A Nelder–Mead algorithm-based inverse transient analysis for leak detection and sizing in a single pipe
Oussama Choura, Caterina Capponi, Silvia Meniconi, Sami Elaoud and Bruno Brunone

ABSTRACT
In this paper the results of an experimental validation of a technique for leak detection in polymeric pipes based on the inverse transient analysis (ITA) are presented. In the proposed ITA the Nelder–Mead algorithm is used as a calibration tool. Experimental tests have been carried out in an intact and leaky high-density polyethylene (HDPE) single pipe installed at the Water Engineering Laboratory (WEL) of the University of Perugia, Italy. Transients have been generated by the fast and complete closure of a valve placed at the downstream end section of the pipe. In the first phase of the calibration procedure, the proposed algorithm has been used to estimate both the viscoelastic parameters of a generalized Kelvin–Voigt model and the unsteady-state friction coefficient, by minimizing the difference between the numerical and experimental results. In the second phase of the procedure, the calibrated model allowed the evaluation of leak size and location with an acceptable accuracy. Precisely, in terms of leak location the relative error was smaller than 5%.

Key words | inverse transient analysis, leak detection, Nelder–Mead algorithm, unsteady-state friction, viscoelasticity

HIGHLIGHTS
- An inverse transient analysis, based on the Nelder–Mead algorithm and laboratory data, is used for viscoelastic parameter estimation of polymeric pipes.
- The same approach is followed for evaluating leak location and size.
- The effect of several parameters, such as the unsteady-state friction, the length in time of the pressure signal, and the location and number of the measurement sections is analyzed.

INTRODUCTION
Leakage plays a crucial role in the behavior of water distribution systems and transmission mains. This fault causes huge water losses (e.g., Ferrante et al. 2014b), degradation of water quality (e.g., Fontanazza et al. 2015) and important wasteful energy consumption (e.g., Colombo & Karney 2002). These consequences highly affect water companies and their customers worldwide. According to Hyman (1998), ‘some of the biggest returns can be made by the supplier reducing the water it wastes’.

The global volume of non-revenue water is estimated to be 346 million cubic meters per day or 126 billion cubic meters per year (Liemberger & Wyatt 2019), and the amount of water lost due to leaks may change depending on countries and transportation systems: from 3–7% in the well-maintained systems (Beuken et al. 2008), from 20 to...
30% in some developing countries and infrequently maintained systems (Lambert 2002). In most drinkable water systems, the problem of aging assets plays a very important role for defining proper strategies towards loss reduction (Beuken et al. 2020). In severe cases, water loss may surpass 50% in underdeveloped countries (Van Zyl & Clayton 2007; Puust et al. 2010; Zahab et al. 2016). In such countries, a successful reduction in water losses may make it possible to increase the population supplied regularly (Schoute & Halim 2010).

Leak detection has attracted the attention of both practitioners and researchers over the last half century. Within the past thirty years, the interest in developing reliable leak detection methodologies has grown considerably. With regard to transmission mains, the performance of transient test-based techniques (TTBTS) is quite encouraging. Within TTBTS, four types of approach have been proposed for the analysis of pressure signals (Colombo et al. 2009; Datta & Sarkar 2016; Xu & Karney 2017; Ayati et al. 2019): the transient reflection-based method (e.g., Brunone 1999; Liou 1998; Ferrante et al. 2014a), the transient damping-based method (Wang et al. 2002; Nixon et al. 2006; Brunone et al. 2019; Capponi et al. 2020), the frequency response-based method (e.g., Mpesha et al. 2001; Covas et al. 2005a; Lee et al. 2006; Duan et al. 2011; Wang et al. 2019; Wang et al. 2020), and the inverse transient analysis (ITA), the latter proposed by Liggett & Chen (1994). In particular, for leak detection, ITA has achieved noticeable success by researchers because of its capability to simultaneously detect leak characteristics and calibrate the unknown parameters of the system (e.g., the pressure wave speed). All the unknowns of the problems are determined by optimizing a merit function, which fits the numerically modeled pressure signals to measurements. To this aim, several optimization methods have been applied (Ayati et al. 2019). For example, it is worth mentioning the Levenberg–Marquardt method (e.g., Liggett & Chen 1994; Covas & Ramos 2010; Soares et al. 2011), genetic algorithms (e.g., Vitkovsky et al. 2000; Kim 2005; 2018; Fathi-Moghadam & Kiani 2020), the shuffled complex evolution method (Stephens et al. 2004; Lee et al. 2005), the model parsimony approach-model error compensation (Vitkovsky et al. 2007), the sequential quadratic programming method (e.g., Shamloo & Haghighi 2009), the central force optimization (Haghighi & Ramos 2012), the simulated annealing approach (Huang et al. 2015), and the particle swarm optimization, (Ranginkaman et al. 2016). A further optimization method, the Nelder–Mead algorithm, has been successfully used for leak detection by Capponi et al. (2017) in series with a genetic algorithm in a branched pipe system. This type of algorithm, suitable for multivariable functions, does not imply calculating the derivatives of the objective function. This reduces the computational time and enhances the rapidity of the algorithm compared to other derivative-free methods, as an example genetic algorithms (Pham et al. 2011).

