

Experimental analysis of the effect of bed-load movement on flow hydraulic characteristics

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ABSTRACT

In this study, the effect of bed-load movement on mean flow characteristics was evaluated in two rigid rectangular flumes. The experiments consisted of creating flow conditions carrying sediments with mean diameters of $D_{50} = 0.5, 0.6,$ and 2.84 mm over both smooth and rough beds. Various sediment concentrations were injected at the upstream end of the flume at non-deposit injection rates to study the effect of various concentrations on flow resistance. The effect of sediment movement on flow resistance was examined by comparing the results with those of clear-water flows (without sediment injection on both smooth and rough beds). The results showed that the sediment transport at maximum injection rate may increase the friction factor up to 50% and 58% in the smooth bed, and up to about 75% and 80% in the rough bed with mean diameter of 0.5 and 0.6 mm. In addition, for $D_{50} = 2.84$ mm, the friction factor decreased in the smooth bed and increased up to 50% in the rough bed. In general, it can be concluded that bed-load transport can be increased by the flow friction factor. The results also showed that bed-loads may decrease the average velocity and increase shear velocity with extraction of momentum from the flow, and both the mentioned factors may increase the flow friction factor. Raising the bed-load concentration in the flow may increase the elevation of the friction factor, approaching a constant value after reaching the aggregation threshold and generation of bed-forms.

Key words: bed-load movement, flow resistance, friction factor, velocity profile

HIGHLIGHTS

- Bed-load movement, regardless of creating bed-forms, may increase flow resistance.
- Bed-load movement in our treatment span shows an increase in the friction factor with $D_{50} = 0.5$ and 0.6 mm up to 50% and 85% for smooth bed and up to 75% and 80% for rough bed, respectively.
- This augmentation started to reduce with increasing sediment diameter, as $D_{50} = 2.84$ changed to 50% for rough bed.

1. INTRODUCTION

Generally, there is an obvious relation between mean flow velocities, depth, energy slope and other flow characteristics, which is known as the resistance equations. The different forms of these equations and their multiplicatives have been identified for rigid boundary channels. The prediction of flow resistance and velocity distribution in alluvial channels is complicated for two reasons: (1) By increasing the average bed shear stress more than the critical value in alluvial channels, the bed sediments start to move. The general form of alluvial bed channel may change. Depending on flow conditions and other effective factors due to the sediment particles, this type of bed deformation increases the flow resistance in comparison with rigid bed channels. This makes it difficult or impossible to predict the flow resistance with a constant ratio. (2) Sediments which are transported either as bed-load or suspended load may change flow characteristics such as velocity distribution, especially, near the bed that has more sediment concentration. Therefore, it is necessary to consider the sediment movement, variation of bed roughness and interaction between them (Abrahams & Li 1998).

Until now, considerable efforts have been made on the first subject to find the relation between mean velocity, depth, energy slope, and particle size. Some researchers, such as Brownlie (1981), tried to estimate the total resistance using flow velocity equations based on power and logarithmic laws. On the other hand, other researchers believed that bed shear

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stress friction factor (or Chézy coefficient) should be divided into two parts: the particle roughness and bed-form resistance (Meyer-Peter & Müller 1948; Einstein & Barbarossa 1952; Engelund & Hansen 1967; Van Rijn 1984).

Moreover, there is an important question about the effect of sediment movement on flow resistance, whether sediment movement as bed-load can increase flow resistance in comparison with clear water or not. This question has given rise to two general points of views among researchers. The first group believed that bed-load movement alone has no effect on flow resistance (Vanoni & Nomicos 1960). In contrast, other researchers such as Kennedy (1895), Owen (1964), Smith & McLean (1977), Dietrich (1982), Grant & Madsen (1982), Wang & Chien (1985), Dyer (1986), Wiberg & Rubin (1989), Song *et al.* (1998), Bergeron & Carbonneau (1999), Gao & Abrahams (2004), and Campbell *et al.* (2005) found that bed-load movement may cause reductions in near-bed velocity, mean velocity and increase of roughness coefficient which are dependent on the thickness of the moving sediment layer.

