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Estimating output flow depth from Rockfill Porous media

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ABSTRACT

To analyze the flow in a rockfill porous media using the Gradually Varied Flows theory (one-dimensional flow analysis) and solving the Parkin equation (two-dimensional flow analysis), calculation of the output flow depth as the downstream boundary condition is of great importance. In most previous studies, the output flow depth has been considered equal to the critical depth. In the rockfill porous media, unlike free surface channels, the fluid weight is exerted to the aggregates in addition to the flow, and therefore, the output flow depth from the rockfill is always greater than the critical depth (flow leaves the rockfill with a specific energy greater than the critical energy), and is expressed as a coefficient (Γ) of the critical depth. In the present study, using dimensional analysis and particle swarm optimization (PSO) algorithm and experimental data in different conditions (a total of 178 experimental data for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, sandy natural materials), an equation was presented to calculate the mentioned coefficient as a function of the physical characteristics of the rockfill porous media as well as the flow that can be used for all experimental conditions with high accuracy. If the output flow depth is considered to be equal to the critical depth, the mean relative error (MRE) in terms of using the experimental data of the mentioned materials separately and for the data of all the mentioned materials together was equal to 84.40, 83.81, 60.62, 67.68, 74.82 and 69.96%, respectively. In the case of using the proposed equation in the present study, the corresponding values of 5.49, 4.72, 6.24, 4.41, 6.42 and 8.99% were calculated, respectively.

Key words: critical depth, dimensional analysis, output flow depth, particle swarm optimization (PSO) algorithm, rockfill porous media, steady flow

HIGHLIGHTS

- Presenting an approach to increase the accuracy of one-dimensional and two-dimensional analysis of steady flow in coarse-grained porous media.
- Using the dimensional analysis, particle swarm optimization (PSO) algorithm and experimental data in different conditions.
- Presenting an equation applicable to all porous media with different conditions to calculate the output flow depth.

1. INTRODUCTION

Coarse-grained gravel (rockfill material) has numerous applications in engineering including filtration, gabion construction, channel lining, stilling basins, ponds, and cobble stone dams as well as flood control. In the fine-grained media, there is a laminar flow with a linear relation between hydraulic gradient and flow velocity so that the flow follows Darcy's law (Equation (1)) (McWhorter & Sunada 1977). However, in the coarse-grained media, due to the presence of voids, flow velocity is high with a tendency to turbulent flow formation (Hansen *et al.* 1995), and there is a nonlinear relation between hydraulic gradient and flow velocity and flow follows non-Darcy law. Hydraulic gradient equations in the non-Darcy media considering steady flow condition are classified into two groups of power and binomial equations, according to Equations (2) and (3) (Forchheimer 1901; Leps 1973; Stephenson 1979).

$$i = \left(\frac{1}{k}\right)V \quad (1)$$

$$i = mV^n \tag{2}$$

$$i = aV + bV^2 \tag{3}$$

where V is flow velocity (m/s), i is hydraulic gradient, m and n are values dependent on the properties of the porous media, fluid and flow, while a and b are coefficients dependent on the properties of the porous media as well as the fluid.

The binomial equation was proved by dimensional analysis (Ward 1964) and by the Navier-Stokes equations (Ahmed & Sunada 1969) and is more accurate and efficient in comparison to the exponential equation (Leps 1973; Stephenson 1979).

SedghiAsl & Rahimi (2011) applied the Darcy-Weisbach friction loss within pipes and combined it with the Manning equation to study flow in the coarse-grained porous media. They proposed a new equation and calibrated the coefficients based on the performed experiments considering the rockfill and flow characteristics. Sadeghian *et al.* (2013) studied the parallel and convergent nonlinear flows through coarse-grained porous media and concluded that both the parallel and convergent flows are non-Darcy flows for which the binomial and exponential equations are still valid. Salahi *et al.* (2015) conducted a series of experiments on the pressure columns in the rockfill media to study different hydraulic gradient equations of the non-Darcy flows. They concluded that there is a nonlinear relationship between hydraulic gradient and pore velocity of flow and proposed an empirical equation between the hydraulic gradient and pore velocity. Sedghi-Asl & Ansari (2016) analytically solved free flows by the Dupuit-Forchheimer assumptions and presented dimensionless results. In addition, they compared the results with a series of experimental data and concluded that the analytical solution is capable of calculating the flow profile inside the rockfill media. Many authors investigated flow through rockfill porous media (e.g., Sidiropoulou *et al.* 2007; Di Nucci 2018). A semi-analytical solution of the nonlinear differential equations constructing a fully saturated porous media was presented by Abbas *et al.* (2021). The effect of different factors on particle transport in porous media (Zhang *et al.* 2016; Cui *et al.* 2017) and solute transport in porous media (Rolston 2007; Agarwal & Sharma 2020) has been investigated. Ostad-Ali-Askari *et al.* (2019) investigated the effect of climate change on groundwater resources in semiarid and arid areas and have shown adverse effects on groundwater recharge and water level. In this study, climate conditions were predicted for the future period under the emission scenarios of RCP2.6, RCP4.5, and RCP8.5, for an aquifer using MODFLOW. Hu *et al.* (2019) Kriging-approximation simulated annealing (KASA) optimization algorithm has been used to optimize flow parameters in the porous media. The particle swarm optimization (PSO) algorithm is a population-based evolutionary algorithm and is used in civil engineering and water resources optimization problems such as reservoir performance (Nagesh Kumar & Janga Reddy 2007), water quality management (Lu *et al.* 2002; Chau 2005; Afshar *et al.* 2011), optimization of the Muskingum method coefficients (Chu & Chang 2009; Moghaddam *et al.* 2016; Bazargan & Norouzi 2018; Norouzi & Bazargan 2020; Norouzi & Bazargan 2021) and optimization of the parameters of the porous media equations (Norouzi *et al.* 2021; Safarian *et al.* 2021). Therefore, in the present study, the Particle Swarm Optimization (PSO) algorithm was used to optimize the coefficients of the proposed equation.