ITA procedures have been developed both in the time (e.g., Liggett & Chen 1994; Kapelan et al. 2003; Brunone & Ferrante 2004; Covas & Ramos 2010; Haghighi & Ramos 2012) and frequency domains (e.g., Lee et al. 2005; Ranginkaman et al. 2016; Capponi et al. 2017). Moreover, traditionally, fitting is checked by considering a least-square objective function; more recently a matched-filter objective function has been proposed to produce a more robust localization in a noisy environment (Keramat et al. 2019). Within leak characterization by means of ITA, one result of all the mentioned methods is better accuracy in terms of leak localization (in many cases a percentage error smaller than the 5%) with respect to leak sizing (an error ranging between 7 and 38%, expressed as the ratio of the leak effective area to the cross-sectional area of the pipes).

In this paper, within a time-domain approach, an ITA-based model is used for leak characterization in a single polymeric pipe by considering pressure signals acquired during laboratory transient tests. The refined model takes into account both the viscoelastic behavior of the pipe material and the unsteady-state friction, with model calibration based on the Nelder–Mead algorithm. In the first stage, the model parameters were determined by considering the experimental pressure signals acquired in an intact pipe. In the second stage, the calibrated ITA solver was used to evaluate leak location and size on the basis of the experimental pressure signals measured in the leaky pipe. With the aim of improving the performance of the calibration procedure and giving an original contribution to this research field, in both stages the objective function has taken into account the pressure traces measured at five sections along the pipe beyond the one at the supply tank.
MATERIAL AND METHODS

Laboratory experiments

Experimental tests were carried out at the Water Engineering Laboratory (WEL) of the University of Perugia, Italy, on a high-density polyethylene pipe (HDPE) with a total length $L = 168.5$ m, an internal diameter $D = 93.3$ mm, and a wall thickness $e = 8.1$ mm (Figure 1). The pipe was supplied by a pressurized tank; during the tests, a pressure head equal to about 20 m was provided by the installed pump. The device simulating the leak (Figure 2) was placed at a distance $L_1 = 63.2$ m downstream of the tank; the effective area of the leak, $A_E$ (=33.6 mm²), drilled in a steel plate, was evaluated by means of steady-state tests.

Transients were generated by the complete and fast closure of a pneumatically actuated DN50 ball valve, installed at the downstream end section of the pipe.

The pressure signals, $H$, were measured by means of piezoresistive transducers with a full scale (f.s.) of 3 or 4 bar, depending on the maximum value of the pressure during the transient tests, at the tank $R$ and five measurement sections (Figure 1 and Table 1) with a sampling rate of 2,048 Hz. The measurement uncertainty was rated at $\pm 0.25\%$ of the f.s. The pre-transient discharge downstream of the leak, $Q_{d,0}$, was measured by means of an electromagnetic flow meter (EF in Figure 1), with an uncertainty of the measured discharge of $\pm 0.2\%$ of the measured value. The value of $Q_{d,0}$ was been adjusted by means of a gate valve installed just upstream of the maneuver valve.

The steady-state discharge through the leak, $Q_{L,0}$, was evaluated by means of the well-known Torricelli’s (or orifice) equation:

$$Q_{L,0} = A_E \sqrt{2gh_L}$$  \hspace{1cm} (1)

by considering the pressure head, $H_L$, measured at section C (Cassa et al. 2010; Ferrante et al. 2013, 2014a; Van Zyl 2014; Van Zyl et al. 2017; Choura et al. 2020).

Three transient tests were carried out in the leaky pipe for different values of $Q_{d,0}$ (=3.3 L/s, 2.3 L/s, 1.2 L/s) and then of $Q_{L,0}$ (=0.648 L/s, 0.659 L/s, 0.671 L/s). Values of $Q_{d,0}$ were set for the tests executed, as a reference, in the intact pipe.

In Figures 3 and 4, as an example, the acquired pressure signals were reported for $Q_{d,0} = 3.3$ L/s for the intact and leaky pipes, respectively.

