


Effect of slope on energy dissipation for flow over a stepped spillway

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ABSTRACT

In a stepped spillway, the spillway face is provided with a series of steps from near the crest to the toe. This type of spillway is attractive for water resource management because it dissipates energy that is within the flow. When energy is dissipated by the steps, it reduces the energy reduction that otherwise must occur at the toe of the spillway. In the present study, nine physical stepped spillway models with slopes of 15, 25, 45° and with a number of steps varying from 5 to 50 have been constructed. The crest of the spillways have standard Ogee profiles and a constant slope extends from the crest to the spillway toe. Two different-sized flumes were used in the experiments (25 and 50 cm wide). Several physical models have been constructed in order to measure the energy dissipation. The purpose of this research is to investigate energy dissipation rate in spillways with different slopes. It was found that the two most important dimensionless factors influencing energy dissipation are the Froude number and $q^2/(gH_{dam}^3)$. The spillway slope and step number have less impact on the rate of energy dissipation. However, for a constant discharge over a stepped spillway, increasing spillway slope, and increasing the number of steps will increase the energy dissipation. Using multiple regressions, some useful relations for determination energy dissipating are obtained.

Key words: energy dissipation, multiple regression, number of step, slope, stepped spillway

HIGHLIGHTS

- Energy dissipation rate is investigated in stepped spillways with different slopes.
- It was found that the two most important dimensionless factors influencing energy dissipation are the Froude number and $q^2/(gH_{dam}^3)$.
- The spillway slope and step number have less impact on the rate of energy dissipation.
- Increasing spillway slope and increasing the number of steps will increase the energy dissipation.

LIST OF MATHEMATICAL SYMBOLS

H_{dam}	Spillway height (m);
g	Acceleration due to gravity (m/s^2);
ΔH	Difference between specific energy upstream and downstream of chute (m);
y_1	Downstream flow depth: i.e. $d = q_w/V_{max}$ (m);
N	Number of steps on the chute (dimensionless);
y_2	Depth of water after hydraulic jump or conjugate depth (m);
q	Discharge per unit width of chute (m^2/s);
h/l	Stepped spillway slope (dimensionless);
h	Height of step (m);
l	Length of step (m);
H_1	Specific energy at downstream of spillway (m);
H_t	Total specific energy at upstream of spillway (m);
R_e	Reynolds number (dimensionless);
F_n	Froude number (dimensionless);
ρ	Water density (kg/m^3);
μ	Water dynamic viscosity ($N.s/m^2$);
y_c	Critical depth: $y_c = \sqrt[3]{\frac{q^2}{g}}$

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INTRODUCTION

Stepped spillways have been used since ancient times, about 3500 years ago. However, some relevant hydraulic aspects of these spillways remain unknown (Chanson 1994a). In recent decades, new dam construction technologies such as the use of roller-compacted concrete (RCC) has increased the development and attention to this type of spillway. Research shows that the amount of energy dissipated in stepped spillways exceeds that of smooth spillways (without steps) with the same dimensions. Significant energy loss allows a reduction of the depth of the cut, the length, and the height of the downstream stilling basin. Consequently, cost savings can be achieved with thoughtful construction.

Most researchers have pointed to two different types of flow regimes called nappe and skimming flow regimes. The first type occurs at low discharge and with high steps. The second type occurs in spillways with high discharge and low step heights (Horner 1971; Chanson 1994b; Salmasi *et al.* 2021). The transition flow regime is intermediate between nappe and skimming flow regimes and is not very important in terms of design. Oscillating flow over a stepped chute is a characteristic in transition flows (Nouri *et al.* 2020; Salmasi *et al.* 2020). In Figure 1, sketches of the three flow regimes are provided.

Horner (1971) and Sorensen (1985) were probably the first researchers to classify flow over stepped spillways into nappe and skimming flow regimes. Based on experiments performed on stepped spillways, the conversion of a nappe flow to a skimming flow occurs at approximately $0.8y_c/h$ (Chanson 1994b) where y_c is the critical depth on the spillway (representative of discharge) and h is the vertical height of the step. Chanson (1994c) showed that in addition to the dimensionless parameter y_c/h , the spillway slope is also effective in converting a nappe flow to a skimming flow. The results of that study showed that with a skimming flow regime, the discharge must be higher than a critical characteristic value $[(y_c)_{\text{onset}}]$. This characteristic

