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A stochastic simulation-based risk assessment method for water allocation under uncertainty

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ABSTRACT

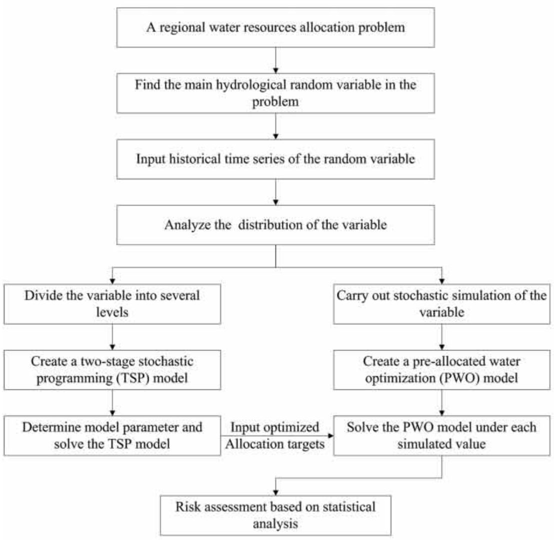
There are a lot of uncertainties in the water resources system, which makes the water allocation plan very risky. In order to analyze the risks of water resources allocation under uncertain conditions, a new methodology called the stochastic simulation-based risk assessment approach is developed in this paper. First, the main hydrological stochastic variable is fitted by a proper probability distribution. Second, suitable two-stage stochastic programming is constructed to obtain the expected benefit and optimized water allocation targets. Third, the Monte Carlo method is used to obtain a suitable stochastic sample of the hydrological variable. Fourth, a pre-allocated water optimization model is proposed to obtain optimized actual benefit. The methodology can give a way for risk analysis of water allocation plans obtained by uncertain optimization models, which provides reliable assistance to water managers in decision-making. The proposed methodology is applied to the Zhanghe Irrigation District and the risk of the water allocation plan obtained under the randomness of annual inflow is assessed. In addition, three different division methods of the annual inflow are applied in the first step, namely three levels, five levels and seven levels, respectively. From the results, the risk of the water allocation scheme obtained by the TSP model is 0.372–0.411 and decreases with the increase of the number of hydrological levels. Considering both the risk and model complexity, seven hydrological levels are recommended when using the TSP model to optimize water allocation under stochastic uncertainty.

Key words: changing environment, Monte Carlo stochastic simulation, two-stage stochastic programming, water allocation risk analysis

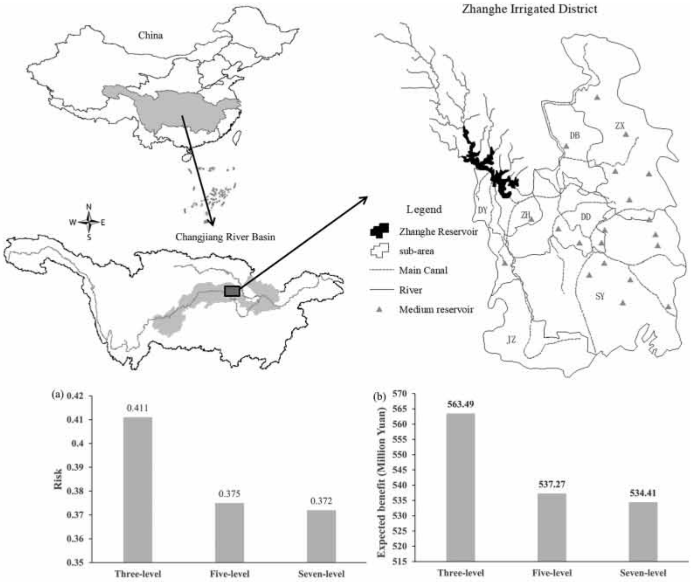
HIGHLIGHTS

- A stochastic simulation-based risk assessment method is proposed to analyze risks in water allocation plans obtained by uncertain models.
- The suitable number of hydrological levels is revealed for using TSP model to optimize water allocation under stochastic uncertainty.

GRAPHICAL ABSTRACT



(1) The flowchart of the proposed method for water allocation under uncertain condition.



(2)The application of the proposed method in the Zhanghe Irrigation District.

1. INTRODUCTION

As one of the necessary natural resources, water plays an important role in human life and socio-economic development. However, because of population growth and economic development, limited water resources are no longer enough to meet increasing water demand (Mapani *et al.* 2016). The resulting water shortage has become a restrictive factor affecting social–economic development in some regions (Kang & Cao 2012). So, various methods should be taken to develop sound management plans to improve water utilization efficiency (Wheater & Gober 2015). Optimization methods can be divided into two categories: deterministic methods and uncertain methods. Deterministic optimization methods do not take into account the various uncertainties that exist in the water resources system. However, uncertain optimization methods can handle one or more uncertainties, such as randomness, imprecision and ambiguity. Among all the uncertainties, the randomness of runoff and rainfall is the most important uncertainty factor affecting water resources allocation.

Given that deterministic optimization methods have difficulties in handling the stochastic uncertainty that exists in water management systems, various uncertain optimization methods have been developed over the past decades. Stochastic dynamic programming was employed to optimize the operation of reservoirs, crop irrigation and so on (Alaya *et al.* 2003; Cervellera *et al.* 2006; Davidsen *et al.* 2015). Then, chance constraint programming was also developed for collaborative management of water quantity and quality (Birhanu *et al.* 2014; Grosso *et al.* 2014; Kaviani *et al.* 2015). In addition, two-stage stochastic programming and multi-stage stochastic programming were successively developed and applied to optimize urban water supply, hydropower scheduling and agricultural planting (Wang & Huang 2011; Fu *et al.* 2018; Guo *et al.* 2019). Among all the above methods, two-stage stochastic programming (TSP) is considered to be a proper, sample method to optimize water resources allocation under stochastic uncertainty. For example, Ortiz-Partida *et al.* (2019) formulated a TSP model to optimize a monthly set of single reservoir releases under the randomness of the streamflow. Khosrojerdi *et al.* (2019) coupled the TSP model and interval-parameter programming to optimize water resources allocation between multi-reservoir and multi-user in Iran.