In order to point out the effect of the leak on the transient response of the pipe, in Figure 5, as an example for $Q_{d,0} = 3.3$ L/s, the pressure signal measured at section V is reported for both the intact and leaky pipes. Such pressure traces confirmed that the leak induced additional damping of the pressure peaks as well as the negative pressure peaks.
waves reflected by the leak in the first pipe characteristic time (the latter highlighted by a circle), on which the transient damping method and the reflected wave method are based, respectively.

Governing equations

According to the literature (Wylie & Streeter 1993; Chaudhry 2014), the governing equations of unsteady flows in pressurized pipe systems are the momentum and continuity equations. Taking into account both unsteady friction and pipe wall viscoelasticity, these equations can be written as:

\[
gA \frac{\partial H}{\partial x} + \frac{\partial Q}{\partial t} + h_f = 0 \tag{2}
\]

\[
gA \frac{dH}{dt} + \frac{\partial Q}{\partial x} + \frac{2A}{a} \frac{\partial \varepsilon_r}{\partial x} = 0 \tag{3}
\]

where \(H\) = pressure head, \(Q\) = discharge, \(g\) = gravity acceleration, \(A\) = pipe cross-sectional area, \(h_f\) = friction term, \(a\) = pressure wave speed, \(\varepsilon_r\) = pipe wall retarded strain, \(x\) = space co-ordinate along the pipe axis, and \(t\) = time.

The friction term, \(h_f\), is given by the sum of the steady, \(h_{fs}\), and unsteady-state component, \(h_{fu}\):

\[
h_f = h_{fs} + h_{fu} \tag{4}
\]

The steady-state component, \(h_{fs}\), was evaluated using the Darcy–Weisbach equation:

\[
h_{fs} = \frac{f}{8gD} \frac{|Q|}{A^2} \tag{5}
\]

where \(f\) = friction factor; in the below simulations, as HDPE pipes can be considered as smooth pipes (Brunone & Berni 2010), \(f\) has been estimated using the Blasius formula.

The unsteady-state component, \(h_{fu}\), can be evaluated using two types of model (Ghidaoui et al. 2003): the instantaneous acceleration-based (IAB) model (Brunone et al. 1995; Bergant et al. 2001; Brunone & Golia 2008; Pezzinga 2009) and the convolution-based one (Zielke 1968; Trikha 1975; Vardy & Brown 1996; 2010; Meniconi et al. 2014). In the executed simulations, \(h_{fu}\) was simulated within the IAB approach:

\[
h_{fu} = \frac{k_{fu}}{gA} \left( \frac{\partial Q}{\partial t} - a \text{Sign}(Q) \frac{|\partial Q|}{|\partial x|} \right) \tag{6}
\]
with the decay coefficient, \( k_{tu} \), given by the Vardy & Brown (1996) formula.

The viscoelastic behavior of the pipe wall is taken into account in Equation (2) by the rate of change in time of the retarded strain, \( \varepsilon_r \). In fact, polymeric pipes exhibit an instantaneous-elastic response, \( \varepsilon_i \), and a retarded-viscous response, \( \varepsilon_r \) (Pezzinga et al. 2014).

The most popular approach in transient simulation of the behavior of polymeric pipes is the combination of conceptual and mathematical elements, i.e., springs and dashpots. Different combinations of such elements have been reported in the literature (Montgomery & MacKnight 2005): in series (Maxwell models) or in parallel (Kelvin–Voigt (KV) models). In this paper, the generalized KV model (Figure 6), is used where the creep function, \( f(t) \), can be written as:

\[
J(t) = J_0 + \sum_{i=1}^{n_{KV}} J_i \left( 1 - e^{-\frac{t}{r_i}} \right)
\]  

with \( J_0 \) = creep compliance of the first spring, \( n_{KV} \) = number of the KV elements, \( J_i \) = creep compliance (defined as \( J_i = 1/E_i \), with \( E_i \) = modulus of elasticity) of the \( i \)-th KV element spring, and \( r_i = \eta_i/E_i \), and \( \eta_i \) are the retardation time and the viscosity of the \( i \)-th KV element dashpot, respectively. In order to accurately simulate the viscoelastic behavior of the pipe (Pezzinga et al. 2016), in the following numerical investigation, five KV elements were used (i.e., \( n_{KV} = 5 \), corresponding to 11 viscoelastic parameters to be calibrated.

### 1-D Numerical model and boundary conditions

The set of partial differential equations, Equations (2) and (3), was been solved using the Method of Characteristics (MOC) (Wylie & Streeter 1993; Chaudhry 2014).