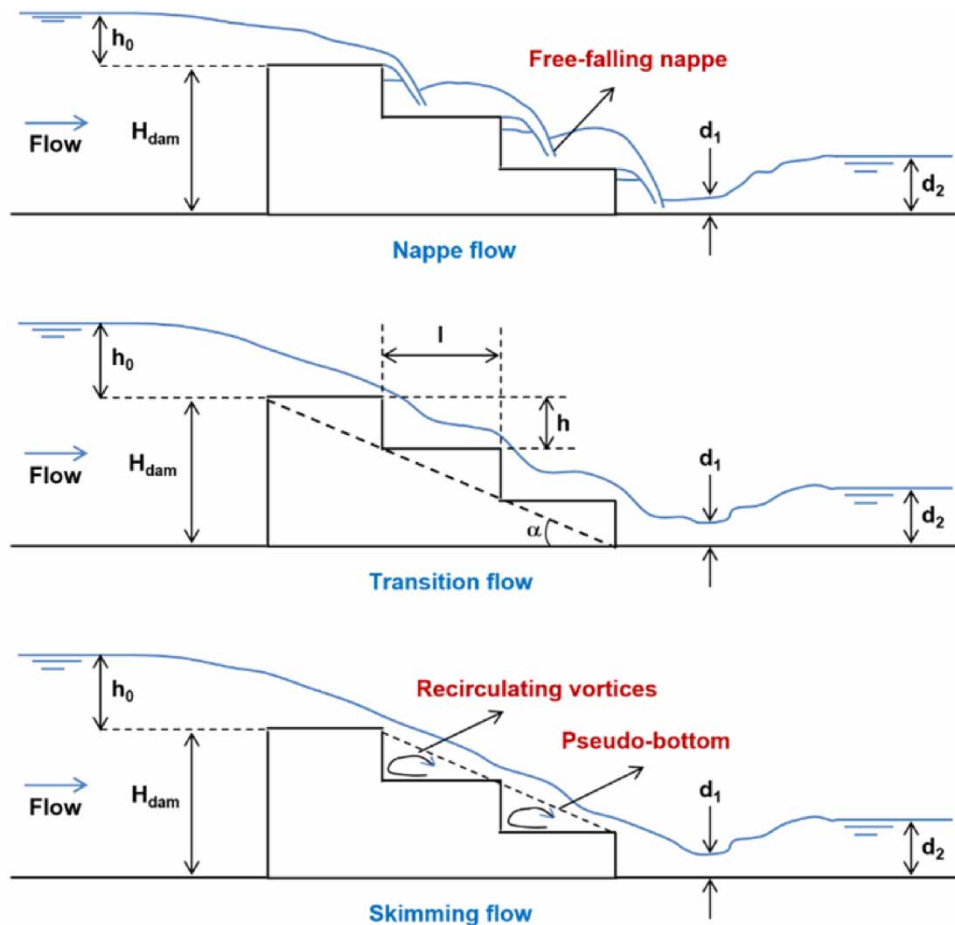


Figure 1 | Schematic representations of the three flow regimes (Salmasi & Özger 2014).

onset of skimming flow regime was presented as follows (Chanson 1994c):

$$(y_c)_{onset}/h = 1.057 - 0.465 (h/l) \quad (1)$$

In Equation (1) l is the horizontal length of the step. Many researchers, including Peyras *et al.* (1992), Chamani & Rajaratnam (1999), and Matos & Quintela (1994), have argued that energy dissipation in the nappe flow regime is greater than in the skimming flow regime. However, according to Chanson (1994b), in long stepped spillways with established flow, the flow energy loss with the skimming flow exceeds that of nappe flow. In response, Matos & Quintela (1994) identified the error caused by air entrainment in the spillway and its effects on measuring flow depth. Consequently, they considered the Chanson (1994a, 1994b)'s conclusions to be questionable. Matos & Quintela (1994) did not consider the use of the aerated water depth downstream of a spillway to calculate energy loss to be correct; they found that because of the use of aerated water depth instead of pure water depth, the amount of flow energy dissipation was overestimated. Determining the amount of energy loss by indirect methods using the conjugate depth of the hydraulic jump by Tazzi (1992) and Pegram *et al.* (1999) has also been reported.

Pegram *et al.* (1999) carried out experiments on stepped and smooth spillways with a slope of 59° and a height of 30 m (model scale was 1:10) and with step heights of 0.25, 0.5, 1.5 and 2 m. Experiments have shown that the energy dissipation contradicts the theoretical estimate of Rajaratnam (1990), who reported this rate to be 89%. The results also showed that the energy dissipation does not depend on the height of the steps. This result is true for the establishment or non-establishment of flow conditions.

Rice & Kadavy (1996) built a model of the stepped spillway of Salado Creek Dam in Texas. The height of the model was 85.35 cm, its slope was 21.8° , and the number of steps was 27. To compare the model, a smooth spillway (without steps) with the same height was made. The slope of the stepped spillway was much smoother than that of Christodoulou (1993) with a slope of 55° .

According to Rice & Kadavy (1996), energy loss should be greater for steep slopes compared to gentle slopes. The results of experiments showed that the energy dissipation in their experiments and results from Christodoulou (1993) show a very good agreement.

Chamani & Rajaratnam (1999) carried out experiments on a stepped spillway with a slope of 59° and with step heights of 125, 62.5 and 31.25 mm and another slope of 51° with step heights of 125 and 31.25 mm. By measuring the concentration of air bubbles entering the flow and its effect on increasing the depth of the flow, they calculated the depth of pure water/clean water (without air) and the pure water depth was used to determine the relative energy loss. The experimental results showed that the relative energy loss in the skimming flow regime varies between 48 and 63%.

Felder & Chanson (2011) performed experiments on a physical model of a stepped spillway with a height of 1 m and a slope of 26.6° . Five different cases were considered for the step heights and the residual energy downstream of the spillways were measured. Although the results showed that the number of steps did not have much effect on the residual downstream energy, a spillway with 10 steps of 10 cm height had less residual energy and thus more energy dissipation. For the five cases, the Darcy-Wiesbach roughness coefficient (f) varied from 0.12 to 0.37, which was consistent with the previous work of Felder & Chanson (2009) in which f varied from 0.1 to 0.35. Shoja *et al.* (2011) used genetic algorithms to determine the optimal dimensions of the Sarough Dam spillway located in West Azerbaijan province, Iran. The optimization process showed that the stepped spillway led to increased energy dissipation compared to a smooth spillway. This in turn led to the elimination or reduction of stilling basin dimensions downstream of the spillway. For fixed slopes, increasing the discharge reduces the relative energy dissipation, increasing the optimal height of the steps. By analyzing the data obtained from a large number of experiments, Salmasi (2009) showed that energy loss in stepped spillways is affected by the number of steps and for a fixed number of steps, a maximum energy loss can be achieved. Of course, this maximum energy dissipation varies from one spillway to another and depends on the slope of the spillway, the design discharge, and the height of the spillways. Salmasi (2009) also mentioned the new step-by-step design method for stepped spillways.

