A comparison of two infiltration models applied to simulation of overland flow over a two-dimensional flume

K. J. B. Mallari, H. Kim, G. Pak, H. Aksoy and J. Yoon

ABSTRACT

At the hillslope scale, where the rill-interrill configuration plays a significant role, infiltration is one of the major hydrologic processes affecting the generation of overland flow. As such, it is important to achieve a good understanding and accurate modelling of this process. Horton’s infiltration has been widely used in many hydrologic models, though it has been occasionally found limited in handling adequately the antecedent moisture conditions (AMC) of soil. Holtan’s model, conversely, is thought to be able to provide better estimation of infiltration rates as it can directly account for initial soil water content in its formulation. In this study, the Holtan model is coupled to an existing overland flow model, originally using Horton’s model to account for infiltration, in an attempt to improve the prediction of runoff. For calibration and validation, experimental data from a two-dimensional flume which is incorporated with hillslope configuration have been used. Calibration and validation results showed that Holtan’s model was able to improve the modelling results with better performance statistics than the Horton-coupled model. Holtan’s infiltration equation, which allows accounting for AMC, provided an advantage and resulted in better runoff prediction of the model.

Key words | hillslope, Holtan model, Horton model, infiltration model, overland flow

INTRODUCTION

As part of the overall watershed management, accurate prediction of rainfall-runoff processes has been an important topic of study over the years. While most of developed mathematical models deal with modelling at the global-catchment scale, the effects of microtopographic configuration such as rills and interrills are rarely incorporated into hydrological analyses of rainfall-runoff partitioning. Microtopography is expected to play an important role in ecohydrological processes of arid and semi-arid systems where the portioning of rainfall between infiltration and runoff at the soil surface is considered important (Thompson et al. 2010). In the local irregularities on the surface of hillslopes, runoff flows into low-lying areas. It then forms concentration flow routes where prediction of runoff generation and soil erosion is significant (Liu et al. 2004).

Developed mathematical models for predicting overland flow are often incorporated into models simulating sediment transport and soil erosion (Liu et al. 2006; Deng et al. 2005; An & Liu 2009; Arguelles et al. 2013) and may also be further incorporated into larger watershed scale models. Furthermore, models of this type may be intended for planning best management practices in areas where solute transport by overland flow is a major cause of pollution. In addition, accurate prediction of overland flow may also help increase the efficient utilization of applied chemicals particularly in agricultural lands.

Overland flow modelling has been undertaken at different scales using different techniques. Kavvas & Govindaraju (1992) performed stochastic averaging of the hydrodynamic processes over a rilled hillslope, though it was found limited due to assumptions of one-dimensional (1D) flow at overland sections, straight and parallel rills, and no interaction between interrill and rill flows. Tayfur & Kavvas (1994, 1998) who first performed spatial averaging to local-scale interrill and cross-section averaged rill flow equations later did longitudinal averaging of the local-scale equations which were further ensemble averaged over the hillslope transect to represent the entire hillslope. However, the equations developed in these studies required significantly more computational effort involved with the solution procedure (Tayfur & Kavvas 1994) or they lacked spatial details on hydraulics along a hillslope (Tayfur & Kavvas...
Liu et al. (2004) developed a grid-based model which simulates the flow in a cell by the kinematic wave model. A different flowline routine is specified in every grid and the inflow and the outflow discharge of the adjacent grids are calculated to estimate the flow from the entire hillslope. In one way or another, these models demonstrate limitations in accounting for rills and interrills explicitly and establishing their flow connection, while at the same time providing adequate spatial details of hydraulics and ease of computation. It is important to overcome these limitations in order to achieve a realistic modelling of the interacting interrill-rill configuration and at the same time, reduce the computational burden with physically based modelling of overland flow.