There are two general methods to analyze steady–non-Darcy flow through rockfill materials:

a. One-dimensional analysis of flow, using gradually-varied flow theory:

Application of this theory in simulation of flow through rockfill materials was investigated by Bari & Hansen (2002) and then by other researchers, such as Bazargan & Shoaei (2006). The results showed that at the end-points of the formed media, particularly at high discharges, there was a significant difference between the observational and computational depth results. Hence, in this method, computations of the longitudinal profile of the water surface are not accurate enough because of the lack of output flow depth as the downstream boundary condition.

b. Two-dimensional analysis of flow, using the Parkin equation:

Parkin, 1969, combined the continuity equation with the exponential function of velocity and hydraulic gradient for the first time and developed an equation as an alternative to Laplace equation. This equation could be solved by knowing the boundary conditions as well as porosity (Arbhabhirama & Dinoy 1973). In other word, without knowing output flow depth as the downstream boundary condition, and the water surface profile as a boundary condition which depends on the output flow depth, accurate solution of the Parkin equation is practically impossible.

Using gradually-varied flow theory in analysis of non-Darcy steady flows is an acceptable engineering method that involves lower computations compared to the Parkin equation and solution of a two-dimensional problem (Bazargan & Shoaei 2006).

In general, the leakage line profile in rockfill materials is important, because:

1. Output leakage level downstream of the rockfill structures and equivalent drainage is immersed in a particular discharge. If the computed output water level considering the maximum discharge is low, the erosive power of aggregates and, consequently, the possibility of destruction of the downstream slope of the structures would be high.
2. The observed leakage level downstream of the rockfill structures is used as a boundary condition in computation of pore pressure in the rockfill dams and computation of hydraulic conductivity (Hansen 1992).

Stephenson (1979) considered the output flow depth through rockfill drainage equal to the critical depth in case of steady flow (Equation (4)).

$$y_c = \sqrt[3]{\frac{q^2}{n^2 g}} \tag{4}$$

where y_c = critical depth, q = discharge in unit width of the rockfill, n = porosity of materials, and g = gravity acceleration.

However, experimental tests conducted by other researchers proved that Equation (4) is inaccurate. In this regard, Sedghi-Asl *et al.* (2010) corrected the equation based on the experimental data by applying Γ coefficient (Equation (5)).

$$y_e = \Gamma \sqrt[3]{\frac{q^2}{n^2 g}} \tag{5}$$

In the previous studies, values of Γ were calibrated by different researchers using experimental data. Using such data, Sedghi-Asl *et al.* (2010) obtained the value of Γ coefficient for the angular and rounded materials as 2.3 and 2.4, respectively. There is no doubt that Γ value for open channels that lack rockfill materials is equal to 1. In other words, the difference between values of Γ is due to the fact that in the angular materials, porosity of media is more than for rounded ones and is closer to 1, which is the same as the porosity of open channels (Sedghi-Asl *et al.* 2010). In another study, Chabokpour & Tokaldani (2018) used their own experimental data and obtained the value of Γ coefficient for a length of 100 cm as 1.83 and 2.05 for aggregate diameters of 16 and 30 mm, respectively; and for a length of 193 cm as 1.58 and 1.84, respectively considering the same aggregate diameters.

In previous studies, coefficient Γ values were presented as numbers that were only accurate enough for the same experimental conditions, and could not be used for different experimental conditions (by changing experimental conditions, the value of the Γ coefficient also changed). In other words, in previous studies, the coefficient Γ value was not presented considering the physical characteristics of the aggregates and the flow characteristics and using dimensional analysis so that it can be used for all experimental conditions. For this reason, in the present study, using dimensional analysis, the particle swarm optimization (PSO) algorithm and experimental data in different conditions (a total of 178 experimental data for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, sandy natural materials), an equation was presented to calculate the coefficient Γ considering the physical characteristics of the rockfill as well as the flow that can be used for all experimental conditions with high accuracy. In other words, using the equation presented in the present study, it is possible to calculate the value of the Γ coefficient and consequently, the output flow depth from the rockfill porous media in different conditions with high accuracy.

2. MATERIALS AND METHODS

2.1. Experimental data

In the present study, in order to conduct the tests, authors designed and installed a steep flume with a square cross-section (width and height of 0.3 m, length of 5 m) in the Technical Faculty of Zanjan Universtiy (Figure 1). Flow discharge was measured using spillovers (rectangular sharp-edged spillways.) The bottom and walls of the flume were made of Plexiglas, which allows the flow to be observed. To record the longitudinal profile of pressure, 23 piezometers were installed on the

Figure 1 | The channel used for the experiments.

bottom with an installation distance of 10 cm. To develop a rockfill porous media, two porous plastic-made sidewalls were installed at both sides of the porous media. Upstream and downstream faces of this container were orthogonal, keeping fixed by sidewalls during the experiments. A schematic view of the channel is shown in Figure 2.