Due to the increasing importance of stepped spillways, many research studies have been carried out to identify factors that affect energy dissipation. These studies have mainly utilized scaled laboratory tests with physical models (e.g. Pegram *et al.* 1999; Roushangar *et al.* 2017). In addition, a few studies have relied upon numerical simulation (e.g. Eghbalzadeh & Javan 2012; Salmasi & Samadi 2018). Salmasi & Abraham (2021) used genetic algorithms for optimizing stepped spillways to

maximize energy dissipation. In that study a stepped spillway with optimal dimensions was proposed as a replacement of the smooth spillway of Sarogh Dam located in West Azerbaijan province, Iran.

In the watershed management plan in Kohkiluyeh city, Iran, a number of mortar stone structures with an annual storage capacity of more than 2 million cubic meters will be implemented to strengthen groundwater aquifers. Figure 2 shows a stepped chute constructed in Kohkiluyeh city.

Another example of the construction of a stepped spillway is seen in the earthen dam of Tang Khon river in Bushehr province, Dashtestan city in Iran. This earthen dam has a design flow of $109 \text{ m}^3/\text{s}$, equivalent to a return period of 100 years, a reservoir volume of 350 million m^3 , a crest width of 5.5 m, and a crest length of 132 m. Figure 3 shows a panoramic image of the stepped spillway with a width of 19.3 m constructed in this dam.

Moradabad earthen dam was built in 2009 in Firoozabad city, Iran. Figure 4 shows a stone stepped chute constructed with cement mortar. The length of the stepped spillway of this dam is 419 m, its width is 5 m, and its height is 14 m from the riverbed.

In the present study, nine physical stepped spillway models with slopes of 15, 25 and 45° and with a number of steps varying from 5 to 50 have been constructed. The crest of the spillways have standard Ogee profiles and a constant slope extends from the crest to the spillway toe. Two different-sized flumes were used in the experiments (25 and 50 cm wide). Several physical



Figure 2 | A stepped chute constructed in Kohkiluyeh city, Iran.



Figure 3 | Tang Khon earthen dam in Bushehr province, Iran and its stepped spillway.



Figure 4 | A stepped chute from Moradabad earthen dam in Firoozabad city, Iran.

models have been constructed in order to measure the energy dissipation. The purpose of this study is to investigate energy dissipation rate in spillways with different slopes.

MATERIALS AND METHODS

This study was carried out in the laboratory of the Water Engineering Department, Faculty of Agriculture, University of Tabriz, Iran. Nine physical models of stepped spillways with slopes of 45, 25 and 15° and with a number of steps of 5, 10 and 15 were constructed and tested. Two flumes were used for these testing, a flume 50 cm in width and length of 10 m and the other flume with 25 cm width and 12 m length. The spillways with large scales had a slope of 45° and were 50 cm wide and 100 cm high. The spillways with small scales had slopes of 25 and 15° including 25 cm wide and 32 cm high. Table 1 lists some of the specifications of the physical models of the constructed spillways.

Figure 5 shows the longitudinal cross section of the flume with a width of 50 cm and a stepped spillway.

The water flow enters the flumes through a tank with a fixed height of 4.5 m by the control valve and is transferred downstream after passing over the spillway crest. To control the downstream water and the location of hydraulic jump, a vertical sliding gate was used at the end of the flume. The vertical sliding gate was adjusted in a manner to create a free hydraulic jump downstream of the stopped spillway. In this way both of the y_1 and y_2 depths (conjugate depths) were measured for the energy dissipation calculation (Equation (5)). The end gate is adjusted to cause a hydraulic jump. Then the conjugate depth (y_2) was measured by point gauge. Then y_1 is calculated using the hydraulic jump relation. Measurement of the conjugate depth (y_2) has the benefit that the air bubbles are lower at y_2 than in the y_1 section. Clear water depth measurements (without air bubbles), will be more accurate (Khatibi *et al.* 2014).

Water depths upstream and downstream of the hydraulic jump was measured by a calibrated point gage with an accuracy of 0.1 mm at three points across the width of the flume. Discharge was measured by a 53° triangular weir. Although volumetric measurement of discharge is the most accurate method, several research studies show that triangular weirs provide

Table 1 | General specifications of the physical model of stepped spillways

Flume width (cm)	Number of experiments	Number of steps	Spillway height (cm)	Spillway slope (degrees)
50	93	5, 10, 15, 20, 35 and 50	100	45
25	22	5, 10 and 15	32	25
25	45	5, 10 and 15	32	15