On the other hand, Arora *et al.* (1986) showed that the friction factor in sediment-laden flows may increase or decrease in comparison with clear-water flows depending on different factors such as sediment concentration, fall velocity of particles, mean flow velocity and energy slope. The comprehensive study of Wiberg & Rubin (1989) showed that bed-load roughness is a function of bed-load layer thickness. Song *et al.* (1998) showed that bed-load movement can actually cause an enhancement of friction factor of up to 50%, increasing the flow depth by up to 14%. Bergeron & Carbonneau (1999) concluded that increasing sediment concentration induces a steepening of the near-bed velocity gradient, which increases the shear stress and the roughness scale. Hu & Abrahams (2004) compared friction factors in mobile and fixed beds and found that the friction factor in mobile beds is higher than in fixed beds. Habibzadeh & Omid (2009) showed that sediment transport may increase the friction factor up to 90% and 60% in smooth and rough beds in supercritical flow, respectively. They also showed that increasing the friction factor is directly related to bed-load concentration and particle size. Omid *et al.* (2010) investigated the effect of bed-load transport on flow resistance of alluvial channels with undulated bed. Their results indicated that the transport of fine particles ($D_{50} = 0.5$ mm) can decrease the friction factor by 22% and 24%, respectively, for smooth and rough beds. Increasing the bed-load size ($D_{50} = 2.84$ mm) can decrease the friction factor by 32% and 39%, respectively, for smooth and rough beds. They concluded that the presence of bed-load increases shear velocity and flow resistance. However, the decrease in flow resistance is more and may reduce the total flow resistance. Wang *et al.* (2011) investigated the effects of bed-load movement on mean flow characteristics in mobile gravel beds. It was found that the sand-waves have an average friction angle of 11.2° and heights ranging from $2.2D_{50}$ to $3.8D_{50}$. Based on the Schultz–Grunow formula, they developed a modified equation to better express the flow resistance factor in terms of the Reynolds number and the ratio of flow depth to roughness height. Hajbabaei *et al.* (2017) studied bed-load transport and flow resistance in alluvial channels carrying fine suspended sediment. The results showed that the bed-load pickup rate increases with an increase in the suspended sediment concentrations. After studying the resistance of turbid flow in a channel with an erodible bed, it was concluded that increasing suspended particles increased the mobility of bed material and led to a higher friction factor. The results also showed that the damping effect of suspended sediment caused lower boundary resistance with increasing suspended load concentration. On the other hand, suspended particles increased the mobility of bed materials and led to a higher mobile-bed resistance. They concluded that the relative importance of mobile-bed resistance is more than that of boundary friction for the high bed-load transport rate. Therefore, the damping effect of the suspended sediment on boundary friction can only be observed for low sediment transport or a fixed bed. Hou *et al.* (2019) performed an experimental study on the fixed-bed resistance of mountain channels. They derived a basic expression of the mobile-bed resistance of steep mountain channels by determining the controlling factors of bed-load movement on the riverbed resistance. Their proposed formula can accurately predict the variation of bed-load resistance. The analysis of Di Stefano *et al.* (2020) showed that for large-size mixtures, the friction factor can be accurately estimated neglecting the effect of the mean concentration of suspended sediments while for small-size mixtures the friction factor decreases when the mean sediment concentration increases. Gladkov *et al.* (2021) concluded that in the simulation of sediment transport and calculation of channel deformations in rivers, it is expedient to use the calculation dependences of the Chézy coefficient for assessing the roughness of the bottom sediment mixture, or the dependences of the form based on the field data.

As can be observed, the aforementioned studies have presented contradictory results about the effect of bed-load sediments on friction factor in alluvial channels. In addition to alluvial channels, which were mostly the focus in the above-mentioned works, sediment transport along with rigid channels is also important for the design and operation of lined canals in irrigation practice and channels in experimental works. Thus, the study of the effect of sediment loads on the friction factor is important in rigid boundary channels such as irrigation channels, sewage ducts, sediment transport channels, etc. In fact, the differences between

sediment transport in alluvial and rigid boundary channels is crucially important and should be considered. The sediments in rigid boundary channels originate from outside of channel networks but those in alluvial channels are from both upstream and bed sediments. Therefore, the main aim of this study is to investigate the effects of bed-load movement on flow resistance experimentally in two rigid rectangular flumes. So the main research gap is how bed-load movement affects mean flow hydraulic characteristics. In order to achieve the main goals, the effect of fine sediment movement on the friction factor over smooth and rough beds was investigated using two experimental rectangular flumes in steady-state and uniform flow conditions.

2. MATERIALS AND METHODS

Experiments were performed in subcritical flow conditions using three different sediment diameters ($D_{50} = 0.5, 0.6,$ and 2.84 mm) in two rigid rectangular flumes of glass sides and smooth bed made of Perspex. Sediments with mean diameters of 0.5 and 2.84 mm were used in flume I with 0.25 m \times 14 m dimensions. Other experiments were performed in flume II with 0.2 m \times 10 m dimensions. A sediment trap and a collecting basket were provided at the downstream end of the flumes. The weight of the collected sediments in the basket was measured by a digital weighing scale. Downstream of the sediment trap was a tailgate consisted of horizontal rotating flaps to provide various openings (Figures 1).

A tailgate was used to adjust the water depth in the flume without backwater in the flume. In each experiment, clear water was supplied at a steady state from an overhead constant head tank. The experiments were carried out with (1) clear water running over either smooth or rough beds, and (2) sediment-laden flow over either smooth or rough beds. For rough bed experiments, a single layer of uniform sand with a diameter of $D_{50} = 0.5, 0.6,$ and 2.84 mm was glued onto the Perspex bed and the same sizes of sediments were used in the sediment-laden experiments.

In the first series of the experiments (clear-water experiments), preliminary tests were carried out to determine the friction factor of the smooth and rough beds. Various channel slopes and flow velocities were considered in these experiments. Consequently, this information was used to set the flow depth prior to injecting bed-load sediments (at the upstream end) in the second series of experiments.

Firstly, the tailgate was adjusted to obtain uniform flow without sediment and prescribed bed depth. Then the experiment was performed with sediment-laden flow, for a given slope of the channel and flow discharge, and various concentrations of sediments were injected into the flow at a constant rate using a screw-feeder system at the upstream end of the flume. The rate of sediment supply was kept constant by adjusting the feeder and keeping the sand level in proportion with the top level of the container. For flume II, a device was utilized for sediment injection having two series holes and creating a constant head behind the second hole and consequently stabilizing the injection rate for the required concentrations at different times.