Sieving method was used to make aggregates that are reasonably uniform. Then, in order to obtain a grading curve, ASTM D422-63 and AASHTO T88-70 practices in sampling were adopted. Table 1 presents characteristics of the rockfill materials that were used in this study for 8 samples of rounded and crashed shapes.

In Table 1, uniformity coefficient (C_u) corresponds to $\frac{D_{60}}{D_{10}}$ and curvature coefficient (C_c) corresponds to $\frac{(D_{30})^2}{D_{10}*D_{60}}$.

Figure 3 shows changes in depth of the input and output flows versus flow discharge for rounded and crashed materials.

In addition, the data registered by Eshkou (2008) in the Laboratory of Amirkabir University, Tehran, Iran, were used in these experiments (Figure 4). Experiments were conducted inside a horizontal flume with a width, height and length of 0.5, 0.75 and 6 meters, respectively. The sidewalls were made of glass; in the middle of the flume, the physical model was developed by putting together single-sized glass balls (marbles) (0.035 diameters) with cubic (porosity of 0.526) and rhomboid structures (porosity of 0.42), or by putting together sandy single-sized natural materials (diameter of 0.011, porosity of 0.42).

Figure 5 shows changes in depth of the input and output flows versus flow discharge for glass artifacts with different structures (porosities) and sandy natural materials.

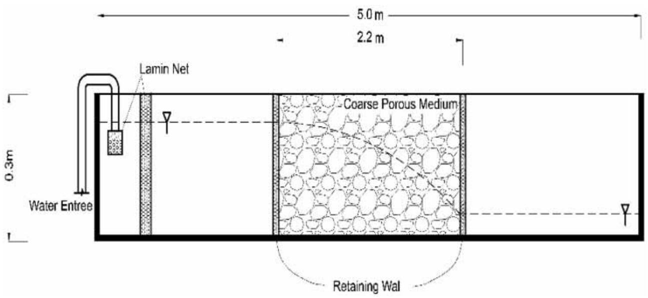

Figure 2 | Longitudinal profile of the experimental model.

Table 1 | Characteristics of the experimental materials

Materials	d_0 (mm)	d_{50} (mm)	d_{100} (mm)	C_u	C_c	Porosity
Rounded	3.35	7	12.7	1.6	0.87	0.368
	3.35	9	25.4	2	0.81	0.396
	6.35	17	50.8	2	0.85	0.362
	12.7	26	50.8	2	0.99	0.401
Crashed	3.35	7	12.7	1.6	0.9	0.432
	3.35	9	25.4	2	0.89	0.396
	6.35	17	50.8	2	0.77	0.454
	12.7	26	50.8	1.6	0.99	0.496

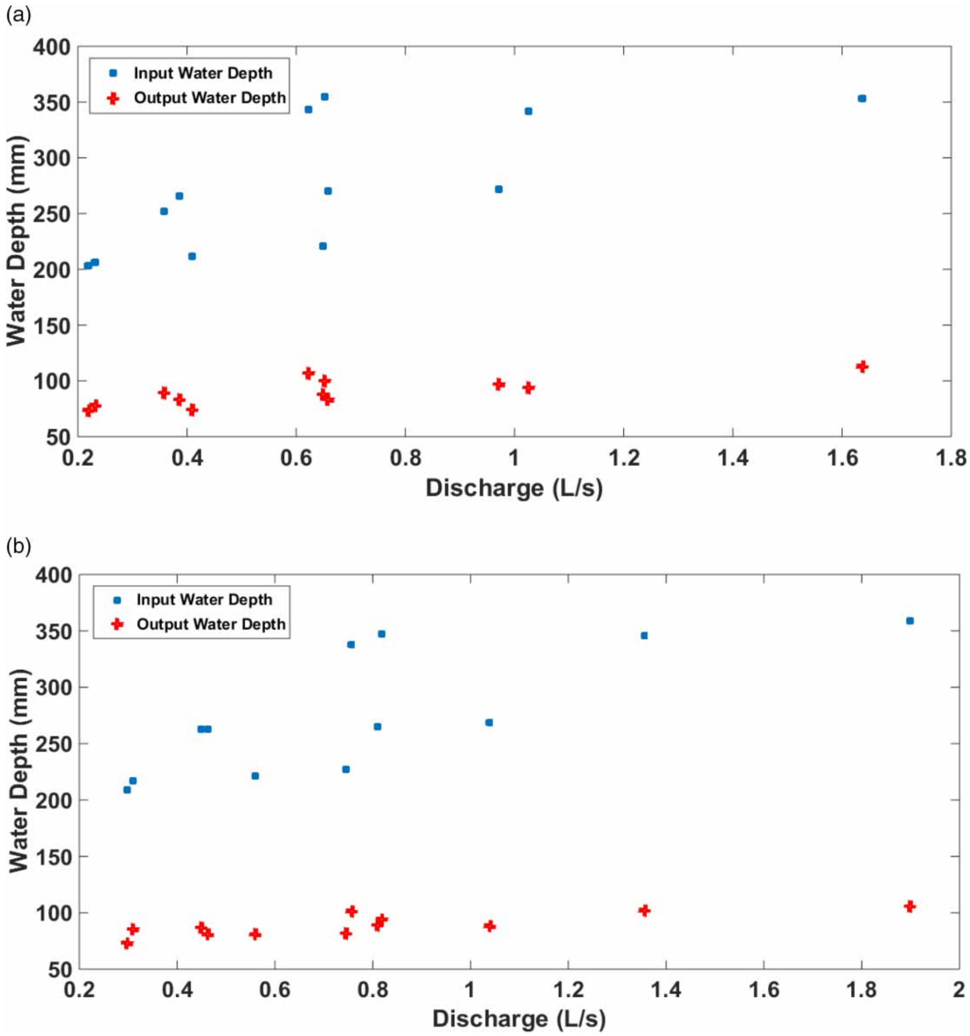

Figure 3 | Depth of the observational input and output flows (experimental) versus flow discharge in rounded and crashed materials. (a) Rounded materials. (b) Crashed materials.