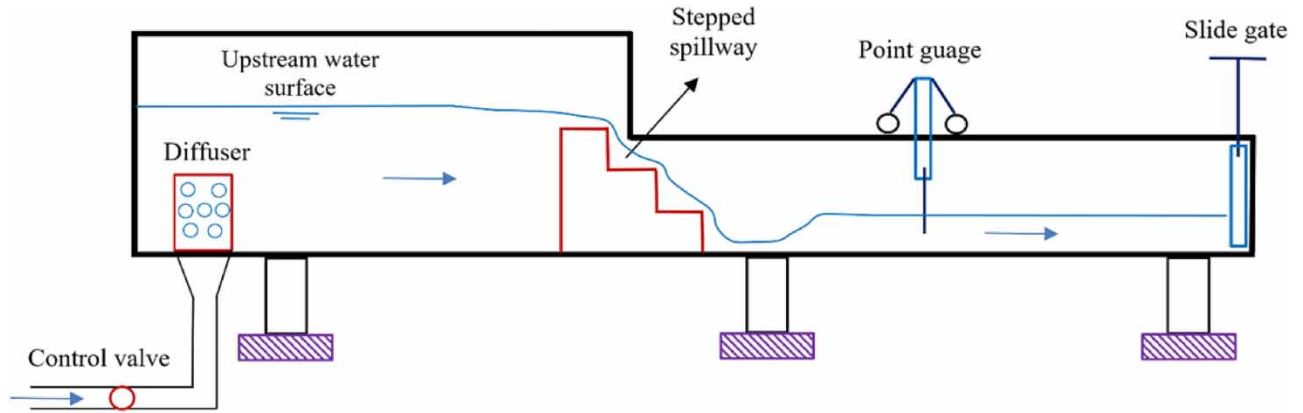


Figure 5 | Longitudinal section of the flume, stepped spillway and flow control system in the flume (dimensions are shown in Table 1).

reasonable accuracy for discharge easements. For example, readers are directed to the studies of Matos & Quintela (1994), Felder & Chonson (2011) and Salmasi & Abraham (2022).

In the water depth section before the hydraulic jump, air was input into the flow (especially at high discharge) and therefore in this section, the two-phase flow (water and air mixture) was measured by a point gauge. Measurement of the water depth after hydraulic jump was performed in a place where small air bubbles were observed in the water flow. Figures 6 and 7 show flow over the physical model of the stepped spillway prepared in this study.

In these experiments, depths before and after hydraulic jump were measured. However, to calculate the relative energy loss, the conjugate depth of the hydraulic jump was used due to the presence of fewer air bubbles in this section compared to the section upstream of the hydraulic jump (Salmasi 2003). The amount of energy upstream of the spillway crest, H_t , was calculated as follows:

$$H_t = H_{dam} + 3/2y_c \quad (2)$$

In Equation (2), H_{dam} is equal to the height of the spillway. Its average value is 1 m for a spillway with a width of 50 cm. Conversely, the value of H_{dam} is 32 cm for a spillway with a width of 25 cm. Also, y_c indicates the critical depth on the spillway. It is obtained from the relation $y_c = (q^2/g)^{1/3}$, where q is the discharge per unit width of the spillway ($\text{m}^3/\text{s}/\text{m}$) and g is the acceleration of the Earth's gravity equal to 9.81 m/s^2 . To calculate the energy downstream of the spillway and before the



Figure 6 | Flow over stepped spillway (downstream view).



Figure 7 | Close-up of flow over stepped spillway (side view), turbulent and energy dissipation at the toe of the spillway (Salmasi & Samadi 2018).

hydraulic jump, conjugate depths of the hydraulic jump were used as follows (Pegram *et al.* 1999):

$$H_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} \quad (3)$$

$$y_1 = \frac{1}{2}y_2 \left(\sqrt{1 + 8(y_c/y_2)^3} - 1 \right) \quad (4)$$

In Equations (3) and (4), y_1 and y_2 are the measured depths before and after the hydraulic jump, respectively. It should also be noted that y_1 and y_2 represent the depths of the ‘water-air mixture/white water’ and the ‘water/clear water’, respectively. The relative energy loss is found from Equation (5):

$$\Delta H/H_t = (H_t - H_1)/H_t \quad (5)$$

In Equation (5), the H_1 is the energy downstream of the spillway and before the hydraulic jump (Equation (3)), and the H_t is the total energy upstream of the spillway (Equation (2)), and the rest of the parameters have already been defined.

Dimensional analysis

The purpose of a dimensional analysis method is to find a relationship between effective physical parameters in a natural phenomenon. The obtained relationship among variables with dimensional analysis is usually simpler and can be transferred among unit systems (SI and atc).

Important and effective parameters of the flow through stepped spillways are grouped as follows:

- Properties related to fluid including: liquid density (ρ), dynamic viscosity (μ), and gravity acceleration (g)
- Flow-related characteristics including: flow depth (y) and flow velocity (V)
- Geometric characteristics of the spillway, including: number of steps (N), height of the steps (h), horizontal length of the steps (l) and total height of spillway (H_{dam}).

The functional relationship can be summarized in Equation (6):

$$f(H_{dam}, h, l, N, g, y, \rho, \mu, V) = 0 \quad (6)$$

To convert Equation (6) to a dimensionless format, three iterative variables are selected: y , V and ρ . The three variables together include the dimensions M, L, T. Dimensional parameters are defined as follows:

$$\pi_1 = h/l$$

$$\pi_2 = N$$

$$\pi_3 = \Delta H/H_t$$

$$\pi_4 = \mu/\rho Vy = 1/R_n \quad (R_n = \text{Reynolds number})$$

$$\pi_5 = gy/V^2 = 1/F_r^2 \quad (F_r = \text{Froud number})$$

$$\pi_6 = H_{dam}/y_c$$

$$\pi_7 = q^2/gH_{dam}^3$$

The fourth dimensionless factor is related to the Reynolds number (R_e), which is avoided due to the turbulence of the flow over stepped spillway and the creation of a large Reynolds number, and eventually Equation (6) will change to Equation (7):

$$\frac{\Delta H}{H_t} = f_1 \left(\frac{y_c}{H_{dam}}, \frac{q^2}{gH_{dam}^3}, \frac{h}{l}, F_r, N \right) \quad (7)$$

RESULTS AND DISCUSSION

The results are presented in four sections:

1. Relative energy dissipation
2. The effect of slope on energy dissipation
3. Comparing the relative energy dissipation of this study with previously published results
4. Statistical analysis of multivariate linear and nonlinear models for stepped spillways with different slopes of 45, 25 and 15°.

