of the objective function for the 10-reservoir system achieved by MSA was 12.5% and 6.6% better than that of HS and ICA, respectively. The calculation time of the 10-reservoir system optimization by MSA, HS and ICA were 722 seconds, 1,963 seconds, and 1,421 seconds, respectively, indicating the superior performance of MSA.

As previously described, for a better comparison of the three utilized algorithms in the optimal operation of multi-reservoir systems, seven statistical indices were employed. According to Table 8, the maximum values of the accuracy parameters $R^2$ and $d$ were obtained by MSA, with values of 0.9511 and 0.9874 for the four-reservoir system and 0.9852 and 0.9963 for the 10-reservoir system. The accuracy indices dropped significantly in HS and ICA for both multi-reservoir systems. In the same way, while the minimum values of error parameters were achieved by MSA in both the four-reservoir (RMSE = 0.5275, MAE = 0.1951, MSE = 0.3506, NMSE = 0.04993 and MAPE = 490.1845) and the 10-reservoir (RMSE = 0.5211, MAE = 0.1357, MSE = 0.4254, NMSE = 0.0150 and MAPE = 282.0782) systems, the corresponding values for HS and ICA were not satisfactory.

Figure 6(a) and 6(b) show the water release patterns for the operation of four- and 10-reservoir systems using MSA, HS and ICA. As can be seen, the operating policies obtained using MSA gave the maximum benefits with a more appropriate release pattern, so that the system does not face shortages. Regarding the higher capability of MSA in calculating near-optimal global solutions, it can be employed by water policymakers as a guide (rule curve) to schedule water releases from multi-reservoir systems in a way that gives the most benefits. It can be seen from Figure 6 that HS and ICA failed to produce reasonable results for multi-reservoir systems.
### Table 6 | Results of 10 runs of the four-reservoir system

<table>
<thead>
<tr>
<th>Number of runs</th>
<th>MSA</th>
<th></th>
<th>CPU time (s)</th>
<th></th>
<th></th>
<th>CPU time (s)</th>
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<td>246.36</td>
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<td>304.14</td>
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<td>8</td>
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Best: 308.83  264.61  306.47
Worst: 307.41  246.36  292.06
Average: 308.25  258.842  296.363
SD: 0.53  5.631914614  4.900340238
Coefficient of variation: 0.0017  0.021758117  0.016534926
Best CPU time (s): 269.71

### Table 7 | Results of the 10 runs of the 10-reservoir systems

<table>
<thead>
<tr>
<th>Number of runs</th>
<th>MSA</th>
<th></th>
<th>CPU time (s)</th>
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<th></th>
<th>CPU time (s)</th>
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<td>1,121.63</td>
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Best: 1,195.58  1,060.76  1,136.22
Worst: 1,192.10  1,024.86  1,091.23
Average: 1,194.19  1,044.325  1,115.158
Standard deviation: 1.34  12.61843823  13.82279341
Coefficient of variation: 0.0011  0.012080552  0.012395368
Best CPU time (s): 722.55

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Table 8 | Comparison of MSA, HS and ICA for optimization of multi-reservoir systems

<table>
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<th>Case study</th>
<th>Algorithm</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>MAE</th>
<th>$d$</th>
<th>MSE</th>
<th>NMSE</th>
<th>MAPE</th>
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<td>0.8442</td>
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<td>0.9087</td>
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<td>0.334</td>
<td>12,073</td>
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</table>

Figure 6 | Water release patterns of MSA, HS and ICA for (a) the four-reservoir system and (b) the 10-reservoir system.
Comparison with previous research

Table 9 compares the results of this study with those of previous studies on the optimization of four- and 10-reservoir system operation. It can be seen that MSA has the highest performance among all evolutionary algorithms. For the four-reservoir system, MSA gave the optimal value of 308.83, the closest to the global optimum. After that, GSA (Bozorg-Haddad et al. 2016), with the optimal value of 308.30, demonstrated better performance than the other algorithms. Next were MFA (Garousi-Nejad et al. 2016), hybrid GA-krill (Ehteram et al. 2017), IBA (Ahmadianfar et al. 2016), HBMO (Bozorg-Haddad et al. 2011), krill (Ehteram et al. 2017), ICA (present study), WCA (Eskandar et al. 2012; Bozorg-Haddad et al. 2015; Qader et al. 2018), FA (Garousi-Nejad et al. 2016), and HS (present study) with the optimum values of 308.21, 308.17, 308.05, 307.50, 307.26, 306.47, 306.39, 305.51 and 264.61, respectively. This confirms the superiority of MSA to the 11 other metaheuristic algorithms in the optimal operation of the four-reservoir system.

For the case of the 10-reservoir system, it was also observed that MSA was the most robust model. It achieved the optimal value of 1,195.58, close to 100% of the global optimum. After MSA, the GA-krill hybrid (Ehteram et al. 2017) solved the problem with the highest percentage of the objective function relative to the global optimal solution. The order of next algorithms in terms of achieving the best solutions IBA, GA, krill, MFA, WCA, HBMO, ICA, FA, and HS, respectively. Comparison of the results indicates the extraordinary capability of MSA in the optimization of a complex 10-reservoir system. It could efficiently reach the absolute global optimum in both multi-reservoir systems.

CONCLUSION

In this study, the capability of the recently introduced MSA for solving benchmark functions as well as the optimal operation of multi-reservoir systems was compared with two robust algorithms: ICA and HS. The results from MSA, HS and ICA were compared with those of nine other metaheuristic algorithms: GA, HBMO, GSA, IBA, FA, MFA, krill, hybrid GA-krill and WCA, which were developed by previous researchers. The results indicated the remarkable performance of MSA compared to all other algorithms in the optimal operation of four-reservoir and 10-reservoir systems. This algorithm successfully reached the absolute global optimum in both multi-reservoir systems. It also had the shortest CPU time for obtaining the optimal value. The findings of this research and the application of the same methodology for the optimal operation of multi-reservoir systems will enable decision makers to make informed choices on water development, conservation, allocation, and use in the context of growing demands for all uses and increased scarcity. In addition to the low cost, easy implementation and simple procedure of the methodology presented, it has many capabilities that make it attractive to be used by water policymakers, water resources planners, and reservoir managers.

ACKNOWLEDGEMENTS

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

REFERENCES


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