Infiltration is a key component in these overland flow models. A large number of models for its determination have been developed over the decades but their suitability for real world data is not very clear. Therefore, it is not always evident as to which model is better and under what conditions (Mishra et al. 2005). Most runoff models transform the infiltration excess into runoff using physically based models, conceptual models or empirical relations (Chahinian et al. 2005). Horton’s infiltration model, one of the earliest developed methods for computing infiltration, has been used in quite a number of hydrologic models such as MARINE and SWMM, among others (Huber & Dickinson 1988; Tayfur & Kavvas 1994; Estupina-Borrell et al. 2002; Tayfur 2007; Lee & Huang 2013). However, one of its limitations is not being able to handle antecedent moisture conditions (AMC) of soil adequately. Horton’s view of infiltration capacity as being largely controlled by surface processes and neglecting the role of capillary potential gradients in the soil has often given the impression that it is purely an empirical equation, curve-fitted to measured data (Beven 2004). The Holtan model, though empirical in nature, is thought to be able to provide a better estimation of infiltration since its formulation is expressed in terms of available water storage, which can directly consider initial soil water content, soil porosity and other soil variables. It has also been employed in hydrologic models such as LISEM and ANSWERS (Dillaha & Beasley 1983; Huggins & Monke 1966; De Roo et al. 1996) which have often been applied to predict overland flow and erosion from a watershed through division of the area into cells. However, these process-based models do not explicitly account for the existence of rill and interrill areas in a hillslope section which is an important aspect of the overall catchment modelling.

In the authors’ previous work (Arguelles et al. 2013), the overland flow model coupled to the Horton’s model had displayed some limitations in accounting for wetter soil conditions resulting in adjustment of calibrated parameters during the validation. Therefore, the objectives of this study were to incorporate the Holtan model into the existing model and compare the performance of the two infiltration models in the prediction of overland flow in an attempt to improve the modelling results of the previous study. The developed model explicitly accounts for the microtopographical features of a hillslope such as rill and interrill areas and their hydraulic connections which allows for a high-resolution overland flow modelling at the hillslope scale. Calibration and validation of results using data from a laboratory flume set up (Aksoy et al. 2012) were done to evaluate model performance.

MATERIALS AND METHODS

Existing overland flow model

The governing equations for overland flow in this study are based on the kinematic wave approximation of both sheet and rill flows. Based on the local-scale interrill flow derivation by Tayfur & Kavvas (1994), the 1D interrill flow equation of this model has been derived as (Arguelles et al. 2013)

$$\frac{\partial h_o}{\partial t} + \frac{\partial}{\partial x} \left( K_x h_o^{3/2} \right) = q_i - \frac{\pi}{2} \frac{3}{2} K_x h_o^{5/2}$$

where \( h_o(x,t) \) = width-averaged interrill flow depth, \( K_x = C_x S_{ax}^{1/2} \left[ 1 + \left( \frac{S_{ox}}{S_{ax}} \right)^2 \right]^{1/4} \), \( K_y = C_y S_{oy}^{1/2} \left[ 1 + \left( \frac{S_{oy}}{S_{ax}} \right)^2 \right]^{1/4} \), \( C_x \) = Chezy’s roughness coefficient, \( S_{ax} \) = bed slope in the x- and y-directions, \( c = (\pi/2)^{3/2} \left( 1/l \right) \int_0^{\pi} \sin^{3/2}(\theta/2l) \, d\theta = 1.09542 \), \( l \) = interrill area width, and \( q_i(t) \) = rainfall excess [rainfall (i) – infiltration (f), see (9) and (10)].

Meanwhile, the rill flow equation is derived from the cross-sectionally averaged flow equation (Chen & Chow 1971)

$$\frac{\partial h_r}{\partial t} + \frac{\partial}{\partial x} \left( K_x \frac{w_r^{1/2} h_r^{1/2}}{(w_r + 2h_r)^{1/2}} \right) = q_i + \frac{\pi}{2} \frac{3}{2} K_x \frac{T_r^{3/2}}{w_r}$$

where \( h_r(x,t) \) = rill flow depth, \( K_x = C_{xr} S_{rx}^{1/2} \), \( C_{tr} \) = rill Chezy’s roughness coefficient, \( S_{ax} \) = bed slope in the x-direction, \( w_r \) = rill width, and \( q_i(t) \) = rainfall excess [rainfall (i) – infiltration (f)]. The second term of the right-hand side of (2) represents the water flux going into the rill from the interrill area.
A four-point implicit scheme (Woolhiser et al. 1990) was employed to discretize these equations, and the following initial and boundary conditions were used.