Relative energy dissipation

Figure 8 shows the results of relative energy dissipation in stepped spillways including 5–50 steps with a slope of 45°. The horizontal and vertical axes are both dimensionless. It should be noted that for the sake of brevity, other shapes for slopes of 25 and 15° are not provided in this section.

According to Figure 8, it can be seen that the relative energy dissipation decreases with increasing dimensionless ratio y_c/h . In other words, with increasing discharge, the effect of step roughness on energy dissipation decreases. Conversely, with increasing number of steps (and a reduction in the height of steps), the effect of steps on flow resistance decreases and therefore energy dissipation decreases.

Figure 9 presents another way to visualize the results, but with a change in the dimensionless factor on the horizontal axis. The interesting result is that by normalizing the dimensionless factor on the horizontal axis in $y_c/(hN)$, one line can be fitted for all test points so that a more compact equation can be obtained.

The effect of slope on energy dissipation

In Figure 10, the effect of a 45° slope (maximum slope) on energy dissipation is compared to 25 and 15° slopes. The effect of the steep slope is probably related to the turbulence, the higher flow velocity, and the incomplete or complete effect of the hydraulic jump on each step (in the case of nappe flow regime). Experimental data show that there is no significant difference in relative energy dissipation between the 25 and 15° slope cases. However, the 25° slope is slightly larger than the 15° slope case. This is probably due to the short height of the spillway. As mentioned before, the spillway models with slopes of 25 and 15° had a height of 32 cm, and therefore the difference in the length of the sloping face on which the steps are located was also small. It seems that the effect of slope on taller spillways (such as the model with a height of 100 cm) will be more apparent. Further experimentation is needed to investigate this issue.

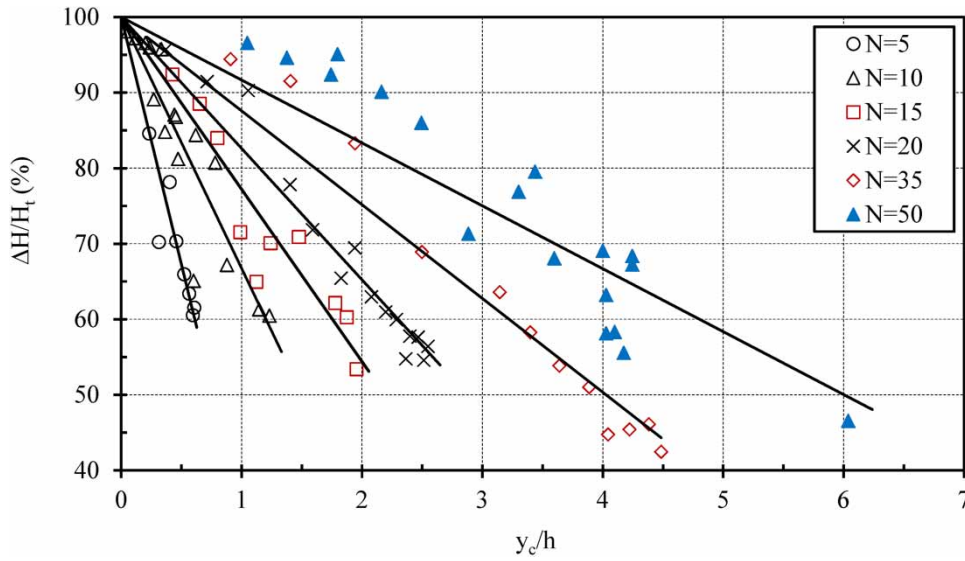


Figure 8 | Relative energy dissipation in stepped spillways of 5–50 steps with a slope of 45°.

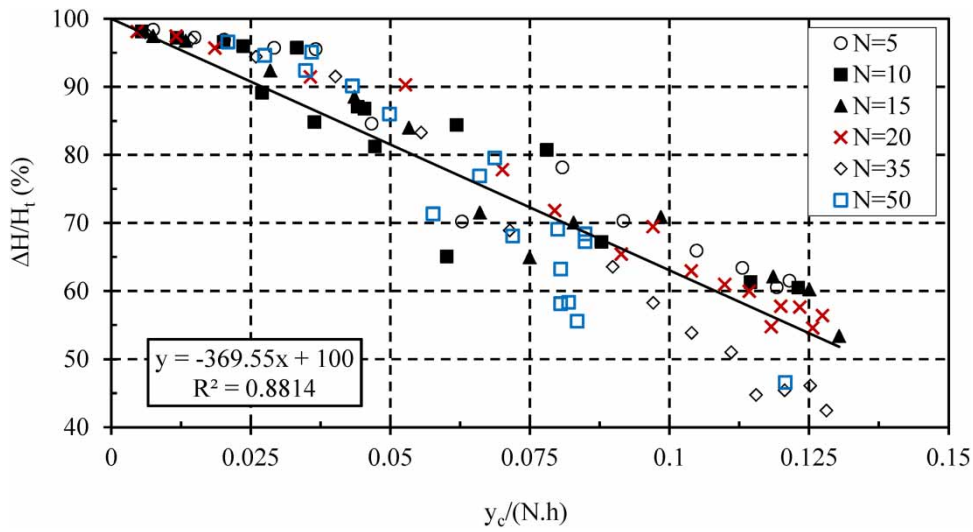


Figure 9 | Relative energy dissipation in 5–50 step (normalized) stepped spillways with a 45° slope.