Figure 4 | Experimental equipment (Eshkou 2008).

In general, the data used in the present study (experimental data of steady flow passing through coarse-grained porous media) include the following:

(1) Experimental data of steady flow through Rounded and Crashed materials that were recorded by the authors of the present study in the Laboratory of Zanjan University, Zanjan, Iran.

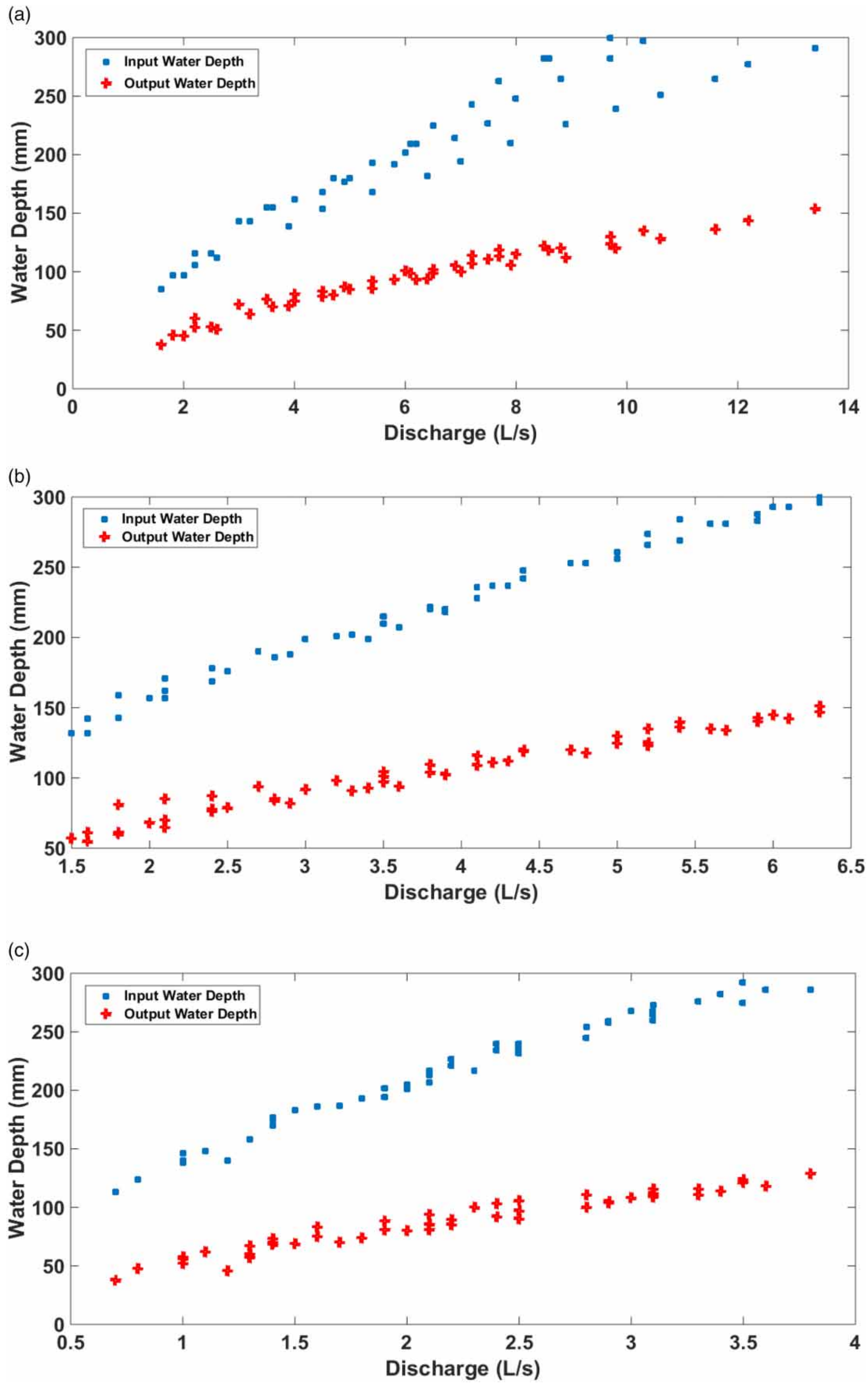

Figure 5 | Changes in depth of the observational input and output flows (experimental) versus flow discharge for artificial and natural materials. (a) Glass artificial materials with cubic structure. (b) Glass artificial materials with rhomboid structure. (c) Sandy natural materials.

(2) Experimental data of other researchers that were recorded in the Laboratory of Amirkabir University, Tehran, Iran, considering steady flow condition through Glass artificial materials with cubic structure, Glass artificial materials with rhomboid structure and Sandy natural materials.

It is worth noting that due to the difference in input and output flow depths (Figures 3 and 5), there is developed flow through the coarse-grained porous media. In addition, for the following reasons, the flow depth across the channel width has not changed.