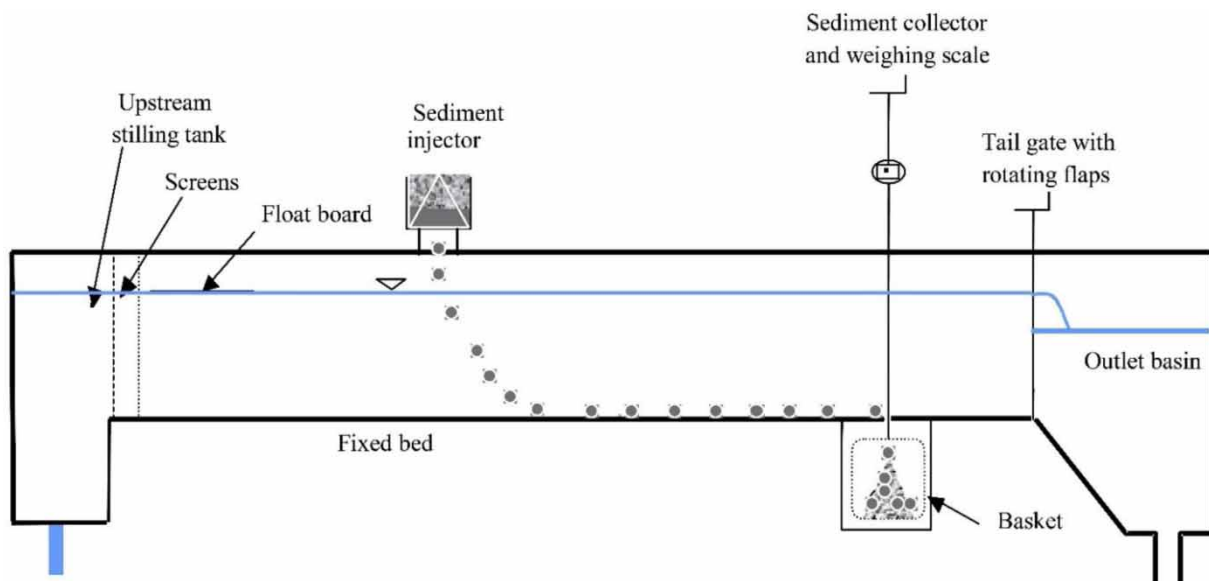


Figure 1 | Schematic view of the laboratory flume.

Table 1 | Experimental conditions

Sediment diameter	0.5, 0.6 and 2.84 mm
Sediment concentration	0.18% to 0.75%
Water depth	25 to 256 mm
Mean velocity	0.35 to 1.2 m/s
Bed slope	1:1,000 to 5:1,000

Flow characteristics were measured for various injection rates less than the rate at which initiation of sediment deposition on the bed was observed. Bed-load particles were collected in the sediment trap provided at the downstream end of the flume and removed periodically. For each injection rate (I , expressed in terms of mass rate), the corresponding volumetric sediment concentration (C) is simply stated as follows:

$$C = \frac{I}{QS_s} \quad (1)$$

where Q is the flow discharge and S_s is sediment density, equal to $2,650 \text{ kg/m}^3$. The experimental conditions are summarized in Table 1.

For a given concentration of bed loads, two or three flow velocity distributions were measured along with the vertical centerline of the flow cross-section at a distance of 8.0 m in flume I and 6.0 m in flume II downstream of the location where the sediment particles were injected into the flow. Velocities were measured at vertical intervals of 2–5 mm up to 20% of water depth and then at 10 mm intervals up to the water surface. A micro-propeller current meter was used for measuring the velocity profiles.

There are many methods to determine the flow resistance. In this study, the logarithmic velocity distribution was used. The averaged velocity distribution of each treatment was used to calculate mean bulk velocity (u) by integrating over the whole flow depth and shear velocity u_* using a logarithmic velocity distribution law:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y}{k_s}\right) + B \quad (2)$$

In this equation, u is the velocity at a distance y above the smooth bed, u_* is shear velocity, κ is the von Kármán constant (≈ 0.4), k_s is equivalent roughness ($=D_{50}$), while B is an integration constant. This law is known as ‘the law of the wall’, that the average flow velocity at a certain point is in proportion with the logarithm of the distance from that point to the ‘wall’, or the boundary of the fluid region (von Kármán 1930). The law of the wall was published for the first time by von Karman (1930). It is just technically applicable to the parts of the flow that are close to the wall (less than 20% of the flow depth). In this region, the velocity distribution is mostly affected by the bed characteristics and it is a good approximation for the entire velocity distribution of natural streams. Values of u_* and k_s were gained by each averaged profile from the slope and intercept of the regression equation obtained by fitting a straight line to the lower points of the profile up to 20% of the flow depth. The Darcy–Weissbach friction factor (f) was then calculated as follows (Omid *et al.* 2003):

$$f = 8\left(\frac{u_*}{U}\right)^2 \quad (3)$$

3. RESULTS AND DISCUSSION

Figures 2–4 show the comparison of velocity distribution for the experiments with and without sediment injection over both smooth and rough beds. As can be seen, the bed-load movement affects the velocity distribution and decreases the velocity in the lower part of the velocity distribution. The two main findings are as follows: (1) decreasing average velocity, (2) increasing slope of the lower part of the logarithmic velocity distribution. To better interpret the results, the values of u are plotted against $\ln y$ in Figures 5 and 6. According to the logarithmic velocity distribution (Equation (2)), the slope of the fitted line to the data equals the ratio of u_*/κ . As can be seen in Figures 5 and 6, the line slopes for the experiments with bed-load movement by increasing the sediment concentrations are enhanced. This means that the value of u_*/κ (and as a result, u_*) is