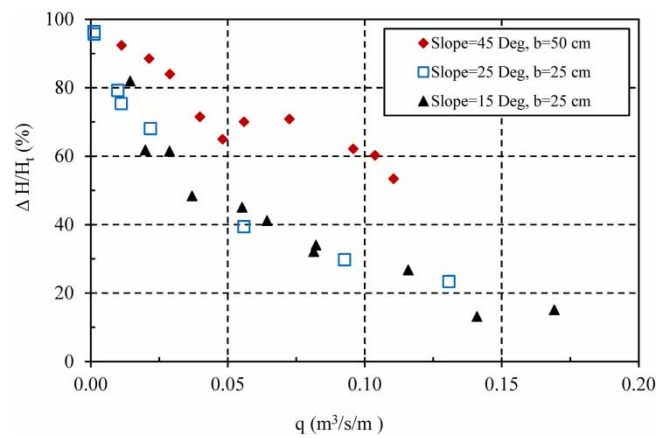


Figure 10 | Effect of slope on relative energy dissipation in a 15-step stepped spillway.

Figure 11 shows the effect of slope on relative energy dissipation in a 15-step stepped spillway. Figure 5 uses the dimensionless factor y_c/h and shows that the test points are closer to each other on three slopes and the slope effect has somehow appeared at the height of the steps. Figure 11 is a normalization of Figure 10.

Comparisons of the present results with prior publications

In Figure 12, the results of relative energy dissipation with a 45° slope from this study are compared with results of prior studies. Because the experiments of others have been performed on different slopes, we have tried to make comparisons for the most similar slopes.

The selected slopes are between 40 and 51°, which are categorized as steep slopes. According to Figure 12, the changes in relative energy dissipation are similar amongst the different datasets. Another result is that increasing the slope increases energy dissipation due to the increase in the amount of exchange movement between the rotational flow under each step

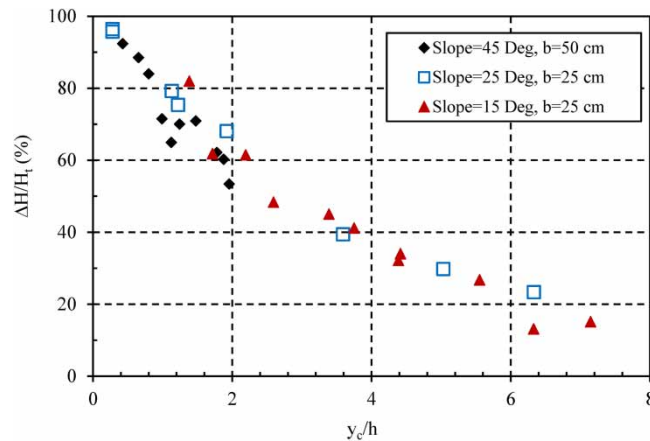


Figure 11 | Effect of slope on relative energy dissipation in a 15-step stepped spillway (normalized).

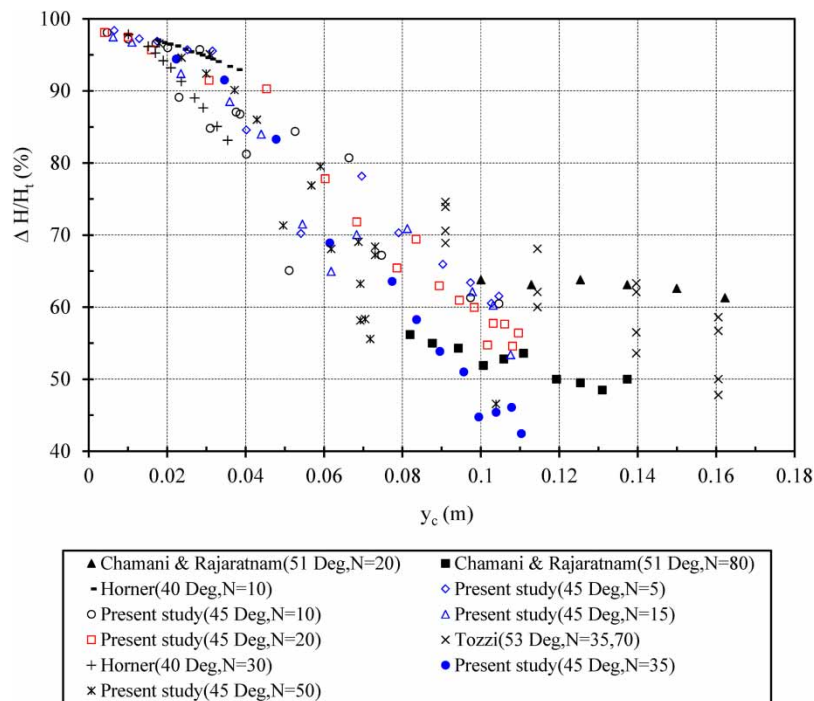


Figure 12 | Comparison between relative energy consumption with other researchers.

and the main flow. This exchange of motion between two currents at different velocities is also expressed by Felder & Chanson (2011).