(1) The side walls of the channel were made of Plexiglas and using photography, changes in the flow depth on both side walls of the channel were recorded and, upon examination, a similar water surface profile was observed.
(2) To study the changes in the piezometric head across the channel width, two piezometers were used. The values recorded by both piezometers were almost the same.

2.2. Particle swarm optimization (PSO) algorithm

This algorithm was first developed and introduced by Ebrahart & Kennedy (1995). PSO is a population-based searching algorithm that is inspired by nature, like the Genetic Algorithm (GA), Ant Colony and Artificial Bee Colony, and is developed based on the collective intelligence and social behavior of bird flocking or fish schooling. The advantages of this algorithm include simplicity of the structure and implementation, small number of controllable parameters, high convergence speed and high computational efficiency. The basic idea of PSO is based on the assumption that potential solutions are flown through hyperspace with acceleration towards more optimum solutions. Each particle adjusts its flying according to the experiences of both itself and its companions. During the process, the overall best value attained by all the particles within the group and the coordinates of each element in hyperspace associated with its previous best fitness solution are recorded in the memory (Chau 2007; Nagesh Kumar & Janga Reddy 2007).

Efficiency, high convergence speed and proper accuracy of the PSO algorithm have been examined and approved in previous studies. Therefore, the PSO algorithm was selected to optimize the coefficients of the proposed equation. The details of PSO can be obtained elsewhere (Clerc & Kennedy 2002; Gurarslan & Karahan 2011; Karahan 2012; Di Cesare *et al.* 2015).

To evaluate the optimum values of the coefficients of the proposed equation, minimization of the Mean Relative Error (MRE), which is defined using Equation (6), was used as the objective function in the Particle Swarm Optimization (PSO) algorithm.

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - Y_i}{y_i}\right| * 100 \tag{6}$$

where y_i and Y_i are the observed and calculated values of the output flow depth. The flowchart used in the present study is presented in Figure 6.

A description of the Particle Swarm Optimization (PSO) algorithm is provided in the appendix.

3. RESULTS AND DISCUSSION

For one-dimensional and two-dimensional analysis of steady flow in a coarse-grained porous media, calculation of the output flow depth as the downstream boundary condition is of utmost importance. In the present study, using dimensional analysis, particle swarm optimization (PSO) algorithm and experimental data in different conditions, an equation was presented to calculate the Γ coefficient in Equation (5), which is highly accurate for all experimental conditions.

In general, the present study consists of the following stages:

(1) According to Stephenson (1979) (the output flow depth from the coarse-grained porous media is equal to the critical depth), the recorded output flow depth from the porous media under different experimental conditions (a total of 178 data recorded under different experimental conditions) was compared with the critical depth (Equation (4)).
(2) Since the output flow depth from the coarse-grained porous media is very different from the critical depth, and according to Equation (5) (the output flow depth from the coarse-grained porous media is as a coefficient of the critical depth and in previous studies, the coefficient was presented as numbers that were only accurate enough for that experimental condition) and studies performed, the Γ coefficient is a function of the average particle size (d_{50}), the length of the rockfill