![Generalized Kelvin–Voigt model](image)

**Figure 6** Generalized Kelvin–Voigt model (from Montgomery & MacKnight 2005).

Regarding the boundary conditions, at the inlet section, according to the characteristics curve of the pump, the pressure at the tank increased slightly with time after the end of the valve complete closure. Consequently, the measured pressure signal at the pressurized tank was assumed as a datum (Meniconi et al. 2012); at the leak, Equation (1) was considered. With regard to the maneuver generating the transients, it was assumed to be linear with the closure time, \( t_c \), evaluated experimentally (\( t_c = 50, 40 \) and 35 milliseconds, for \( Q_{d,0} = 3.3 \text{ L/s}, 2.3 \text{ L/s} \) and \( 1.2 \text{ L/s} \), respectively). This behavior of \( t_c \) vs. \( Q_{d,0} \) is justified by the fact that the duration of the maneuver of the pneumatically actuated valve – driven by a compressible fluid pressurized by a constant pressure valve (about 8 bar) – depends on the inertial forces, due to fluid action on the body valve, that increases with \( Q_{d,0} \).

### The Nelder–Mead optimization method

Within the frequency-domain approach, the Nelder–Mead algorithm has been already used (in series with a genetic algorithm) for the calibration of the viscoelastic parameters (Ferrante & Capponi 2018a, 2018b) and leak detection (Capponi et al. 2017) on a branched pipe system. In this paper, the experimental pressure signals acquired in the intact single pipe at six measurement sections were used to calibrate both the viscoelastic parameters and unsteady-state friction coefficient within an ITA procedure based uniquely on the Nelder–Mead algorithm (Nelder & Mead 1965), in the time domain. Successively, the calibrated ITA solver, with the same optimization tool, was utilized to analyze the experimental pressure signals measured in the leaky pipe for leak detection.

The Nelder–Mead algorithm was used for minimizing a nonlinear function \( F \) of \( n \) variables for unconstrained multidimensional optimization cases. It compared the values of the objective function \( F \) at the \( (n + 1) \) vertices of a general simplex (polygon). Unlike classical gradient methods, the Nelder–Mead algorithm does not imply calculating derivatives. In contrast, it creates a geometric simplex and uses its movement to guide its convergence. At the \( i \)-th iteration, the algorithm starts by ordering the vertexes (points generated at the previous iteration) based on the values of the objective function, and determines the best and the worst.
vertex (the point with the highest objective function value). Afterwards, it calculates the centroid of the simplex considering all points but the worst and performs a reflection of the latter with respect to the calculated centroid. At this point, the objective function is evaluated and compared to the best vertex. If the objective function value at the new vertex is larger than the latter at the best vertex and smaller than the objective function values at the other vertices, the reflection vertex substitutes the worst one and the algorithm skips to the evaluation. Otherwise, if it is smaller than the objective function at the best vertex, the algorithm performs an expansion. Again, based on the objective function values of the two vertices (the reflected and expanded), the algorithm selects the best vertex of these two (i.e., the one with the minimum value of the objective function), affects its value to the worst vertex and skips to the evaluation. If objective function value at the reflected vertex is larger than the latter at the other points, the algorithm proceeds to contraction (Lagarias et al. 1998). Based on a comparison between the objective function values at the reflected vertex and the worst vertex, the algorithm decides which type of contraction, between the worst vertex and the centroid, to follow. If the reflected value is larger than the worst vertex, the algorithm chooses an inside contraction, otherwise, it chooses an outside contraction. Eventually, it evaluates the objective function value of the contracted vertex and compares it to the worst and reflected vertex. If the new solution is minimal, it substitutes the worst vertex with the contracted vertex, otherwise it performs a shrinkage around the best vertex. In other words, the function values at each vertex are evaluated iteratively, and the worst vertex with the largest function value is replaced by a new vertex. Otherwise, a simplex will be shrunk around the best vertex, and this process will be continued until a desired minimum is met or no further improvement of the objective function are achieved. In order to minimize properly function $F$, four scalar parameters must be specified as an input: the coefficients of reflection, $\rho$, expansion, $\chi$, contraction, $\gamma$, and shrinkage, $\sigma$; in this paper the following values were into account: $\rho = 1$, $\chi = 2$, $\gamma = 0.5$, and $\sigma = 0.5$.

For the sake of clarity, some comments are reported below about the role of constraints and boundary conditions within the used optimization procedure.