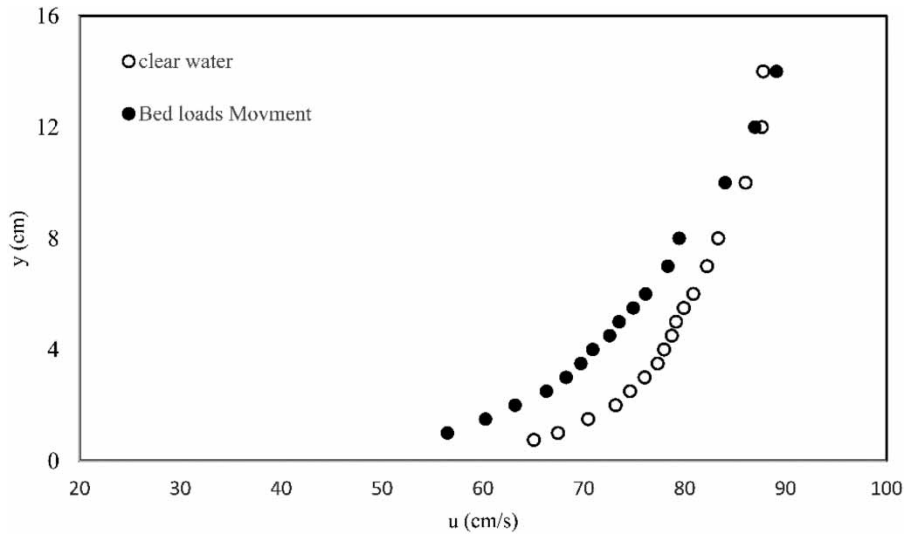


Figure 2 | Velocity distributions in clear water and flow with non-deposit injection in flume I.

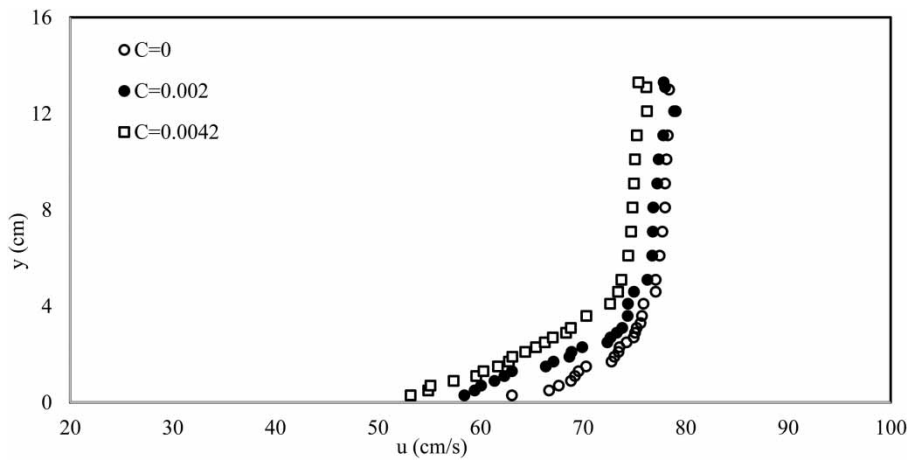


Figure 3 | Velocity distributions in clear water and flow with non-deposit injection in flume II (smooth bed).

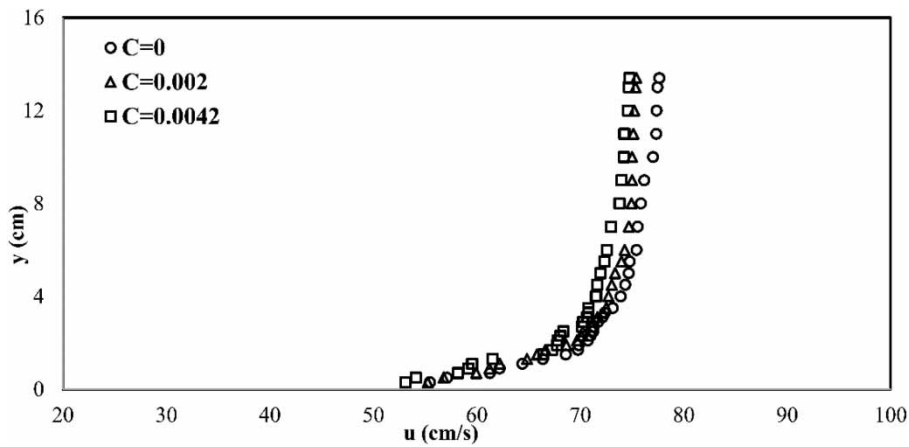


Figure 4 | Velocity distributions in clear water and flow with non-deposit injection in flume II (rough bed).

increased. According to Equation (2), it is clear that increasing u_* and decreasing average velocity (u) may generally increase the flow friction factor (f). As is found from the results, decreasing the average flow velocity will increase the flow depth.