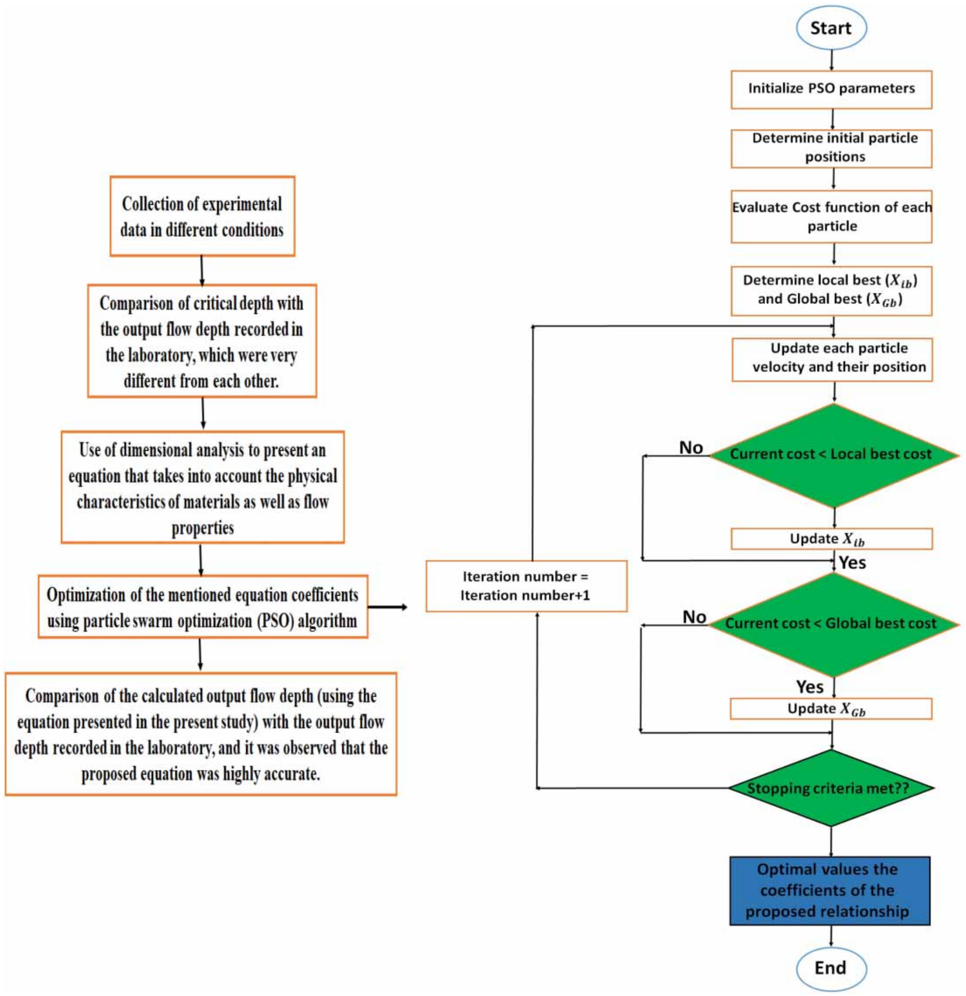

Figure 6 | Flowchart of the present study stages.

perimeter (L), flow discharge (q), porosity (n), upstream depth (Y_{up}) and critical depth (Y_c). For this reason, in the present study, using dimensional analysis, PSO algorithm and experimental data in different conditions, an equation was presented to calculate the Γ coefficient with high accuracy and efficiency in all experimental conditions.

It is worth noting that in both Sections 1 and 2, the calculated output flow depth was compared with the recorded one in the experimental condition for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, and sandy natural materials first separately and, then, for all materials together.

According to the equation developed by Stephenson (1979), output flow depth from rockfill porous media in steady flow condition is equal to the critical depth (Equation (4)). Figure 7 shows changes in depth of the observational output flow and critical depth versus flow discharge for all experimental data.

As can be seen from Figure 7, there is a significant difference between the observational output flow depth and critical depth. Values of MRE for rounded, crashed, glass artificial materials with cubic and rhomboid structure, as well as sandy natural materials are presented in Table 2.

According to Table 2, if the output flow depth from the coarse-grained porous media is considered to be equal to the critical depth, the mean relative error (MRE) for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, and sandy natural materials was calculated as 84.40, 83.81, 60.62, 67.68 and 74.82%, respectively, and 69.96% for all experimental data together. As such, output flow depth through rockfill porous media is not equal to the critical depth.

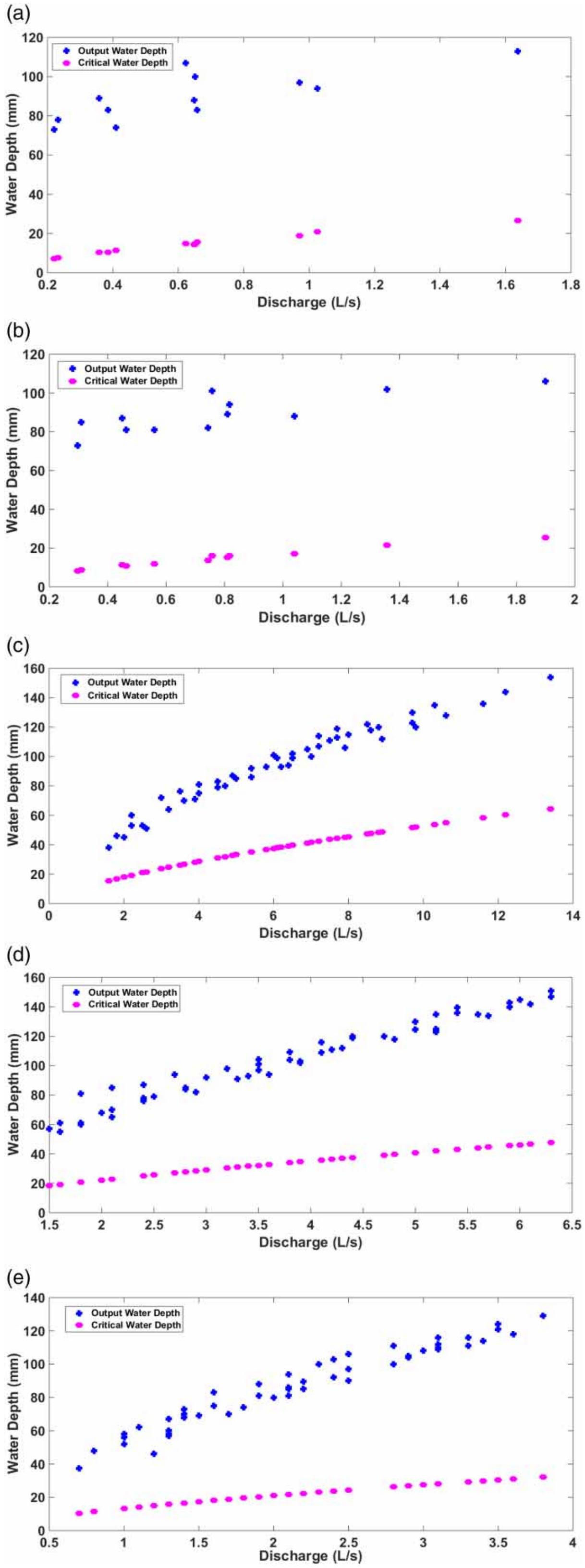

Figure 7 | Changes in depth of the observational output flow and critical depth versus flow discharge. (a) Rounded materials. (b) Crashed materials. (c) Glass artificial materials (cubic structure). (d) Glass artificial materials (rhomboid structure). (e) Sandy natural materials.