In TSP, a pre-decision is made to maximize the ‘benefits’ before the stochastic events have occurred; then, a second decision is undertaken to minimize ‘penalties’ due to the target not being met after the value of the random variable is known (Huang & Loucks 2000). However, the method is optimized using expectation as an evaluation criterion in the second stage, which indicates that it is a kind of expected risk method. So, conditional value-at-risk has been employed and integrated into uncertain optimization models to reduce the risk (Shao *et al.* 2011; Syme 2014; Hu *et al.* 2016). The risk of a water resource allocation scheme obtained by an uncertain method is lower than that obtained by a deterministic method. Uncertain methods still have some certain risks, making the water resources allocation scheme unable to achieve the expected results. To provide reliable assistance to water managers in decision-making, risk analysis should be carried out for the water allocation scheme obtained by uncertain optimization models like the TSP model. So far, many studies have been conducted to analyze the risks in water management plans obtained by deterministic optimization models (Gu *et al.* 2009; Lambourne & Bowmer 2013). However, there are few studies on risk assessment of water resource management plans obtained by uncertain methods like TSP model.

In this paper, a new methodology named the stochastic risk assessment method is proposed to carry out risk analysis for water allocation schemes obtained by the TSP model. The methodology mainly includes four contents: (1) uncertain optimization – a proper TSP model is constructed to obtain the expected benefit and optimized water allocation targets after probability distribution analysis of the hydrological random variable; (2) stochastic simulation – a suitable stochastic sample of the random variable is obtained through the Monte Carlo method; (3) deterministic optimization – a pre-allocated water optimization model is formulated to obtain the actual benefit based on the optimized allocation targets as input; (4) risk assessment – the risk is calculated through statistical analysis. The developed methodology is used for a water allocation plan obtained under uncertainty in the Zhanghe Irrigation District.

2. RESEARCH METHODOLOGY

To carry out risk analysis of a water allocation scheme obtained by the TSP model under uncertainty, a stochastic simulation-based risk assessment (SSRA) method is developed (see Figure 1). The methodology mainly includes four contents: create a suitable two-stage stochastic programming model, carry out stochastic simulation, create a proper pre-allocated water optimization model, and carry out risk assessment, which will be described separately as follows.

2.1. Two-stage stochastic programming model

Although the TSP method can handle continuous variables, it usually estimates discrete random variables to simplify model solutions. Assume ξ is the selected main hydrological random variable in the water allocation system. After fitting the suitable probability distribution of the hydrological random variable, divide ξ into Nr levels and let ξ take on value ξ_k with probability P_k for $k = 1, 2, \cdots, Nr$. Thus, following the simplified method by Huang & Loucks (2000), the TSP model for water allocation is presented as follows:

$$\max f^* = \sum_{i=1}^{Nu} T_i B_i - \sum_{i=1}^{Nu}\sum_{k=1}^{Nr} P_k S_{ik} C_i$$
$$s.t. \begin{cases} WA_k \geq \sum_{i=1}^{Nu} (T_i - S_{ik}), \forall k \\ T_i^{\max} \geq T_i \geq S_{ik} \geq 0, \forall i, k \end{cases} \tag{1}$$

where f^* represents the expected net benefit of the system (yuan), yuan represents the legal currency of China; i represents a water user; Nu represents the number of water users participating in the allocation; T_i represents the water allocation target for user i (m^3), which is the decision variable in the first stage; B_i represents the net benefit of water allocated for user i (yuan/m^3); S_{ik} represents the water shortage for user i under level k (m^3), which is the decision variable in the second stage; C_i represents the penalty of water shortage for user i (yuan/m^3); WA_k represents the available water of the system under level k (m^3); $T_i^{\max}$ represents the maximum water demand of user i. After model–solving, let T_i^* represent the optimized water allocation target for user i.

2.2. Stochastic simulation

The Monte Carlo method is a well-known method focused on stochastic modeling (Golasowski *et al.* 2015) and is used for stochastic simulation of the hydrological random variable. There are three steps to achieve the goal. Firstly, 10,000 stochastic samples with 1,000 simulated values in each sample can be obtained after fitting a suitable probability distribution of the random variable. Each simulated value is a possible value of the random variable. After that, probability distribution analysis is carried out for each sample to obtain its distribution parameters. Finally, the sample with the most suitable distribution parameters is chosen as the optimized simulation sample and the simulated values in the sample are used as the inputs for the following calculation.