Figure 7 shows the friction factor f versus Reynolds number Re for clear water over a smooth bed. In this figure, values of f due to the Colebrook–White formula are also shown as a straight line. As can be observed, there is an acceptable correlation between the two sets of results.

Figures 8 illustrates the friction factor versus the ratio of h/k_s (relative submergence) for a rough bed with clear-water flow as a straight line and based on $B = 4.5$ for flume I and $B = 2.5$ for flume II (Song *et al.* 1998; Gao & Abrahams 2004). The bed roughness for these treatments is shown as particle roughness. The equations for clear-water flow over the rough bed are defined as follow:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y}{k_s}\right) + 4.5 \quad \text{flume I} \tag{4}$$

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y}{k_s}\right) + 2.5 \quad \text{flume II} \tag{5}$$

The variations of flow resistance versus Reynolds number of sub-critical flow in all experimental data with different sediment concentrations in flumes I and II are shown in Figure 9. As can be observed, there is a good agreement among all experiments indicating the effect of bed-load injection or no injection over both smooth and rough beds.

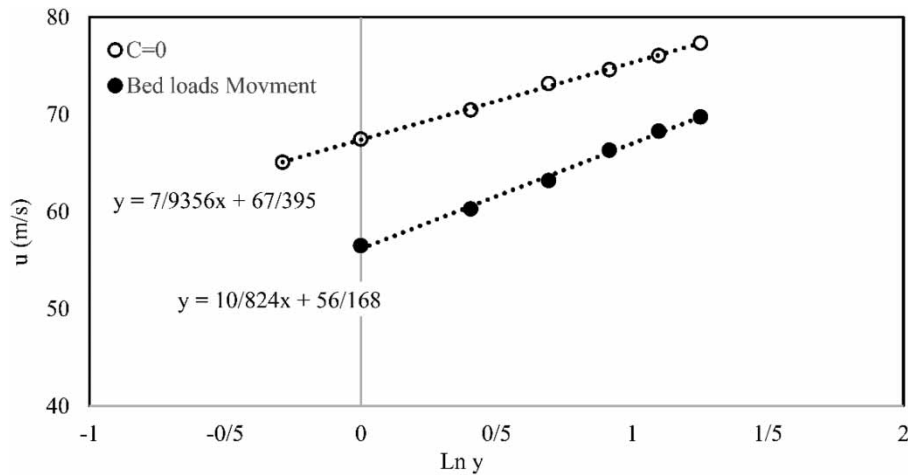


Figure 5 | Effect of non-deposit sediment injection on shear velocity (flume I).

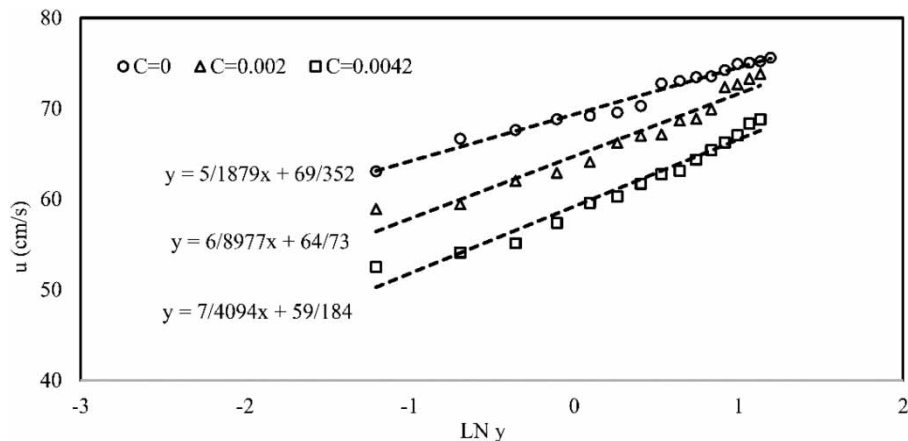


Figure 6 | Effect of non-deposit sediment injection on shear velocity (flume II).

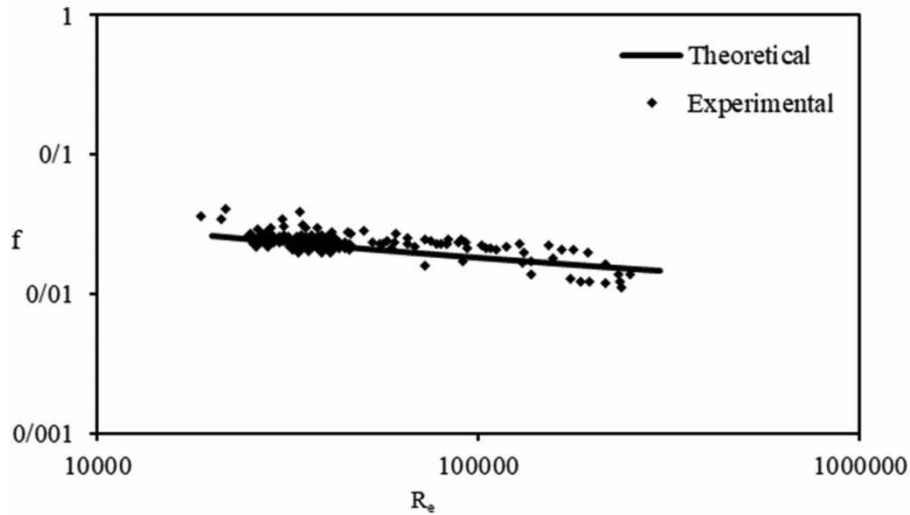


Figure 7 | Friction factor versus Reynolds number (comparison of Colebrook-White with experimental data).