Table 2 | Values of MRE between the observational output flow depth and critical depth for different materials

Materials	Number of tests	MRE %
Rounded	12	84.40
Crashed	12	83.81
Glass artificial (cubic structure)	50	60.62
Glass artificial (rhomboid structure)	53	67.68
Sandy natural materials	51	74.82
All materials	178	69.96

Studies performed by SedghiAsl *et al.* (2010), Chabokpour & Tokaldani (2018) suggested that output flow depth through rockfill media has significant difference with the critical depth (Equation (4), developed by Stephenson 1979). Therefore, the modified Γ was multiplied by the critical depth in SedghiAsl *et al.* (2010). Investigations have shown that the mentioned coefficient is a function of particle size (d_{50}), length of the rockfill media (L), flow discharge (q), porosity (n), upstream depth (Y_{up}) and critical depth (Y_c). In other words, the coefficient may be rewritten as:

$$\Gamma = f(d_{50}, L, q, n, Y_{up}, Y_c) \tag{7}$$

Then, Equation (8) was presented to calculate Γ using dimensional analysis and experimental data (a total of 178 experimental data).

$$\Gamma = J*\left(\frac{d_{50}}{Y_c}\right) + B*\left(\frac{Y_c}{L}\right) + H*\left(\frac{gd_{50}Y_{up}^2 n^{\frac{2}{3}}}{q^2}\right) + N*\left(\frac{Y_{up}n^{\frac{1}{3}}}{d_{50}}\right) \tag{8}$$

To optimize the coefficients of Equation (8) (J, B, H, N), the Particle Swarm Optimization (PSO) algorithm and Mean Relative Error (MRE) were adopted as objective functions (Equation (6)). As such, the above-mentioned coefficients were optimized in such a way that the computational and observational output flow depths result in minimum possible MREs.

The optimized values of J, B, H, N and values of MRE, which were obtained by using the proposed equation in the present study (Equation (5) in which Γ is computed using Equation (8)), are presented in Table 3 both separately for each material category and all materials as a whole (or all experimental data).

Unlike open channels, flow leaves coarse-grained porous media with a specific energy greater than the critical energy and, consequently, with a greater depth than the critical depth. In order to accurately calculate the output flow depth from the coarse-grained porous media, the Γ coefficient must be multiplied by Equation (4), according to Equation (5). In previous studies, the value of the Γ coefficient has been calculated as numbers that were accurate only for that special experimental conditions. While in the present study, using dimensional analysis, the PSO algorithm, and experimental data in different conditions, an equation was presented to calculate the Γ coefficient with high accuracy and efficiency for all experimental

Table 3 | Optimized values of the coefficients presented in the present study and MRE of the computational and observational output flow depths

Materials	Number of tests	J	B	H	N	MRE %
Rounded	12	1.542	8.339	0.0017	0.138	5.49
Crashed	12	0.852	7.874	0.0032	0.109	4.72
Glass artificial (cubic structure)	50	1.226	4.687	0.0015	0.164	6.24
Glass artificial (rhomboid structure)	53	1.024	3.937	0.0012	0.312	4.41
Sandy natural materials	51	1.563	5.842	0.0035	0.148	6.42
All materials	178	1.106	6.732	0.002	0.167	8.99