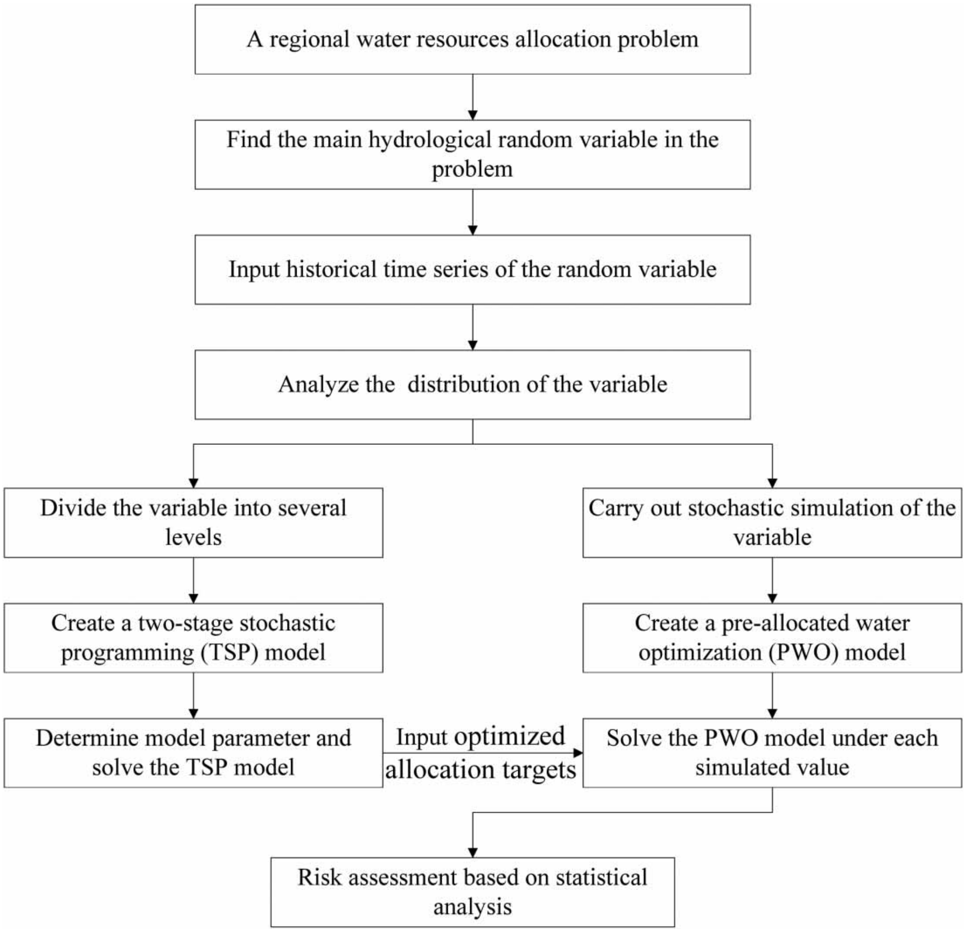


Figure 1 | The flowchart of the SSRA method for water allocation under an uncertain condition.

2.3. Pre-allocated water optimization model

In order to calculate actual benefit under each simulated value, a pre-allocated water optimization (PWO) model is developed. Based on the optimized water allocation target obtained by Equation (1), the PWO model is used to maximize the benefit of the system by optimizing the water shortage of each water user under each simulated value. The model is expressed as follows:

$$\max f_j = \sum_{i=1}^{Nu} T_i^* B_i - \sum_{i=1}^{Nu} S_{ij} C_i$$
$$s.t. \begin{cases} WA_j \geq \sum_{i=1}^{Nu} (T_i^* - S_{ij}) \\ T_i^* \geq S_{ij} \geq 0, \forall i \end{cases} \tag{2}$$

where f_j represents the actual net benefit of the system under a simulated value j (yuan); T_i^* represents the optimized water allocation target to user i obtained by Equation (1) (m^3); S_{ij} represents the water shortage for user i under a simulated value j (m^3), decision variable; WA_j represents the available water of the system under a simulated value j (m^3).

2.4. Risk assessment

Let Nr represent the number of times that f_j is less than f^*. The risk of the water allocation scheme (R) obtained by the TSP model can be calculated by Equation (3):

$$R = \frac{Nr}{Nj} \tag{3}$$

where Nj represents the total number of the simulated values.

3. CASE STUDY

3.1. Problem description

Zhanghe Irrigation District (ZID) is located in the middle reaches of the Yangtze River. It can be divided into seven subregions with some small and medium reservoirs in each subregion (see Figure 2). In addition, Zhanghe Reservoir, which is a large reservoir, is the main irrigation water source for three main crops (semi-late rice, winter rape, and cotton) in the ZID. In order to utilize the water resources reasonably and efficiently in the ZID, the manager is responsible for making a water allocation scheme to optimize water allocation between the above main crops in each subregion every year beforehand. Considering that the annual inflow of Zhanghe reservoir (AIZR) is a stochastic variable, the TSP method can be used to obtain the water allocation scheme. However, the actual benefit from the above water allocation scheme may less than the expected benefit in some years. Thus, there is a need to carry out a risk analysis of the water allocation scheme obtained by the TSP method.

3.2. Problem solution

The SSRA method is applied to solve the above problem. The main steps are as follows: analyze the probability distribution of the AIZR; set the hydrological level; create the TSP model; simulate the AIZR; create the PWO model; solve the model and perform statistical analysis.

3.2.1. Probability distribution analysis

In this study, the Pearson type III distribution is selected to fit the historical series of the AIZR, which has the length of 54 years (from 1963 to 2016). The Pearson type III distribution has been widely used to fit hydrological variables, such as runoff and precipitation in China. Following Wilks (1993), the normal quantile–quantile diagram of residuals (Q–Q diagram) is applied to assess the degree-of-fit. Figure 3(a) demonstrates the Q–Q diagram between the AIZR historical series and the best-fitted Pearson type III distribution, which indicates that the degree-of-fit is satisfactorily good.

3.2.2. Hydrological level setting

Based on the fitted probability distribution, the AIZR can be divided into several levels by selected percentiles as in Chen *et al.* (2021). In order to analyze the impact of the number of levels on the risk of the TSP model, three scenarios of