Initial condition: initially dry condition

\[ h_o(x, 0) = 0; h_f(x, 0) = 0 \]  

(3)

Boundary condition: zero depth at the upstream end

\[ h_o(0, t) = 0; h_f(0, t) = 0 \]  

(4)

Starting from the upstream end, the flow depth at the next time step was computed toward the downstream end by solving the resulting non-linear equation using the Newton–Raphson method as had been done in Woolhiser et al. (1990) and Tayfur & Kavvas (1994).

In deriving (1) and (2), the discharge per unit width for the interrill and rill, \( q_i \) and \( q_r \), in the continuity equation were expressed using the Chezy relationship

\[ q_i = Kx \cdot h_o^{3/2} \]  

(5)

\[ q_r = K_r \cdot \frac{w_i^{1/2} h_f^{3/2}}{(w_r + 2h_f)^{1/2}} \]  

(6)

Multiplying this expression by the respective width will then give the simulated discharge at the interrill \( Q_i \) and rill \( Q_r \) as given in (7) and (8), respectively. The total discharge \( Q_t \) is the sum of these two equations.

\[ Q_i = q_i \cdot l \]  

(7)

\[ Q_r = q_r \cdot w_r \]  

(8)

Infiltration plays a vital role in the formation of rainfall excess. In the model, infiltration is computed at specified time and spatial steps assigned by the user. To generate rainfall excess in (1) and (2), the rate of infiltration is subtracted from the rate of rainfall which occurs for a specified duration. In the original algorithm, Horton’s model was used to account for infiltration given through the following expression (Chow et al. 1988):

\[ f = f_c + (f_0 - f_c)e^{-kt} \]  

(9)

where \( f \) = infiltration capacity, \( f_0 \) = initial infiltration rate, \( f_c \) = final infiltration rate, and \( k \) = decay constant. As Horton’s model is empirical in nature, all three parameters, \( f_0, f_c \) and \( k \) will be calibrated.

**Incorporation of Holtan model into existing overland flow model**

Holtan’s model is an empirical equation based on a storage concept. The infiltration capacity is proportional to the available storage to hold water in the surface layer of the soil. As water infiltrates, the available storage is reduced, and the infiltration capacity decreases accordingly (Akan & Houghtalen 2003). The Holtan equation (Holtan & Lopez 1971) solves for infiltration capacity as follows:

\[ f = GI \cdot a \cdot S_A^{1/4} + f_l \]  

(10)

where \( f \) = infiltration capacity, \( GI \) = growth index, \( a \) = vegetative factor, \( S_A \) = available storage to hold water in the surface layer of the soil, and \( f_l \) = final infiltration capacity. The available storage \( S_A \) is reduced during the rainfall-infiltration process as rainwater infiltrates into the soil. Over a discrete time interval \( \Delta t \), this reduction is calculated as \( \Delta S_A = \Delta t \cdot (f - f_l) \) where \( f \) = infiltration capacity during the time interval. To start the calculations, the initial value of \( S_A \) is determined as the product of the depth of the surface soil layer \( (d) \) and \( \phi (1-S_i) \), where \( \phi \) = effective porosity of the soil and \( S_i \) = initial degree of saturation. The parameters calibrated were \( a, S_A \) and \( f_l \).

**Laboratory flume data**

To evaluate model performance, laboratory data from flume experiments by Aksoy et al. (2012) were used for the calibration and validation of the model. The experimental design consists of a two-dimensional (2D) flume 650 cm long, 1.36 cm wide and 17 cm deep, which can be given both lateral and longitudinal slopes. In this study, the slopes used were 10% and 5% in the x- and y-directions \( (S_{ox}, S_{oy}) \), respectively. The rainfall simulator, equipped with four or five VeeJet nozzles spaced at 125–145 cm depending on the rainfall intensity, was designed to provide natural rainfall characteristics over the flume. The experiments were performed using four different rainfall intensities (105, 85, 65 and 45 mm/hr). Of these four, the highest and lowest were used for calibration and validation, respectively, in order to cover for a wide range of rainfall intensities.
Figure 1 shows how the microtopography of rills and interrill areas was represented in the flume. Using similarity principles, only one half of channel, along with one rill ($w_r = 46$ cm) and one interrill ($l = 90$ cm) area, were modelled in the laboratory experiment. Most of the interrill area contributes to flow in the rill while a small portion of the interrill area flows directly to the channel. As seen in the right-hand side of Figure 1, two outlets were formed: outlet (1) is used for collecting flow from interrill area and rill together, while outlet (2) is used for direct contribution from the interrill area to the channel.