In Figure 13, the results of this study are compared with the Felder & Chanson (2011) experiments. The dimensionless factor H_1/y_c was used on the vertical axis (a comparison was not possible in Figure 12), where H_1 is obtained from Equation (3). Based on Figure 13, H_1/y_c varies on average from 2.5 to 4.5 and also decreases with increasing discharge. Another point is that the results of this study show excellent agreement with the results of Felder & Chanson (2011).

Statistical analysis

Statistical analysis of multivariate linear models for slopes of 45, 25, and 15° will now be presented. All of the information was obtained from the experiments (test spillways in the flume with a widths of 25 and 50 cm), for slopes of 45, 25 and 15° and are provided. The degree and number of different steps ranging from 5 to 50 will be used in multivariate regression equations. Equation (7) obtained from dimensional analysis is considered. Regression equations were obtained by SPSS software. The results of the equations using a single dependent variable are presented in Table 2.

According to Table 2, it can be seen that the largest regression coefficient is related to the presence of dimensionless parameters $q^2/(gH_{dam}^3)$. Figure 14 shows this relationship.

For brevity, not all of the different regression combinations between other dimensionless parameters are shown; only selected equations are presented in Table 3.

According to Table 3, the correlation between relative energy dissipation and the Froude number (F_r) and $q^2/(gH_{dam}^3)$ is very high and the determination coefficient (R^2) is 903 (Equation (15)).

Figure 15 shows the scatter plot for observed and predicted relative energy dissipation using regression relation (Equation (15)). For improved accuracy, the first equation in Table 3 in which the R^2 is equal to 0.928 can be used. It can be seen that by increasing the slope of the spillway or the number of steps, the energy dissipation will also increase. Future work should be performed to investigate the suitability of using a spillway with 1 m height.

CONCLUSIONS

Nine physical models of stepped spillways with slopes of 45, 25, and 15° and with the number of steps ranging from 5 to 50 have been constructed and tested. The height of the models was 32 and 100 cm. The energy difference between upstream and

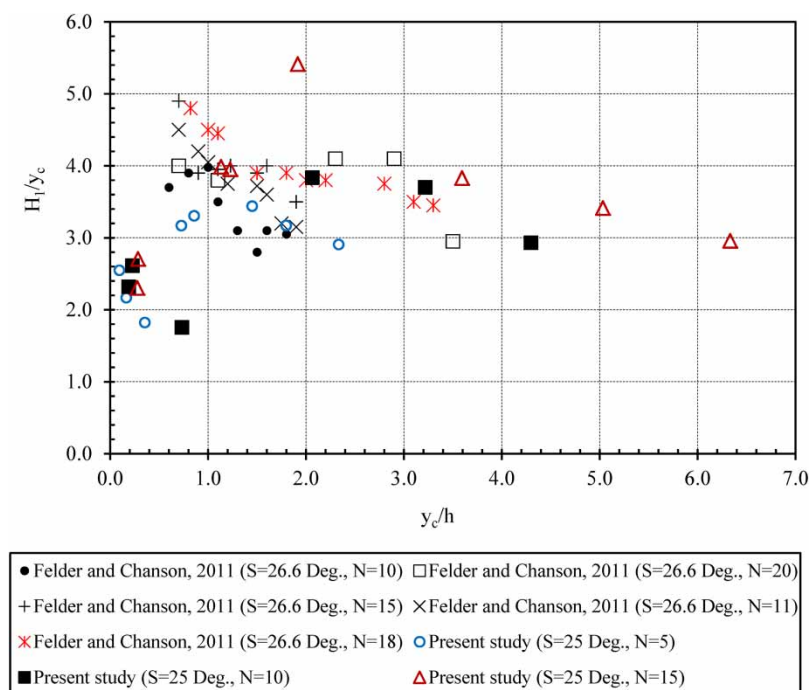


Figure 13 | Comparison of energy in downstream of stepped spillway in present study with Felder & Chanson (2011) results.

Table 2 | Correlation between dependent variable with one independent dimensionless variable

Equation	Regression equation	R ²
8	$\frac{\Delta H}{H_t} = -8.37 \times \text{Ln}\left(\frac{q^2}{gH_{dam}^3}\right) + 1.993$	0.898
9	$\frac{\Delta H}{H_t} = -16.4 \times \text{Ln}\left(\frac{y_c}{h}\right) + 67.45$	0.477
10	$\frac{\Delta H}{H_t} = -13.5 \times \text{Ln}(F_r) + 77.36$	0.106
11	$\frac{\Delta H}{H_t} = 0.007 \times (N)^2 - 0.055 \times N + 55.98$	0.052
12	$\frac{\Delta H}{H_t} = 20.17 \times (S)^{0.285}$	0.086