As mentioned, the Nelder–Mead optimization algorithm, implemented in MATLAB©, was used for unconstrained optimization problems. However, some constraints can be introduced by properly defining the objective function and/or setting some options for the optimization in the used code. Therefore, this type of constraints concerns only the decision variables. With regard to the boundary conditions, they were defined properly and preserved regardless of the optimization process. Moreover, there were constraints on the decision variables – the leak size must be non-negative and the location ratio must range between 0 and 1 – that can be taken into account reliably in the code. In fact, a negative value of the leak size would cause a positive reflected pressure wave that is the contrary of the leak transient response (i.e., a negative reflected pressure wave). This difference increases the error in the optimization procedure. Mathematically speaking, the curve of the leak size vs. the error would take the shape of a parabola of a directrix parallel to the leak size axis and a focus somewhere in the interval (the real value ± a small variation). This would guide the optimization process towards a value in the right interval. For this reason, there was no need for a constraint on the leak size (if this parameter leaves the interval, the object function value will increase and this disagrees with the fact that the problem is a minimization one). Regarding the leak location ratio, this parameter must be in the interval $[0, 1]$, equivalent to $[0, 100\%]$ of the total length ($0\%$ – at the reservoir, $100\%$ – at the valve). For any value outside such a range, the code will not run. In fact, the algorithm relies on testing a flawed section of the pipe as if it is a junction of pipes with a default. In MOC, sections are numbered from 1 to $N + 1$. If the flawed section is 1 or $N + 1$, the code will not run as no pipe junction exists. Starting from 2 to $N$, the algorithm splits the pipe into two pipes with a flawed junction at any section (for example, if the flawed section is 1, i.e., the leak is at the reservoir, the first part of the pipe would be of zero length whereas the second would have a length equal to $L$. For this case, and similarly for the $N + 1$ section, the algorithm gives an error). For this reason, the code has been written inside a try-catch loop.

Within the used calibration procedure, the mean squared error (MSE) was assumed as the objective function $F$ to minimize. In terms of pressure head, $H$, it can be
written:

\[ \text{MSE} = \frac{1}{N_t \times N_p} \sum_{t} \sum_{p} (H_p^m(t) - H_p(t))^2 \]  

(8)

where the superscripts ‘s’ and ‘m’ indicate the simulated and measured values, respectively, \( p \) = location (i.e., the measurement sections V, E, C, B and A), \( N_p \) = number of the measurement sections, and \( N_t \) (= \( \Delta T/\Delta t \)) is the number of the considered instances of time, \( \Delta T \) = sample length of the pressure signals, \( \Delta t \) (= \( \Delta x/a \)) is the MOC time-step, and \( \Delta x \) = space-step.

Even if it is obvious, for the sake of completeness, it needs to be pointed out that, as it will be discussed below, the accuracy and computational expense of the proposed ITA procedure is a result not only of the used Nelder–Mead optimization algorithm, but also of the characteristics of the model simulating the transient response of the system, particularly the temporal and spatial discretization criteria within MOC, the sample length of the pressure signals, the pipe material behavior and the unsteady-state energy dissipation mechanisms.

**MODEL CALIBRATION**

As mentioned, the model calibration procedure was based on the pressure signals acquired during transient tests executed in the intact pipe. Particularly, within the KV parameters calibration, three different approaches have been followed: (i) \( k_{fu} \) is assumed as a further parameter to be calibrated; (ii) \( k_{fu} \) is evaluated by the Vardy & Brown (1996) formula; and (iii) the unsteady friction is neglected (\( k_{fu} = 0 \)). The sample length of the pressure signals, \( \Delta T \), used for the calibration was set equal to 20 s as in this time interval the pressure waves generated during the executed transients dissipate significantly in the considered tank–pipe–valve system (Figure 3). Moreover, the MOC spatial discretization was fixed to \( N = 100 \) pipe elements, with \( \Delta x = 1.68 \) m. Finally, the values obtained by Covas et al. (2004, 2005b) were assumed as initial guess values of the KV parameters.

**Calibration of the KV parameters and \( k_{fu} \)**

The results of the calibration reported in Table 2 show quite close values of the KV parameters irrespective from the value of \( Q_{d,0} \). In this table, the symbol ‘–’ indicates that the values of the retardation time and the corresponding creep compliance of the \( i \)-th KV element have been disregarded. This happens when the retardation time, \( \tau_i \), exceeds the length of the sample time, which means that \( \Delta T \) is assumed as the maximum of the retardation time.

The Nelder–Mead algorithm (1965) was successful in the calibration of the creep parameters in terms of the resulting MSE and time and computational efficiency. The differences between the results of the calibrations executed considering the three tests on the intact pipe can be due to the fact that the derivative-free optimization methods are stochastic in determining the minimum and capable only of delimiting