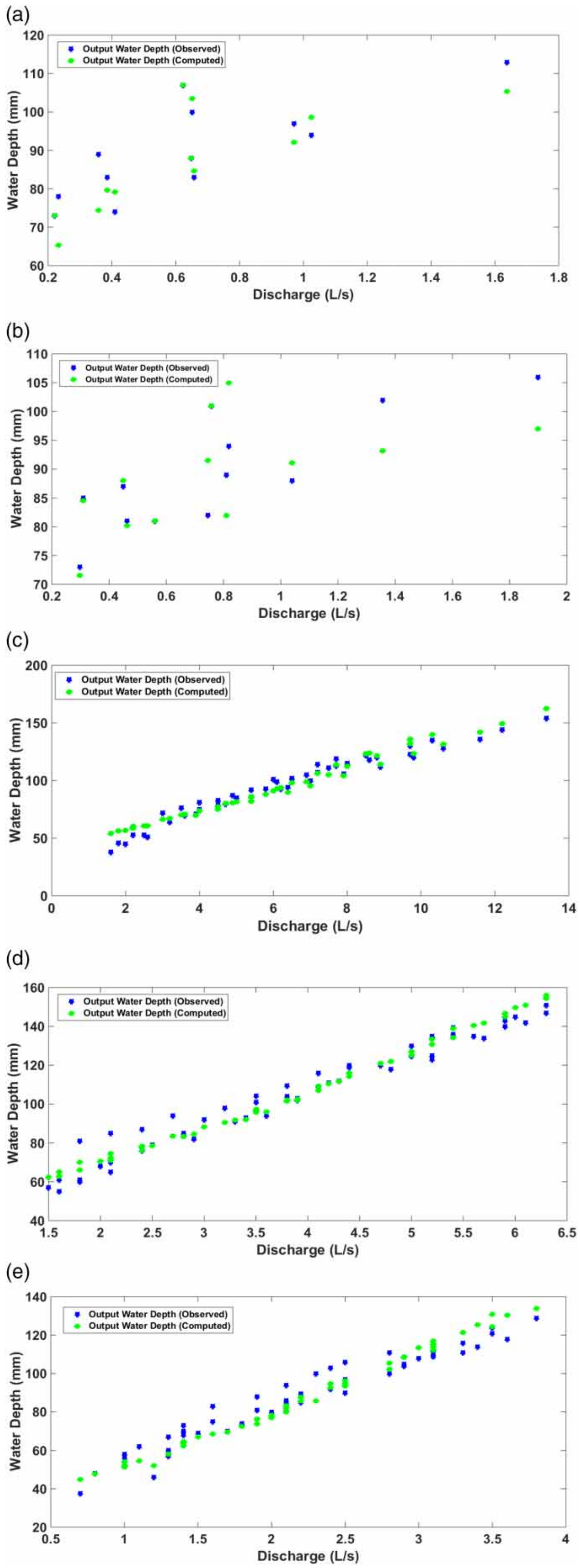

Figure 8 | Changes in the observational and computational output flow depths (using the proposed equation in the present study) versus flow discharge for different materials. (a) Rounded materials. (b) Crashed materials. (c) Glass artificial (cubic structure). (d) Glass artificial (rhomboid structure). (e) Sandy natural materials.

Table 4 | Mean values of each term of the proposed equation relative to the computational output flow depth

Materials	$\left(J*\left(\frac{d_{50}}{Y_c}\right)/Y_e\right)*100$	$\left(B*\left(\frac{Y_c}{L}\right)/Y_e\right)*100$	$\left(H*\left(\frac{gd_{50}Y_{up}^2n^{\frac{2}{3}}}{q^2}\right)/Y_e\right)*100$	$\left(N*\left(\frac{Y_{up}n^{\frac{1}{3}}}{d_{50}}\right)/Y_e\right)*100$
Rounded	13	1	55	31
Crashed	24	1	40	35
Glass artificial (cubic structure)	50	16	4	30
Glass artificial (rhomboid structure)	37	10	7	46
Sandy natural materials	22	8	16	54
All materials	28	12	19	41

conditions. According to Table 3, if the presented equation is used to calculate the Γ coefficient and consequently the output flow depth, the mean relative error (MRE) for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, and sandy natural materials was calculated as 5.49, 4.72, 6.24, 4.41 and 6.42%, respectively and as 8.99% for all experimental data together.

Figure 8 shows changes in the observational output flow depth (observed in experiments) and computational output flow depth (using the proposed equation in the present study) versus flow discharge for different materials.

Mean values of each term of the proposed equation (Equation (8)) relative to the computational output flow depth (Y_e) are presented in Table 4.

As can be seen from Table 4, these terms have the least effect on computation of the output flow depth: the second term in rounded and crashed materials, the third terms in glass artificial materials with cubic and rhomboid structures, and the second term in sandy natural materials. When just one of the J, B, H, N coefficients was optimized for all materials, all terms of the proposed equation in the present study (Equation (8)) affected computation of the output flow depth, and this is why Equation (8) is presented for computation of the Γ coefficient.

Comparison of flow behavior in open channels and through rockfill materials revealed that since in open channels there is no rockfill to withstand the flow, it is directly affected by the fluid weight. For this reason, flow with minimum specific energy (critical energy) leaves the cross-section and therefore, output flow depth is equal to the critical depth. In other words, fluid weight makes the output flow depth equal to the critical depth. In flow through rockfill media, however, rather than total weight of the fluid, a fraction of it is exerted to the aggregates and therefore, the output flow depth is increased more than that of open channels, and it is steadily higher than the critical depth.

4. CONCLUSIONS

Coarse-grained (rockfill) porous media is of great importance in various fields such as civil engineering and hydraulic structures. Calculation of the output flow depth from the rockfill media is of great importance in one-dimensional and two-dimensional analysis of steady flow. The presence of rockfill causes flow to leave the coarse-grained media with a higher specific energy than the critical energy and, consequently, a greater depth than the critical depth. For this reason, the Stephenson hypothesis (equality of the output flow depth from the rockfill (according to Equation (4)) to the critical depth) is not highly accurate in calculation of the output flow depth from the rockfill. To increase the accuracy of Equation (4), the coefficient Γ was multiplied by the critical depth (Equation (5)). In previous studies, the coefficient Γ has been presented as different numbers that were suitably accurate and efficient only for that special experimental condition. While in the present study, using dimensional analysis, the PSO algorithm and experimental data in different conditions (a total of 178 experimental data for rounded, crashed, Glass artificial materials with rhomboid structure, Glass artificial materials with cubic structure, and sandy natural materials), an equation was presented to calculate the mentioned coefficient in all experimental conditions with suitable accuracy and efficiency. In other words, the relationship presented in the present study includes the physical characteristics of the coarse-grained porous media and flow characteristics and is highly accurate in calculation of the output flow depth from the coarse-grained porous media in different conditions.

The results of the present study include the following:

(1) If, according to Stephenson, the output flow depth from the coarse-grained porous media was considered to be equal to the critical depth, the mean relative error (MRE) for rounded, crashed, Glass artificial materials with rhomboid structure,

Glass artificial materials with cubic structure, and sandy natural materials was calculated as 84.40, 83.81, 60.62, 67.68, and 74.82%, respectively and as 69.96% for all experimental data together.

(2) If the equation presented in the present study (Equation (8)) was used to calculate the coefficient Γ and, consequently, the output flow depth from the coarse-grained porous media, the mean relative error (MRE) of 5.49, 4.72, 6.24, 4.41 and 6.42% was calculated for the mentioned materials, respectively, and 8.99% for all experimental data together. In other words, the equation presented in the present study improved the MRE by 93, 94, 90, 93, 91, and 87%, respectively, compared to the case of using the equation presented by Stephenson.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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