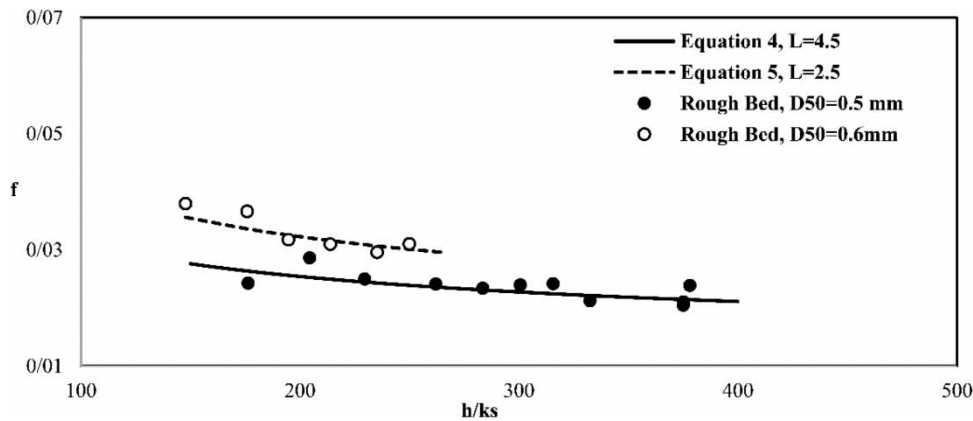


Figure 8 | Friction factor, f , against relative submergence, h/k_s for clear-water experiments.

Figure 10 shows the variations of friction factor versus relative submergence in subcritical flow for all experiments in the two flumes.

In addition, the relation between friction factor and bed-load concentration in both smooth and rough beds is illustrated in figure 11. This figure demonstrates how bed roughness, injection and concentration of sediment increase the flow resistance. For the studied range of friction factors, the results showed that the sediment transport at the maximum injection rate may increase the friction factor up to 50% and 58% for the smooth bed, and up to about 75% and 80% for the rough bed with mean diameter of 0.5 and 0.6 mm. In addition, for $D_{50} = 2.84$ mm, the friction factor decreased in the smooth bed and increased up to 50% in the rough bed.

Similarly to Song *et al.* (1998), Figure 12 illustrates the effect of sediment concentration on relative flow friction factor, f/f_c , in which f is the friction factor for flows with sediment transport and f_c is the friction factor in clear water with the same Reynolds number for smooth bed experiments and with the same relative submergence in rough bed experiments. As can be seen, the values of f/f_c increase with increasing the concentration for all experiments and f/f_c values for rough bed injections are higher than for smooth bed injections.

4. CONCLUSIONS

In this study, the effect of bed-load movement on the friction factor was experimentally investigated by considering grain size and sediment concentrations in two rectangular flumes with steady flow and sediment particles with approximately uniform grain size for bed-load. The measured friction factor in clear-water flow over both smooth and rough bed channels well fits the

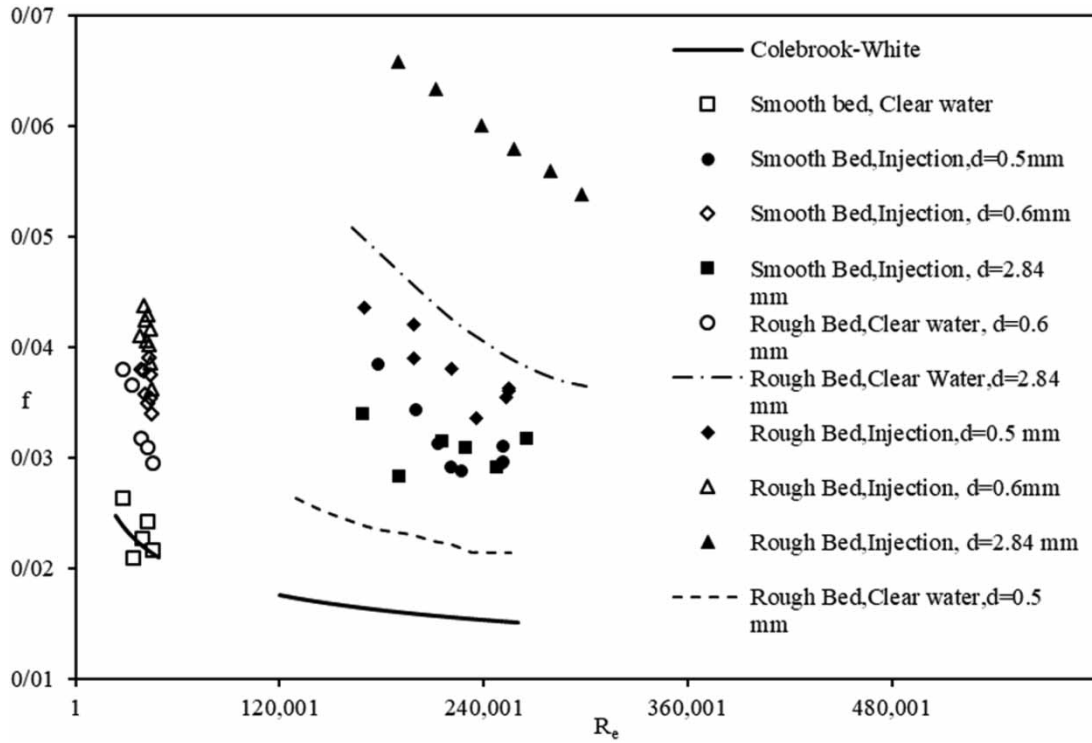


Figure 9 | Variation of friction factor versus Reynolds number.