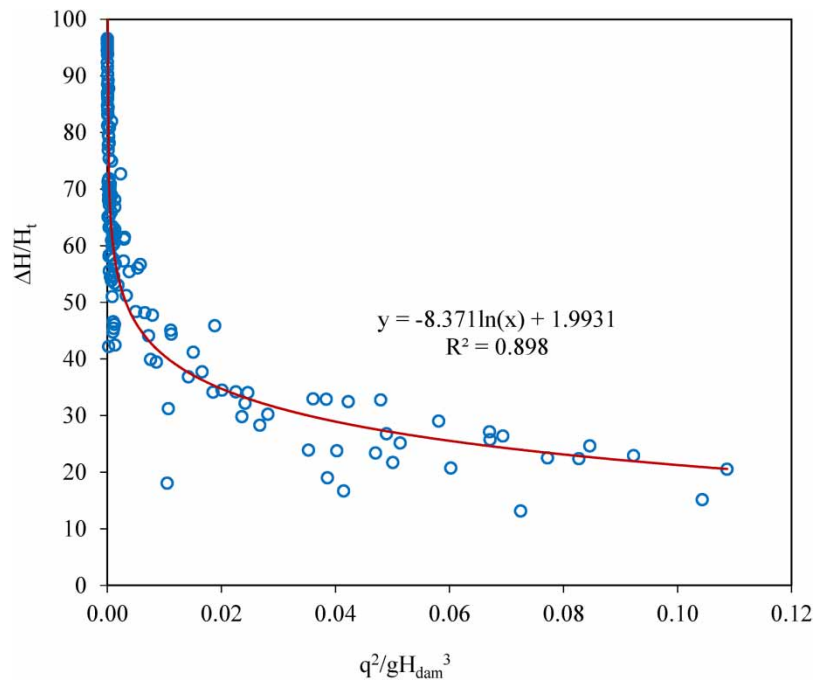


Figure 14 | The best relationship between relative energy dissipation vs. $q^2/(gH_{dam}^3)$.

Table 3 | Selected equations of different combination between dimensionless parameters for stepped spillways with a slope of 45, 25 and 15°

Equation	Regression equation	R ²
13	$\frac{\Delta H}{H_t} = -1.063 \times (F_r)^{-2.119} + 350.365 \left(\frac{y_c}{h}\right)^{-0.02} + 473.031 \left(\frac{q^2}{gH_{dam}^3}\right)^{-0.014} + 61.544N^{0.052} - 858.114S^{0.007}$	0.928
14	$\frac{\Delta H}{H_t} = 26.991 \times (F_r)^{0.075} \times \left(\frac{y_c}{h}\right)^{-0.298} \times \left(\frac{q^2}{gH_{dam}^3}\right)^{-0.048} \times N^{0.255} \times S^{-0.063}$	0.865
15	$\frac{\Delta H}{H_t} = -172464.174 \times (F_r)^{1.791 \times 10^{-5}} + 172471.549 \left(\frac{q^2}{gH_{dam}^3}\right)^{-4.747 \times 10^{-5}}$	0.903
16	$\frac{\Delta H}{H_t} = 21.748 \times (F_r)^{0.028} \times \left(\frac{q^2}{gH_{dam}^3}\right)^{-0.133}$	0.845

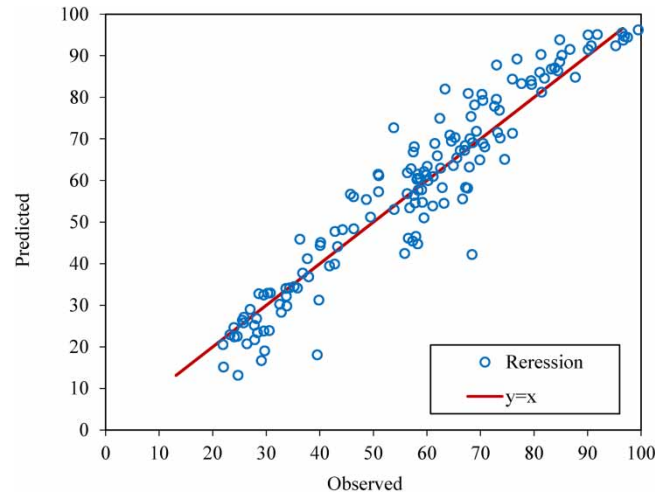


Figure 15 | Comparison between experimental data and the proposed relationship (Equation (15)) for estimating relative energy dissipation.

downstream of the spillways was measured and then the relative energy dissipation was calculated. Air entrainment inflates the water flow over stepped spillways; the bulking of water due to high air entrainment forces designers to use a greater height for sidewalls in stepped spillways and for the downstream stilling basins.

The results showed that slope alone is not an effective factor for energy dissipation and should be used in combination with other factors. With increasing slope and spillway height, relative energy dissipation increases. It is suggested that the model of stepped spillways with a height of more than 1 m be made and the effect of slope be examined again to verify the robustness of the conclusions.

The most important factors are the Froude number and the dimensionless term $q^2/(gH_{dam}^3)$. Also, multivariate regression analysis providing relationships between the independent and dependent dimensionless variables was performed and useful relationships were provided to calculate energy dissipation. In engineering applications, for convenience, a smaller number of parameters can be used to calculate energy dissipation (although with acceptable accuracy). The following relationship is recommended for this purpose:

$$\frac{\Delta H}{H_t} = -172464.174F_r^{1.791 \times 10^{-5}} + 172471.549 \left(\frac{q^2}{gH_{dam}^3} \right)^{-4.747 \times 10^{-5}}$$

ACKNOWLEDGEMENTS

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. This paper is the outcome of a research project supported by the University of Tabriz research affairs office.

ETHICAL APPROVAL

This article does not contain any studies with human participants or animals performed by any of the authors.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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First received 6 February 2022; accepted in revised form 27 April 2022. Available online 10 May 2022