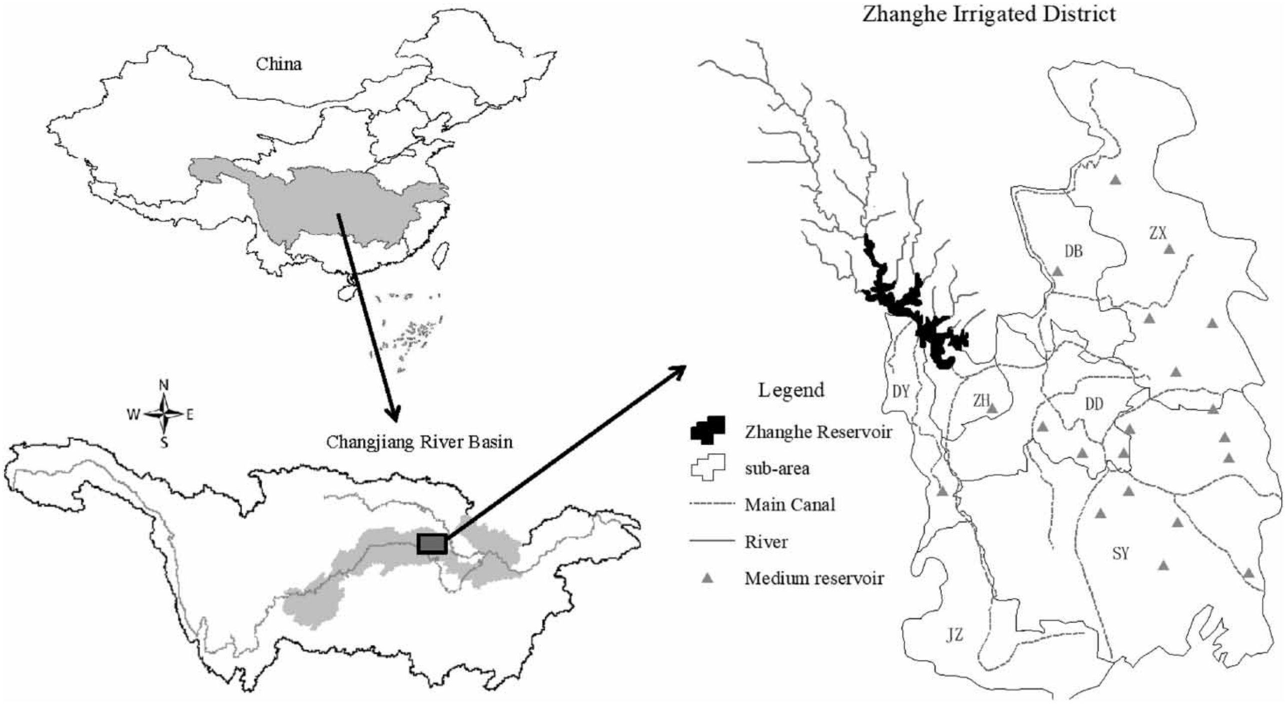


Figure 2 | Schematic of the Zhanghe Irrigation District. DB, Dongbao district; DD, Duodao district; DY, Dangyang county; JZ, Jingzhou city; SY, Shayang county; ZH, Zhanghe district; ZX, Zhongxiang county.

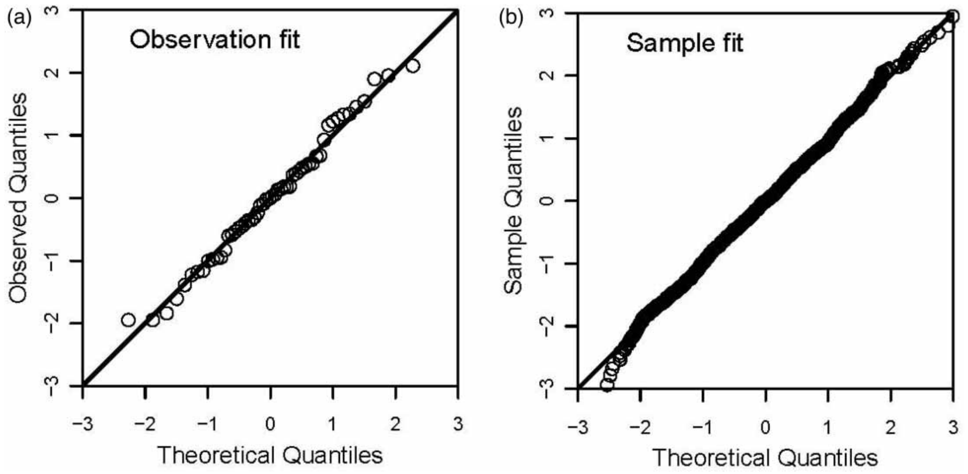


Figure 3 | Diagnostic diagram showing goodness-of-fit of the fitted distribution to the AIZR historical series: (a) fitting the observation series; (b) fitting the simulated series.

hydrological level setting are applied as follow: (a) two percentiles (25th and 75th) are applied to divide the AIZR random variable into three levels. The probabilities are 0.25, 0.5 and 0.25, respectively; (b) four percentiles (12.5th, 37.5th, 62.5th, and 87.5th) are applied to divide the AIZR random variable into five levels. The probabilities are 0.125, 0.25, 0.25, 0.25 and 0.125, respectively; (c) six percentiles (12.5th, 25th, 37.5th, 62.5th, 75th and 87.5th) are applied to divide the AIZR random variable into seven levels. The probabilities are 0.125, 0.125, 0.125, 0.25, 0.125, 0.125 and 0.125, respectively. Under each inflow level, the inflow is an interval number and the expected inflow is calculated according to the method proposed by Chen *et al.* (2021), which is shown in Table 1.

Table 1 | The expected inflow of each inflow level in each scenario

Inflow level	Probability	Inflow range (10^4 m^3)	Expected inflow (10^4 m^3)
Three-level scenario			
T 1	0.25	Less than 56,199.4	42,934.5
T 2	0.5	[56,199.4, 99,294.8]	76,344.0
T 3	0.25	More than 99,294.8	124,065.9
Five-level scenario			
F 1	0.125	Less than 44,726.9	35,146.3
F 2	0.25	[44,726.9, 65,998.3]	55,937.5
F 3	0.25	[65,998.3, 86,320.9]	75,852.9
F 4	0.25	[86,320.9, 118,549.8]	100,283.0
F 5	0.125	More than 118,549.8	140,083.6
Seven-level scenario			
S 1	0.125	Less than 44,726.9	35,146.3
S 2	0.125	[44,726.9, 56,199.4]	50,722.7
S 3	0.125	[56,199.4, 65,998.3]	61,152.9
S 4	0.25	[65,998.3, 86,320.9]	75,852.9
S 5	0.125	[86,320.9, 99,294.8]	92,517.9
S 6	0.125	[99,294.8, 118,549.8]	108,048.2
S 7	0.125	More than 118,549.8	140,083.6