**Evaluation criteria for model performance**

To evaluate calibration and validation, the Nash–Sutcliffe efficiency (NSE) given in (11) and coefficient of determination ($r^2$) were used.

\[
NSE = 1 - \frac{\sum_{t=1}^{n} (x_{s,t} - x_{o,t})^2}{\sum_{t=1}^{n} (x_{o,t} - \mu_o)^2} \tag{11}
\]

where $n$ is the total number of time steps, $x_{s,t}$ is the simulated value at time step $t$, $x_{o,t}$ is the observed value at time step $t$, and $\mu_o$ is the mean of the observed values.

These statistical criteria are widely used for evaluation of hydrologic models with observed data (Gupta et al. 2009). NSE ranges from $-\infty$ to 1.0, with NSE = 1 being the optimal value. Values between 0.0 and 1.0 are generally viewed as acceptable levels of performance, whereas values <0.0 indicate that the mean observed value is a better predictor than the simulated value, which indicates unacceptable performance. $r^2$ ranges from 0 to 1, with higher values indicating less error variance, and typically values greater than 0.5 are considered acceptable (Moriasi et al. 2007).

**RESULTS AND DISCUSSION**

Data from the higher intensity rainfall (105 mm/hr) were used to calibrate the model. The calibration was carried out by trial and error procedure, where simulations were run to locate the optimum set of model parameters. Since the infiltration equations have empirically determined parameters, the parameters values used in this study were obtained from previous literature for similar soil properties under consideration (Holtan & Lopez 1971; Pit et al. 1999; Mishra et al. 2005). After calibration of infiltration parameters, the roughness coefficient used in the overland flow equations was also calibrated. Different values were assigned for the interrill area and rill to distinguish broad sheet flow and concentrated flow in these two areas, respectively. The range of parameters and calibrated values are listed in Table 1.
During the calibration of the infiltration parameters, the \( f_c \) parameter for Horton’s model and \( S_A \) were found to have the most influence on the runoff hydrograph. Changing the values of these two parameters significantly affected the amount of runoff volume, peak, and overall shape of the runoff hydrograph as well. Conversely, the roughness coefficient affected the steepness of the rising and receding limb such that changing its values shifted the hydrograph to the left or right.

A comparison of calibration results shows that using the Holtan equation for infiltration produced slightly favourable results over the Horton infiltration model in terms of NSE as shown in Table 2. Although the NSE values for the interrill are slightly lower, it can still be said that the overall results are better for the Holtan model because of the higher NSE in the rill and combined flow. Moreover, in terms of the amount of runoff, the contribution from the rill to the total measured flow is considerably greater than that of the interrill. Therefore, the lower values for interrill only had minimal effects on the overall statistics and model performance.

The total simulated discharge \( Q_t \), using both infiltration equations with the overland flow model, is fitted to the observed points as shown in Figure 2. The most pronounced difference can be found in the rising limb of the two hydrographs where it can be observed that the start of the runoff for Horton is more delayed than that of the Holtan results. The concept of ponding time is incorporated in Horton infiltration model’s formulation. Therefore, varying the parameters can easily adjust the time when runoff begins. As for Holtan’s model, the time when runoff begins is largely dependent on the available storage parameter. Increasing its value can indeed delay the time runoff begins but this will also decrease the amount of runoff quite significantly.