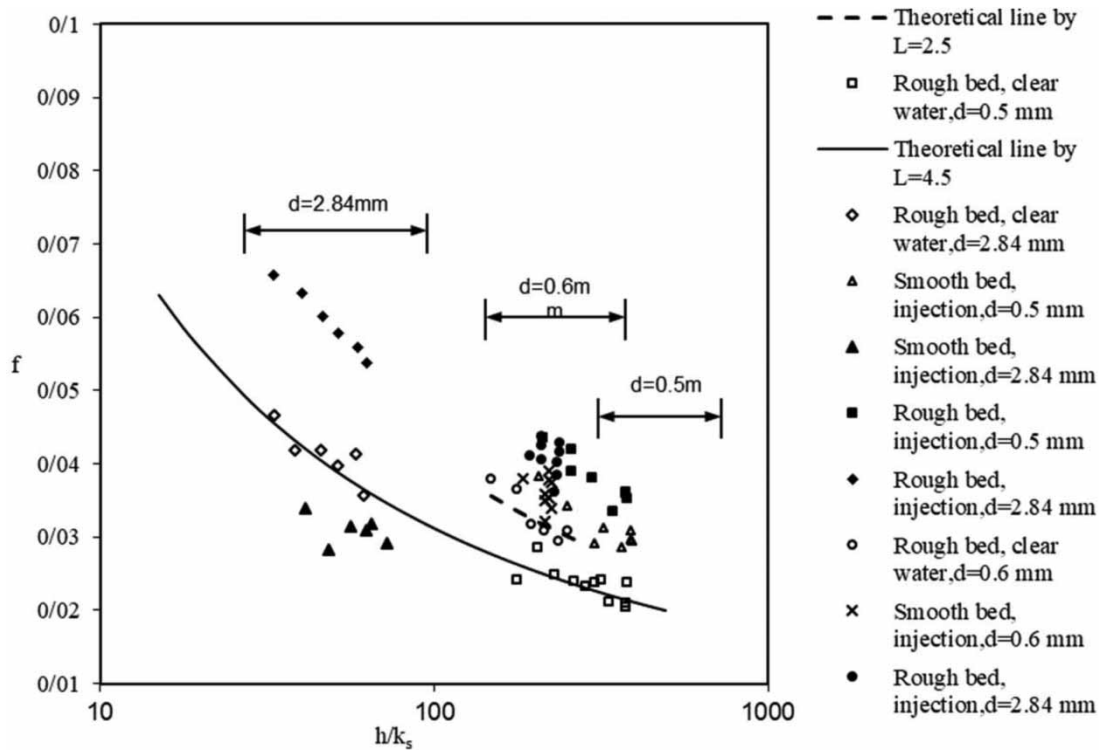


Figure 10 | Variation of friction factor against relative submergence.