3.2.3. Creation of the TSP model

According to the actual situation of Zhanghe Irrigation District, the suitable TSP model for this water allocation problem is given as follows:

$$\max f^* = \sum_{r=1}^{7}\sum_{t=1}^{3} T_{rt}B_{rt} - \sum_{r=1}^{7}\sum_{t=1}^{3}\sum_{k=1}^{Nl} P_k S_{rtk} C_{rt}$$

$$s.t.\left\{\begin{array}{l} WA_k^{\mathrm{zh}} \geq \sum\limits_{r=1}^{7} I_{rk},\ \forall k \\ I_{rk} = \left\{\begin{array}{l} 0,\ \sum\limits_{t=1}^{3}(T_{rt} - S_{rtk}) < WA_r^{\mathrm{in}}\eta_{dr} \\ \left(\sum\limits_{t=1}^{3}(T_{rt} - S_{rtk}) - WA_r^{\mathrm{in}}\eta_{dr}\right)/(\eta_{dr}\eta_{cr}),\ others \end{array}\right. ,\ \forall r,\ k \\ T_{rt}^{\max} \geq T_{rt} \geq S_{rtk} \geq 0,\ \forall r,\ t,\ k \end{array}\right. \tag{4}$$

where f^* represents the expected net benefit of the ZID (yuan); T_{rt} represents the water allocation target of crop t in subregion r (m^3), decision variable; B_{rt} represents the net benefit of crop t in subregion r (yuan/m^3); S_{rtk} represents the water shortage for crop t in subregion r under inflow level k (m^3), decision variable; C_{rt} represents the penalty of water shortage for crop t in subregion r (yuan/ m^3); WA_k^{zh} represents the available irrigation water supplied by Zhanghe Reservoir under inflow level k (m^3); I_{rk} represents the irrigation water allocated for subregion r under inflow level k (m^3); WA_r^{in} represents the internal available irrigation water supplied by reservoirs in subregion r (m^3); η_{dr} represents the water use efficiency in subregion r; η_{cr} represents the conveyance efficiency of the canal between subregion r and Zhanghe Reservoir; $T_{rt}^{\max}$ represents the maximum water demand of crop t in subregion r (m^3).

3.2.4. Stochastic simulation

Based on the best-fitted probability distribution, an optimized stochastic sample with 1,000 simulated inflows is obtained by the method described in section 2.2. Then, the normal quantile–quantile plot of residuals is applied to show the degree-of-fit of the optimized stochastic sample, which is shown in Figure 3(b). It demonstrates satisfactorily good agreements between the best-fitted probability distribution and optimized stochastic sample.

3.2.5. Creation of the PWO model

Based on Equations (2) and (4), the suitable PWO model is given as follows:

$$\max f_j = \sum_{r=1}^{7}\sum_{t=1}^{3} T_{rt}^* B_{rt} - \sum_{r=1}^{7}\sum_{t=1}^{3} S_{rtj} C_{rt}$$

$$s.t.\left\{\begin{array}{l} WA_j^{\mathrm{zh}} \geq \sum\limits_{r=1}^{7} I_{rj} \\ I_{rj} = \left\{\begin{array}{l} 0,\ \sum\limits_{t=1}^{3}(T_{rt} - S_{rtj}) < WA_r^{\mathrm{in}}\eta_{dr} \\ \left(\sum\limits_{t=1}^{3}(T_{rt} - S_{rtj}) - WA_r^{\mathrm{in}}\eta_{dr}\right)/(\eta_{dr}\eta_{cr}),\ others \end{array}\right. ,\ \forall r \\ T_{rt}^{\max} \geq T_{rt} \geq S_{rtj} \geq 0,\ \forall r,\ t \end{array}\right. \tag{5}$$

where j represents a simulated inflow; f_j represents the actual net benefit of the ZID under a simulated inflow j (yuan); T_{rt}^* represents the optimized water allocation target obtained by Equation (4); S_{rtj} represents the pre-allocated water not satisfied for crop t in subregion r under a simulated inflow j (m^3), decision variable; WA_j^{zh} represents the available water supplied by Zhanghe Reservoir under a simulated inflow j (m^3).

The PWO model is optimized under each simulated inflow within the optimized stochastic sample.

3.2.6. Parameter determination

In order to run the above two models, some parameters should be determined. Table 2 demonstrates the maximum demands, benefits, and penalties for three crops. The maximum water demands are calculated by multiplying the planting area and crop irrigation quotas. The benefit and penalty for each crop are approximated from socio-economic indicators, for example crop price, potential crop yield, and irrigation benefit coefficient. In addition, the available irrigation water supplied by Zhanghe Reservoir under each inflow level is obtained through expected inflow minus domestic and industrial water consumption and water loss. Table 3 shows the internal available irrigation water (WA_r^{in}), the water use efficiency (η_{dr}) and conveyance efficiency (η_{cr}) for each subregion.

3.3. Results analysis

3.3.1. Results of model optimization

Just as presented in section 3.2, the AIZR is converted into discrete variables under three scenarios (three-level, five-level and seven-level), respectively. For each scenario, the TSP model is first applied to obtain the expected benefit (f^*) and optimized target (T_{rt}^*). Then, the PWO model is used to calculate the actual benefit (f_j) under all 1,000 simulated inflows based on the optimized target as input. By statistical analysis of 1,000 optimization results, the frequency of each actual benefit can be calculated and is shown in Figure 4.