<table>
<thead>
<tr>
<th>Discharge ( Q_{d,0} ) (L/s)</th>
<th>( J_i \times 10^{-16} \text{ Pa} )</th>
<th>( \tau_i ) (s)</th>
<th>( k_{fu} )</th>
<th>MSE (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>6.284</td>
<td>0.811</td>
<td>0.091</td>
<td>0.000b</td>
</tr>
<tr>
<td></td>
<td>( \tau_i )</td>
<td>0.128</td>
<td>0.189</td>
<td>1.054</td>
</tr>
<tr>
<td></td>
<td>( k_{fu} )</td>
<td>0.0442</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>6.416</td>
<td>0.839</td>
<td>0.018</td>
<td>0.793</td>
</tr>
<tr>
<td></td>
<td>( \tau_i )</td>
<td>0.125</td>
<td>0.638</td>
<td>1.127</td>
</tr>
<tr>
<td></td>
<td>( k_{fu} )</td>
<td>0.0129</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>6.294</td>
<td>0.708</td>
<td>0.657</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>( \tau_i )</td>
<td>0.120</td>
<td>0.574</td>
<td>0.768</td>
</tr>
<tr>
<td></td>
<td>( k_{fu} )</td>
<td>0.0536</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a J_3 = 1.614 \times 10^{-16} \text{ Pa.} \)
the closest local minima to the suggested initial guess, $IG$. Moreover, such differences can also be a natural effect of the conceptual nature of the KV model (Covas et al. 2004; Weinerowska-Bords 2007; Pezzinga et al. 2016). As a consequence, the same result (i.e., the same pressure signal) can be obtained by assuming different values of the KV parameters. In other words, from a mathematical point of view, several combinations of the decision variables (i.e., the KV parameters and $k_{fu}$) correspond to almost the same value of the objective function. Finally, the fact that $MSE$ decreases with $Q_{d,0}$ is reasonable since the larger $Q_{d,0}$, the larger the generated pressure waves and then the more reliable the ITA.

Calibration of the KV parameters with a calculated $k_{fu}$

At this stage, the calibration process includes only the KV parameters (i.e., $J_i$ and $\tau_i$) whereas $k_{fu}$ has been incorporated into the model as a calculated variable by the Vardy & Brown (1996) equation. The calibration process outcome is summarized in Table 3. With respect to the approach where both KV parameters and $k_{fu}$ were calibrated, it can be noted that in this approach $MSE$ slightly increases.

### Calibration of the KV parameters with a calculated $k_{fu}$

Within the third approach, the KV parameters are assumed as a decision variable whereas the unsteady-state friction is neglected ($k_{fu} = 0$). The results of such a calibration procedure are shown in Table 4 and in Figure 7, where, as an example for $Q_{d,0} = 1.2$ L/s, the calibrated pressure signals at the measurement sections along the pipe are compared with the experimental ones. The very good performance of the model when unsteady-state friction is neglected – the experimental and calibrated pressure signals are almost indistinguishable – confirms the negligible relevance of $h_{fu}$ in polymeric pipes pointed out in Duan et al. (2010).

#### Table 3 | Calibration results with $k_{fu}$ calculated

<table>
<thead>
<tr>
<th>Discharge $Q_{d,0}$ (L/s)</th>
<th>$J_i$ (10^-10 Pa), $\tau_i$ (s)</th>
<th>$MSE$ (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l = 0$</td>
<td>$l = 1$</td>
</tr>
<tr>
<td>3.3</td>
<td>$J_i$</td>
<td>6.200</td>
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<tr>
<td></td>
<td>$\tau_i$</td>
<td>0.117</td>
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<td>2.3</td>
<td>$J_i$</td>
<td>6.233</td>
</tr>
<tr>
<td></td>
<td>$\tau_i$</td>
<td>0.125</td>
</tr>
<tr>
<td>1.2</td>
<td>$J_i$</td>
<td>6.168</td>
</tr>
<tr>
<td></td>
<td>$\tau_i$</td>
<td>0.139</td>
</tr>
</tbody>
</table>

$^aJ_4 = 0.904 \times 10^{-16}$ Pa.

$^bJ_4 = 0.494 \times 10^{-15}$ Pa.

#### Table 4 | Calibration results when the unsteady-state friction is neglected ($k_{fu} = 0$)

<table>
<thead>
<tr>
<th>Discharge $Q_{d,0}$ (L/s)</th>
<th>$J_i$ (10^-10 Pa), $\tau_i$ (s)</th>
<th>$MSE$ (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>$l = 1$</td>
</tr>
<tr>
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<td>6.382</td>
</tr>
<tr>
<td></td>
<td>$\tau_i$</td>
<td>0.115</td>
</tr>
<tr>
<td>2.3</td>
<td>$J_i$</td>
<td>6.438</td>
</tr>
<tr>
<td></td>
<td>$\tau_i$</td>
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<tr>
<td>1.2</td>
<td>$J_i$</td>
<td>5.411</td>
</tr>
<tr>
<td></td>
<td>$\tau_i$</td>
<td>0.007</td>
</tr>
</tbody>
</table>

$^aJ_4 = 1.273 \times 10^{-16}$ Pa.