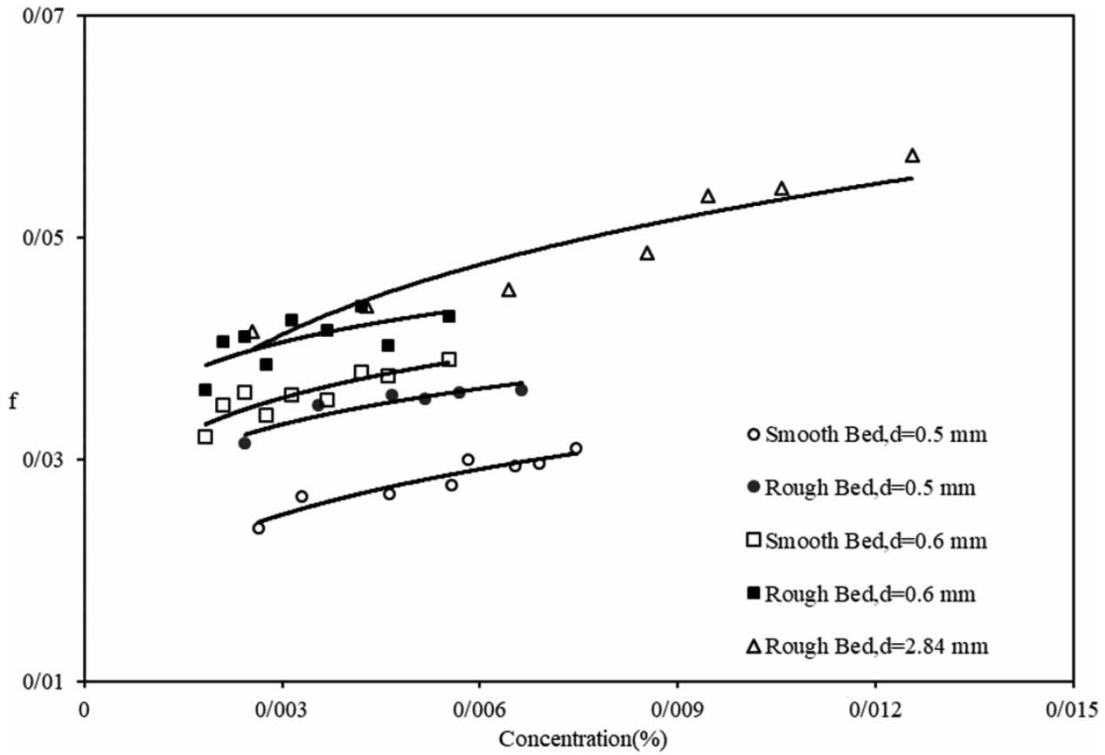


Figure 11 | Effect of sediment concentration on friction factor of both smooth and rough beds.

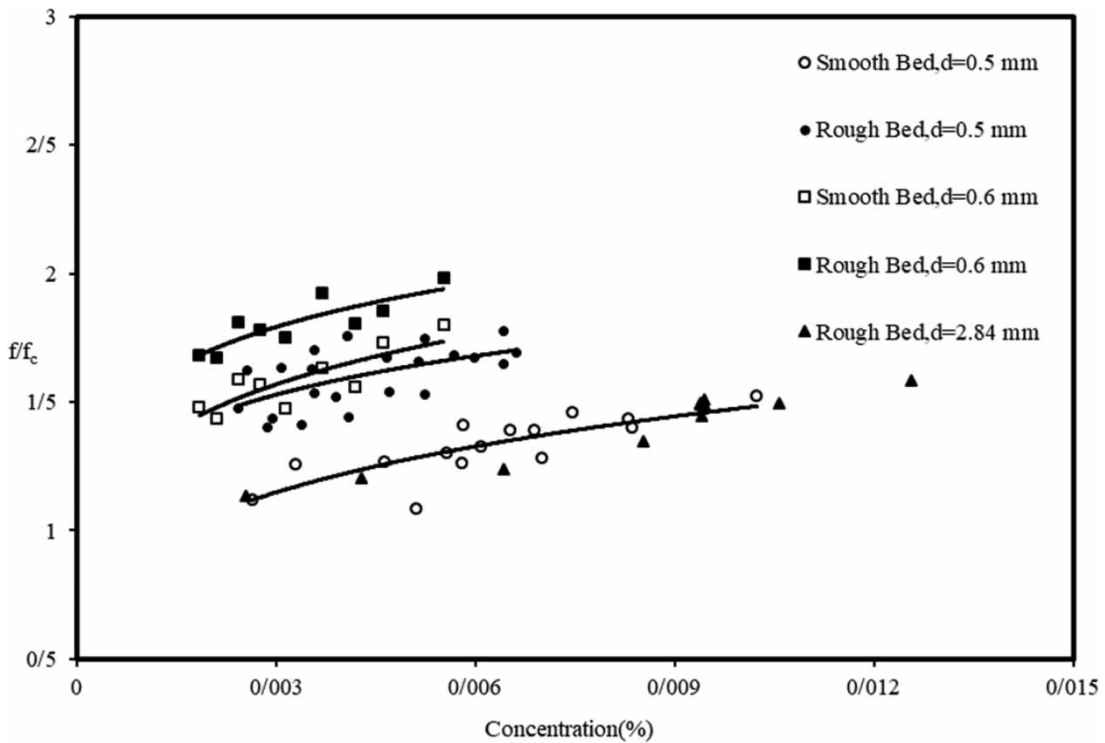


Figure 12 | Variation of f/f_c versus sediment concentration.

Colebrook–White equation. The results of the present study are consistent with the observations of Song *et al.* (1998) and Bergeron & Carbonneau (1999), who found that sediment movement over the bed may affect the flow characteristics. It should be mentioned that bed-load movement, regardless of creating bed-forms, may increase flow resistance. Additionally, the bed-load movement in our treatment span showed an increase in the friction factor with $D_{50} = 0.5$ and 0.6 mm up to 50% and 85% for the smooth bed and up to 75% and 80% for the rough bed, respectively. This augmentation started to reduce with increasing sediment diameter, as $D_{50} = 2.84$ changed to 50% for the rough bed. As the sediments in irrigation canals are often fine-grained, it is critically important to consider this type of sediments in the design of irrigation canals. Results indicated that in low sediment concentrations, bed-load affects only the near-bed region of the flow, where it may cause a slight reduction in mean flow velocities with a consequent increase in shear velocity as well as roughness height. In addition, increasing non-deposit concentration of sediment rates may cause significant increase in the friction factor for $D_{50} = 0.5$ and 0.6 mm and decrease for $D_{50} = 2.84$ mm in a smooth or rough rigid bed channel.

Generally speaking, as the bed-load concentration increases, the friction factor increases to a constant value. After this stage, further increase in the sediment concentration will lead to the deposition point and the generation of bed forms, which will further increase the flow hydraulic resistance. Further studies are needed with a wider range of hydraulic conditions and diameter of sediment particles.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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