It can be seen that the frequency distributions of the actual benefits are similar under the three scenarios. In each plot, the frequency first increases and then decreases with the actual benefit, and finally increases to the maximum. However, when the AIZR is divided into three levels, the first peak is obtained at 400 million yuan (see Figure 4(a)). when the AIZR is divided into five levels or seven levels, the first peaks are both obtained at 450 million yuan (see Figures 4(b) and 4(c)).

3.3.2. Risks of optimized allocation schemes

According to the expected benefit and the distribution of the actual benefits, the risks of the water allocation scheme under the three scenarios are obtained by Equation (3) and the results are shown in Figure 5(a). The larger the risk, the less likely it is to reach the expected benefit. It is demonstrated that the risk of water allocation by the TSP method in the ZID is 0.372–0.411. In addition, the risk is the largest when the AIZR is divided into three levels. However, the risk is the smallest when the

Table 2 | The maximum demands, benefits and penalties for three crops in seven subregions

Water user		Maximum water demand (10^4 m^3)	Benefit (yuan/m^3)	Penalty (yuan/m^3)
Dongbao	Semi-late rice	2,044.96	1.58	2.77
	Winter rape	213.25	5.49	9.62
	Cotton	16.20	6.48	11.35
Duodao	Semi-late rice	4,698.67	1.50	2.77
	Winter rape	490.15	5.21	9.62
	Cotton	37.40	6.15	11.35
Dangyang	Semi-late rice	3,152.37	1.61	2.77
	Winter rape	336.80	5.58	9.62
	Cotton	50.37	6.59	11.35
Jingzhou	Semi-late rice	3,154.15	1.39	2.77
	Winter rape	349.31	4.84	9.62
	Cotton	26.66	5.71	11.35
Shayang	Semi-late rice	19,881.18	1.34	2.77
	Winter rape	2,088.47	4.65	9.62
	Cotton	159.05	5.49	11.35
Zhanghe	Semi-late rice	1,353.98	1.74	2.77
	Winter rape	141.20	6.05	9.62
	Cotton	10.75	7.14	11.35
Zhongxiang	Semi-late rice	4,586.73	1.42	2.77
	Winter rape	478.00	4.93	9.62
	Cotton	36.52	5.82	11.35

Table 3 | The internal available irrigation water, the water use efficiency and conveyance efficiency for each subregion

Subregion	Internal available water (10^4 m^3)	Water use efficiency (η_{dr})	Conveyance efficiency (η_{cr})
Dongbao	1,681.3	0.65	0.92
Duodao	1,425.5	0.61	0.92
Dangyang	526.1	0.63	0.96
Jingzhou	519.9	0.64	0.82
Shayang	1,544.3	0.6	0.83
Zhanghe	757.7	0.66	0.98
Zhongxiang	2,440.7	0.62	0.85

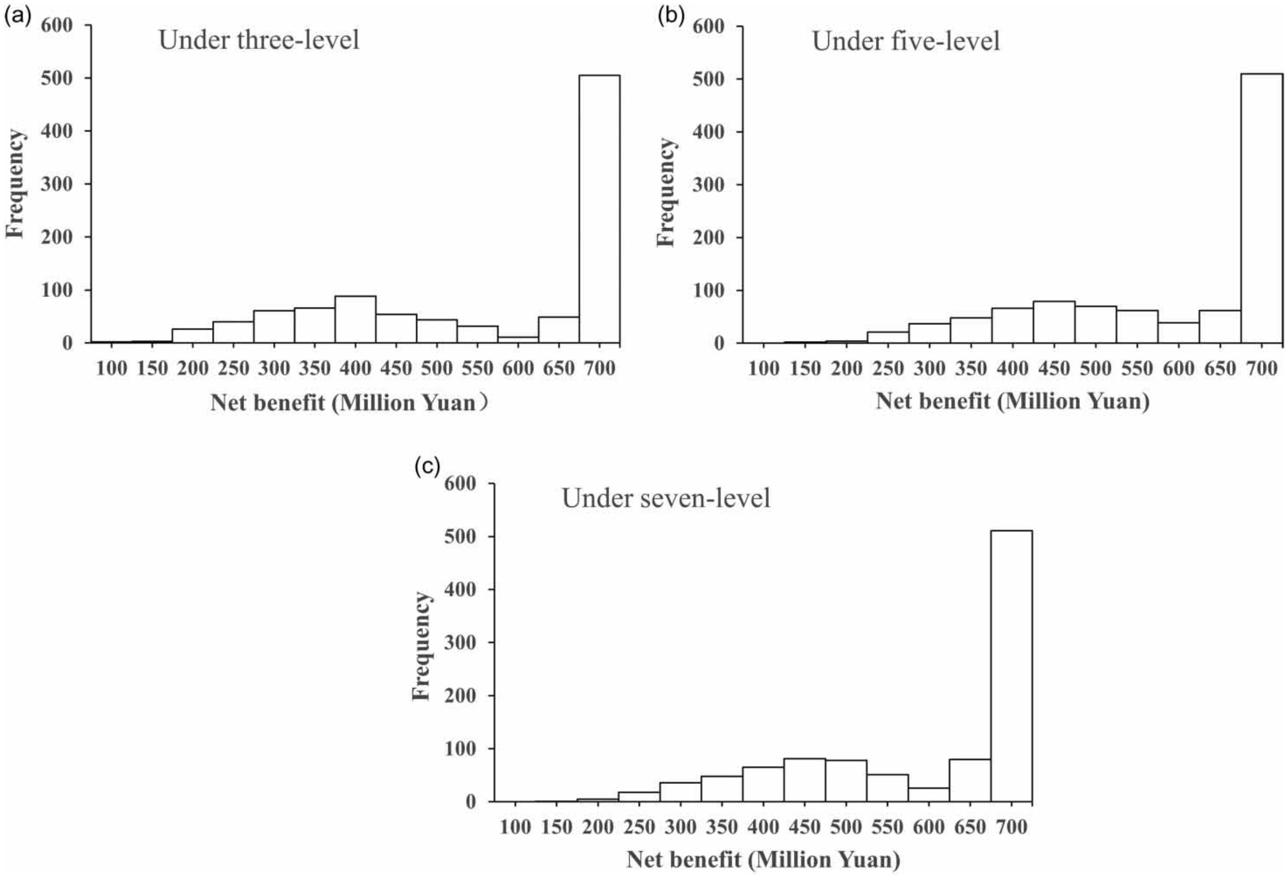


Figure 4 | The frequency of actual benefits obtained by the PWO model under three scenarios.