$^bJ_4 = 0.645 \times 10^{-15}$ Pa.
According to the KV approach, the pressure wave speed, $a$, depends on the value of the creep compliance of the first spring, $J_0$, through the one of the Young modulus of elasticity. As a consequence, different values of $J_0$, resulting from different methods of calibration, imply different values of $a$. It is worth pointing out that within the above calibration procedures the value of $a$ did not change significantly and no clear trend of $a$ with a given calibration procedure was identified. The obtained values of $a$, ranging from 368.9 m/s to 399.9 m/s, were compatible not only with the mechanical characteristics of the installed pipes but also with the measured travel times of the pressure waves.

**LEAK LOCATION AND SIZING**

As for the procedure followed for the calibration of the viscoelastic parameters and unsteady-state friction coefficient, in the ITA procedure for leak characterization, the leak size and location were calibrated by minimizing the MSE given by Equation (8). Moreover, the performance of the algorithm was evaluated by means of the relative mean errors:

$$\varepsilon_L = \left| \frac{L_s - L_1}{L_1} \right| \times 100$$  \hspace{1cm} (9a)

$$\varepsilon_S = \left| \frac{A_s - A_E}{A_E} \right| \times 100$$  \hspace{1cm} (9b)

with the subscripts $L$ and $s$ indicating the leak location and size, respectively. Within such an ITA procedure, for accuracy enhancement purposes, the number of pipe elements doubled ($N = 200$), with $\Delta x = 0.84$ m.

In the below subsections, the role of the initial guesses, length of the sampling time, and considered measurement sections is examined, as factors influencing the performance of the proposed procedure.

**The role of the initial guesses, IG**

As a preliminary step, a series of direct simulations was executed by means of MOC for evaluating MSE as a two-variables function of the leak location and size, within the intervals $0\%L$–$100\%L$ and $0$–$100$ mm$^2$, respectively. In the carried-out simulations, the steps were 0.1% for the leak dimensionless location and 0.1 mm$^2$ for its size. As an example, the case of $Q_{d,0} = 3.3$ L/s with $k_{fu}$ calculated is discussed below. As a result of the numerical simulations, the values associated with the global minimum of MSE (indicated by the square in Figure 8) were $L_1 = 64.87$ m and $A_1 = 34.7$ mm$^2$. The associated relative errors with respect to the exact values (a cross in Figure 8) were $\varepsilon_L = 0.78\%$ and $\varepsilon_S = 3.27\%$, respectively. Such values, i.e., the global minimum and the exact ones, allow an estimation of the performance of the ITA procedure based on the Nelder–Mead algorithm.

Because of the mentioned sensitivity of the results to initial guesses, several $IG$ for leak location have been considered, whereas for leak size it was assumed that $IG = 0$.

In the ITA procedure, firstly, the pipe length was divided into 11 equidistant nodes and the algorithm evaluates MSE for all nodes. Then, the two nodes with the lowest MSE values were selected, the in-between value is chosen and the optimization procedure for the leak parameters...
estimation restarts. In Figure 8, the up-pointing triangle symbol corresponds to the local minima of \( \text{MSE} \) given by the optimization algorithm for \( IG = \) 30\% (\( \text{MSE} = 0.32 \)). In the same figure, the down-pointing triangle symbol indicates the local minima for \( IG = \) 40\% (\( \text{MSE} = 0.31 \)). The relative mean errors associated with \( IG = \) 30\% (40\%) for the leak location and size are \( \epsilon_L = 20.3\% \) (12.8\%) and \( \epsilon_S = 82.85\% \) (18.59\%) that correspond to \( L_1 = 50.7 \) m (71.4 m) and \( A_E = 61.44 \) mm\(^2\) (27.35 mm\(^2\)), respectively.

The role of the length of the sampling time, \( \Delta T \)

To analyze the relevance of \( \Delta T \), four sample time lengths were considered (\( \Delta T = 20, 10, 5 \), and 2 s). For \( Q_{d,0} = 3.3 \) L/s, \( IG = 35\% \), and with \( k_{fu} \) assumed as a parameter in the calibration, the results (Table 5) point out the crucial role of \( \Delta T \). Precisely, the smaller the \( \Delta T \), the smaller the \( \epsilon_L \). This behavior is due to the fact that the effect of the leak location on the pressure signals is very visible in the first characteristic period. In contrast, the smaller the \( \Delta T \), the larger the \( \epsilon_S \) as the important effect of the leak size on the damping of pressure peaks (Capponi et al. 2020). Moreover, these simulations confirm that \( \epsilon_S \) is significantly larger that \( \epsilon_L \).