Overall, good agreements were found for the observed and simulated flows of the overland flow model during calibration. From the performance statistics and runoff hydrographs produced, the results of the study are comparable to those of other overland flow studies which similarly displayed

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Range</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial infiltration rate</td>
<td>( f_0 )</td>
<td>cm/hr</td>
<td>11.0–371.0</td>
<td>25.4</td>
</tr>
<tr>
<td>Final infiltration rate</td>
<td>( f_c )</td>
<td>cm/hr</td>
<td>1.00–64.00</td>
<td>2.92</td>
</tr>
<tr>
<td>Decay constant</td>
<td>( k )</td>
<td>1/s</td>
<td>0.017–0.550</td>
<td>0.030</td>
</tr>
<tr>
<td>Vegetative factor</td>
<td>( a )</td>
<td>–</td>
<td>0.10–1.00</td>
<td>0.45</td>
</tr>
<tr>
<td>Available storage</td>
<td>( S_A )</td>
<td>cm</td>
<td>0.00–6.66</td>
<td>5.00</td>
</tr>
<tr>
<td>Final infiltration capacity</td>
<td>( I_f )</td>
<td>cm/hr</td>
<td>0.000–1.143</td>
<td>1.140</td>
</tr>
<tr>
<td>Chezy roughness coefficient (interrill)</td>
<td>( C_z )</td>
<td>( \text{m}^{0.5}/\text{s} )</td>
<td>1.18–32.36</td>
<td>5.52</td>
</tr>
<tr>
<td>Chezy roughness coefficient (rill)</td>
<td>( C_{zr} )</td>
<td>( \text{m}^{0.5}/\text{s} )</td>
<td>4.36–21.49</td>
<td>11.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Interm</th>
<th>Rill</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horton</td>
<td>NSE</td>
<td>0.396</td>
<td>0.848</td>
</tr>
<tr>
<td></td>
<td>( r^2 )</td>
<td>0.761</td>
<td>0.905</td>
</tr>
<tr>
<td>Holtan</td>
<td>NSE</td>
<td>0.126</td>
<td>0.897</td>
</tr>
<tr>
<td></td>
<td>( r^2 )</td>
<td>0.580</td>
<td>0.903</td>
</tr>
</tbody>
</table>

Figure 2 | Calibration results showing combined interrill-rill runoff hydrograph.
an abruptly rising limb at the start of rainfall and continuously increased until a relatively constant discharge was achieved (Deng et al. 2005; Liu et al. 2006; An & Liu 2009). In the resulting hydrographs, the equilibrium state is followed by a sudden falling limb at the end of the rainfall duration. Although some structural mismatches were observed between the measured and simulated runoff, these may be attributed to a combination of several factors including variation of infiltration rates in space and time, variation in surface slope and slope shape of the surface which has also been found in a similar study on surface runoff by Haque (2002). In addition, non-uniformity of soil moisture in the laboratory model which has not been fully taken into account by the numerical model may have also resulted in the deviations between observed and simulated values.

The smaller intensity (45 mm/hr) experimental data were used for the validation of the model. In the series of experiments, rainfall intensity was varied wherein the smaller intensity was performed last. The soil in the flume was not entirely replaced for each run. Some eroded parts were replaced with new soil instead. In this sense, it can be said that by the end of the series of experiments, the soil was already wetter than it was initially. To account for this wetter condition, the initial degree of saturation was increased for the validation run of the Holtan model. In De Roo & Riezebos (1992) study of infiltration experiments on loess soils, consecutive simulated rainstorms applied to the samples showed a decrease in cumulative infiltration explained by Holtan’s model decrease in infiltration control zone depth. This example shows that change in soil parameters is possible as a result of rainfall conditions and can therefore be said to have a physical basis. Maintaining the rest of the parameters used in calibration, validation was performed and statistics are presented in Table 3.

The validation results produced better performance statistics for the Holtan model. The overall shape of the hydrograph was also better simulated by the model coupled with Holtan infiltration as seen in the rising limb, peak runoff and receding limb (Figure 3). As seen in the plots of overland flow, the increase in runoff is more closely replicated by Holtan’s model as compared to Horton’s result which delays ponding for some time before runoff begins. It can also be noticed that equilibrium is reached more quickly by the Horton’s model and continues until the end of rainfall duration.