AIZR is divided into seven levels. The above results indicate that the more the number of dividing levels of the AIZR, the smaller the risk of the TSP method.

In addition, the difference between the risk under five levels and seven levels is much smaller than that between the risk under three levels and five levels. It is expected that the difference between the risk under seven levels and nine levels may be smaller than that between the risk under five levels and seven levels. The more the number of dividing levels of the random variable, the more the decision variables of the TSP model and the more difficult it is to solve the model. Therefore, it is recommended to divide the random variable into seven levels when using the TSP model to optimize water allocation under uncertainty.

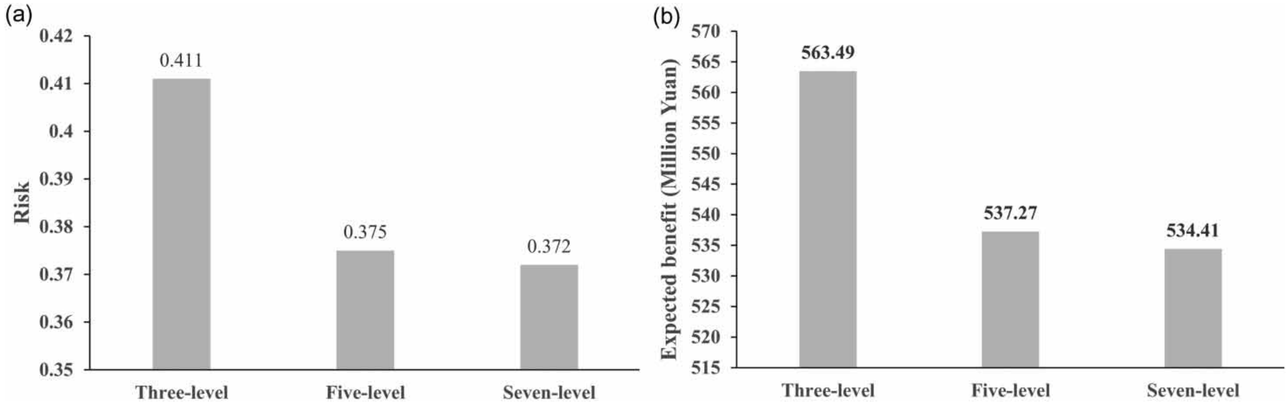


Figure 5 | The risk and expected benefit of the water allocation schemes obtained by the TSP model under each scenario.

4. DISCUSSION

To investigate the reason for the above results, expected benefit and optimized targets obtained by the TSP model under each scenario are compared. The optimized water allocation target for each crop (T_{rt}^*) can be written as $Z_{rt}T_{rt}^{\max}$ by introducing optimized allocation coefficients (Z_{rt}). Table 4 shows the optimized allocation coefficients of three crops obtained by the TSP model under the three scenarios. The optimized allocation coefficients under three levels are larger than that under five levels and seven levels. It indicates that in a wet year in which the available water is enough for all water users, larger benefit could be obtained under three levels than that under the other two scenarios, while in a dry year in which the available water is not enough for all water users, higher recourse cost may be achieved under three levels than that under the other two scenarios. Therefore, actual benefit at the first peak under three levels is lower than that under the other two scenarios (see Figure 4).

Figure 5(b) shows the expected benefits obtained by the TSP model under the three scenarios. The expected benefit is the largest under three levels, while it is the smallest under seven levels. The above results are similar to those of the risks (see Figure 5(a)). Because the expected benefit is larger and recourse cost is higher in a dry year, the expected benefit is harder to achieve under three levels. Therefore, the risk is larger under three levels than that under the other two scenarios.

Table 4 | The optimized allocation coefficients for three crops obtained by the TSP model under the three scenarios

Scenario	Sub-district	Semi-late rice	Winter rape	Cotton
Three-level	Dongbao	1	1	1
	Duodao	1	1	1
	Dangyang	1	1	1
	Jingzhou	0.77	1	1
	Shayang	0.77	1	1
	Zhanghe	1	1	1
	Zhongxiang	1	1	1
Five-level	Dongbao	1	1	1
	Duodao	1	1	1
	Dangyang	1	1	1
	Jingzhou	0.77	1	1
	Shayang	0.77	1	1
	Zhanghe	1	1	1
	Zhongxiang	0.87	1	1
Seven-level	Dongbao	1	1	1
	Duodao	1	1	1
	Dangyang	1	1	1
	Jingzhou	0.77	1	1
	Shayang	0.77	1	1
	Zhanghe	1	1	1
	Zhongxiang	0.81	1	1

5. CONCLUSIONS

This study develops a methodology to carry out risk analysis of the water allocation scheme obtained by the TSP model. The method mainly includes four parts: uncertain optimization, stochastic simulation, deterministic optimization, and risk assessment. After probability distribution analysis of the hydrological stochastic variable, a suitable TSP model is constructed to calculate the expected benefit and optimized water allocation targets. Then, the Monte Carlo method is used to generate the optimized stochastic sample. After that, a deterministic optimization model called a pre-allocated water optimization model is proposed to obtain the actual benefit under each simulated value in the optimized stochastic sample. Finally, the risk of the TSP model is calculated through comparing all the actual benefits and the expected benefit.