The role of the considered measurement sections

Because of the poor accessibility of pipe systems, particularly transmission mains, it is of interest to analyze the effect of the number and location of the measurement sections on the accuracy in the leak parameters estimation. The results of the executed numerical simulations in which different measurement sections were considered (Table 6) point out that, when a single section is taken into account, the closer the measurement section to the transient source the better the results of the ITA procedure for leak location. Such a behavior confirms the results reported in Wang (2021). Moreover, the use of a combination of

### Table 5

<table>
<thead>
<tr>
<th>( \Delta T ) (s)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_S ) (%)</td>
<td>38.10</td>
<td>38.09</td>
<td>37.93</td>
<td>22.00</td>
</tr>
<tr>
<td>( \epsilon_L ) (%)</td>
<td>0.49</td>
<td>0.49</td>
<td>0.49</td>
<td>3.40</td>
</tr>
<tr>
<td>\text{MSE} \ (m^2)</td>
<td>0.61</td>
<td>0.71</td>
<td>0.54</td>
<td>0.29</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Number of sections</th>
<th>Notation</th>
<th>( \epsilon_S ) (%)</th>
<th>( \epsilon_L ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 A(^a)</td>
<td>3.89</td>
<td>46.20</td>
<td></td>
</tr>
<tr>
<td>1 B(^a)</td>
<td>3.50</td>
<td>18.68</td>
<td></td>
</tr>
<tr>
<td>1 C(^a)</td>
<td>3.40</td>
<td>44.95</td>
<td></td>
</tr>
<tr>
<td>1 E(^b)</td>
<td>3.40</td>
<td>23.67</td>
<td></td>
</tr>
<tr>
<td>1 V(^b)</td>
<td>0.02</td>
<td>27.06</td>
<td></td>
</tr>
<tr>
<td>2 (E, V)(^h)</td>
<td>3.40</td>
<td>21.41</td>
<td></td>
</tr>
<tr>
<td>2 (C, E)(^h)</td>
<td>3.40</td>
<td>23.76</td>
<td></td>
</tr>
<tr>
<td>2 (B, C)(^h)</td>
<td>3.40</td>
<td>21.41</td>
<td></td>
</tr>
<tr>
<td>3 (C, E, V)</td>
<td>3.40</td>
<td>21.41</td>
<td></td>
</tr>
<tr>
<td>3 (A, B, C)(^h)</td>
<td>5.34</td>
<td>35.75</td>
<td></td>
</tr>
<tr>
<td>3 (A, C, V)</td>
<td>3.40</td>
<td>20.21</td>
<td></td>
</tr>
<tr>
<td>5 All</td>
<td>3.40</td>
<td>22.00</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Section between the tank and the leak.  
\(^h\)Section between the leak and the maneuver valve.  
\(^b\)Section close to the leak.
measurement sections (i.e., two, three or the five sections upstream or downstream of the leak the leak or close to the transient source) do not give rise to any enhancement in locating the leak (Table 6), but it improves the leak sizing. This result, quite different with respect to Wang’s outcomes, merits further analysis by considering different pipe materials and system layouts. As for the above numerical experiments, values reported in Table 6 point out that $\varepsilon_S \gg \varepsilon_L$.

CONCLUSIONS

An ITA procedure for model calibration in a polymeric pipe test facility at the WEL of the University of Perugia is presented. A five-elements generalized KV model was chosen to characterize the pipe wall behavior during transients. The Nelder–Mead algorithm was used for minimization of the chosen objective function for three values of the discharge. The proposed calibration procedure allowed estimation of 11 viscoelastic parameters and the unsteady-state friction decay coefficient by considering pressure signals acquired at six measurement sections in an intact pipe. Successively, the calibrated model was used for leak detection (location and size) on the basis of the pressure signals measured in a leaky pipe. The role of the initial guesses, length of the sample period and characteristics of the considered measurement sections on the accuracy of the procedure were investigated.

As shown, the proposed ITA procedure was successful in localizing the leak with acceptable accuracy; in contrast, as found for most of the procedures available in the literature, the performance in terms of leak sizing was quite worse. The analysis of this different behavior for the ITA procedure in terms of leak location with respect to leak sizing merits more in-depth analysis with further numerical and experimental tests. As preliminary working hypotheses, the ambiguity of some features of the pressure signal as a mark of a given leak at a given location, pointed out in Brunone et al. (2019) and Capponi et al. (2020), and the role of the pipe material, as conjectured in Meniconi et al. (2013), could be considered as possible causes of the poor performance in leak sizing. In contrast, the good match between simulated and real leak location depends on the fact that this sort of feature is obtained based on the travel time of the pressure wave reflected by the leak, that is quite easy to capture.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

REFERENCES


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