In agreement with other researchers’ claims, the use of the Holtan equation has an advantage because cumulative infiltration rather than time enters its formulation (Chen 1975). In view of varieties in rainfall intensity, the use of cumulative infiltration as an independent variable in the infiltration equation eliminates the need to compute for the time of ponding \( t_p \), to determine at which time rainfall excess begins. The better performance of the Holtan model can be attributed to its available storage concept, where soil moisture accounting prior to the rainfall event allows the soil moisture deficit and initial surface infiltration

### Table 3 | Model validation performance statistics

<table>
<thead>
<tr>
<th></th>
<th>Intermill</th>
<th>Rill</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horton</td>
<td>NSE 1.845</td>
<td>0.244</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>( r^2 ) 0.339</td>
<td>0.765</td>
<td>0.756</td>
</tr>
<tr>
<td>Holtan</td>
<td>NSE 0.618</td>
<td>0.838</td>
<td>0.860</td>
</tr>
<tr>
<td></td>
<td>( r^2 ) 0.837</td>
<td>0.877</td>
<td>0.895</td>
</tr>
</tbody>
</table>

![Figure 3](https://iwaponline.com/wst/article-pdf/71/9/1325/468814/wst071091325.pdf)

**Figure 3** | Validation results showing combined interrill-rill runoff hydrograph.
rate to vary during the event dependent upon the magnitudes of antecedent precipitation. As the moisture input to the soil continues during the event, the soil column is further wetted and the soil moisture deficit decreases to zero. Concurrently, the surface infiltration decays to a minimum value of $f_l$.

Conversely, considerable degradation was observed between the calibration and validation results of the Horton simulation. The formulation of the Horton equation used in the overland flow model is quite limited in the sense that the AMC of the soil is not directly incorporated. The lack of physical basis of its parameters is thought to be a limitation of the model so none of its parameters were adjusted in order to get a better fit to the observed data.

The calibration performance statistics in Table 2 show that both the Horton and Holtan approaches appeared to perform almost equally. For the calibration event, the moisture condition of the soil in the flume was initially dry and higher intensity rainfall was applied. Several studies have found that for high intensity events, the runoff response is more uniform and not dependent on initial moisture conditions (Castillo et al. 2003). It can therefore be assumed that for the calibration event, the influence of initial conditions was not that significant to the model compared to the validation event where runoff from the lower intensity rainfall was more controlled by the moisture conditions of the surface soil layers and is dependent of initial conditions.

The results of the study showed that the amount of overland flow produced by the model is affected largely by the infiltration parameters, suggesting the importance of accounting for infiltration more accurately in the model. Although the generated flow in one or two interrill-rill areas did not have a very big difference, this difference may be magnified by a significant amount when an entire hillslope is considered, more so in large areas characterized by this kind of topography.

CONCLUSION

In this study, Holtan’s infiltration model was coupled to an existing overland flow algorithm in an effort to improve its performance in predicting runoff in hillslopes. The original algorithm used Horton’s model to account for infiltration. The study was able to directly compare the performance of the two infiltration models as applied to overland flow modelling. Model calibration and validation were performed using data from a 2D experimental flume. Infiltration parameters were calibrated to obtain the best fit of simulated results to observed data. Calibration and validation for both infiltration models were able to produce acceptable results based on NSE and $r^2$ values.

Generally, the model calibration and validation suggest that incorporation of the Holtan equation into the overland flow model improved its prediction results. Although both infiltration models are empirical in nature, the storage concept of the Holtan model, which directly considers initial moisture conditions of soil, is thought to provide an advantage over the Horton model which expresses infiltration as a function of time and considers only surface conditions rather than soil moisture content and other essential soil properties. By being able to account for AMC more accurately, the overland flow model prediction results are improved significantly. Results from this study can be used as a tool in predicting runoff from hillslopes as part of an erosion and sediment transport model, which can be further incorporated into a broader model which covers a wider scope in the watershed. As a suggestion for future study, the model could be applied to varying field conditions which may be able to also test the Holtan model’s performance in accounting for different types of vegetation.

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