The methodology has been applied to the ZID to carry out risk analysis of a water allocation scheme obtained under the randomness of AIZR. The results show that the risk of the water allocation scheme obtained by the TSP model is 0.372–0.411 and decreases with the increase of number of hydrological levels in the ZID. The obtained water allocation plan and its corresponding risk can provide a good foundation for water resources management in the ZID. Moreover, it is recommended to divide the random variable into seven levels when using the TSP model to obtain a water allocation scheme under uncertainty.

There are various uncertainties in the process of water allocation, such as the randomness of hydrologic features, imprecision of water demand, and ambiguity of social–economic parameters. This paper only considers the impact of stochastic uncertainty on the risk of water allocation. Furthermore, this paper only discusses the allocation risk from the perspective of water quantity, and does not include the risk of joint allocation of water quality and quantity. The risk of the joint allocation of water quality and quantity needs further research.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

REFERENCES

Alaya, A. B., Souissi, A., Tarhouni, J. & Ncib, K. 2003 Optimization of Nebhana reservoir water allocation by stochastic dynamic programming. *Water Resources Management* **17** (4), 259–272.

Birhanu, K., Alamirew, T., Dinka, M. O., Ayalew, S. & Aklog, D. 2014 Optimizing reservoir operation policy using chance constraint nonlinear programming for Koga irrigation dam, Ethiopia. *Water Resources Management* **28** (14), 4957–4970.

Cervellera, C., Chen, V. C. P. & Wen, A. 2006 Optimization of a large-scale water reservoir network by stochastic dynamic programming with efficient state space discretization. *European Journal of Operational Research* **171** (3), 1139–1151.

Chen, S., Xu, J., Li, Q., Wang, Y., Yuan, Z. & Wang, D. 2021 Nonstationary stochastic simulation method for the risk assessment of water allocation. *Environmental Science: Water Research & Technology* **7**, 212–221.

Davidsen, C., Pereira-Cardenal, S. J., Liu, S., Mo, X., Rosbjerg, D. & Bauer-Gottwein, P. 2015 Using stochastic dynamic programming to support water resources management in the Ziya River Basin, China. *Journal of Water Resources Planning & Management* **141** (7), 04014086.

Fu, Q., Li, T., Cui, S., Liu, D. & Lu, X. 2018 Agricultural multi-water source allocation model based on interval two-stage stochastic robust programming under uncertainty. *Water Resources Management* **32** (4), 1261–1274.

Golasowski, M., Litschmannová, M., Kuchař, S., Podhorányi, M. & Martinovič, J. 2015 Uncertainty modelling in rainfall–runoff simulations based on parallel Monte Carlo method. *Neural Network World* **25** (3), 267–286.

Grosso, J. M., Ocampo-Martínez, C., Puig, V. & Joseph, B. 2014 Chance-constrained model predictive control for drinking water networks. *Journal of Process Control* **24** (5), 504–516.

Gu, W., Shao, D., Huang, X. & Tao, D. 2009 Multi-objective risk assessment on water resources optimal allocation. *Journal of Hydraulic Engineering* **39** (3), 339–345 (in Chinese with English abstract).

Guo, S., Zhang, F., Zhang, C., Wang, Y. & Guo, P. 2019 An improved intuitionistic fuzzy interval two-stage stochastic programming for resources planning management integrating recourse penalty from resources scarcity and surplus. *Journal of Cleaner Production* **234**, 185–199.

Hu, Z., Wei, C., Yao, L., Li, L. & Li, C. 2016 A multi-objective optimization model with conditional value-at-risk constraints for water allocation equality. *Journal of Hydrology* **542**, 330–342.

Huang, G. H. & Loucks, D. P. 2000 An inexact two-stage stochastic programming model for water resources management under uncertainty. *Civil Engineering Systems* **17** (2), 95–118.

Kang, G. D. & Cao, Y. M. 2012 Development of antifouling reverse osmosis membranes for water treatment: a review. *Water Research* **46** (3), 584–600.

Kaviani, S., Hassanli, A. M. & Homayounfar, M. 2015 Optimal crop water allocation based on constraint-state method and nonnormal stochastic variable. *Water Resources Management* **29** (4), 1003–1018.

Khosrojerdi, T., Moosavirad, S. H., Ariafar, S. & Ghaeini-Hessaroeyeh, M. 2019 Optimal allocation of water resources using a two-stage stochastic programming method with interval and fuzzy parameters. *Natural Resources Research* **28** (3), 1107–1124.

Lambourne, M. & Bowmer, K. H. 2013 Investigating over-allocation of water using risk analysis: a case study in Tasmania, Australia. *Marine and Freshwater Research* **64** (8), 761–773.

Mapani, B., Meck, M., Makurira, H., Magole, L., Mashauri, D., Mazvimavi, D. & Mul, M. 2016 Water: the conveyor belt for sustainable livelihoods and economic development. *Physics and Chemistry of the Earth* **92**, 1–2.

Ortiz-Partida, J. P., Kahil, T., Ermolieva, T., Ermoliev, Y., Lane, B., Sandoval-Solis, S. & Wada, Y. 2019 A two-stage stochastic optimization for robust operation of multipurpose reservoirs. *Water Resources Management* **33** (11), 3815–3830.

Shao, L. G., Qin, X. S. & Xu, Y. 2011 A conditional value-at-risk based inexact water allocation model. *Water Resources Management* **25** (9), 2125–2145.

Syme, G. J. 2014 Acceptable risk and social values: struggling with uncertainty in Australian water allocation. *Stochastic Environmental Research and Risk Assessment* **28** (1), 113–121.

Wang, S. & Huang, G. H. 2011 Interactive two-stage stochastic fuzzy programming for water resources management. *Journal of Environmental Management* **92** (8), 1986–1995.

Wheater, H. S. & Gober, P. 2015 Water security and the science agenda. *Water Resources Research* **51** (7), 5406–5424.

Wilks, D. S. 1993 Comparison of three-parameter probability distributions for representing annual extreme and partial duration precipitation series. *Water Resources Research* **29** (10), 3543–3549.

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