Estimating parameters of the variable infiltration capacity model using ant colony optimization

JiaJia Yue, Bo Pang and ZongXue Xu

ABSTRACT

Because hydrological models are so important for addressing environmental problems, parameter calibration is a fundamental task for applying them. A broadly used method for obtaining model parameters for the past 20 years is the evolutionary algorithm. This approach can estimate a set of unknown model parameters by simulating the evolution process. The ant colony optimization (ACO) algorithm is a type of evolutionary algorithm that has shown a strong ability in tackling combinatorial problems and is suitable for hydrological model calibration. In this study, an ACO based on the grid partitioning strategy was applied to the parameter calibration of the variable infiltration capacity (VIC) model for the Upper Heihe River basin and Xitiaoxi River basin, China. The shuffled complex evolution (SCE-UA) algorithm was used to test the applicability of the ACO. The results show that ACO is capable of model calibration of the VIC model; the Nash–Sutcliffe coefficient of efficiency is 0.62 and 0.81 in calibration and 0.65 and 0.86 in validation for the Upper Heihe River basin and Xitiaoxi River basin respectively, which are similar to the SCE-UA results. Despite the encouraging results obtained thus far, further studies could still be performed on the parameter optimization of an ACO to enlarge its applicability to more distributed hydrological models.

Key words | ant colony optimization, distributed hydrological models, parameter calibration, VIC model

INTRODUCTION

Hydrological models are effective ways to understand hydrological systems. Model parameters, which reflect some of the hydrological properties, greatly influence the efficiency of hydrological models (Thomas et al. 1996; Montanari & Young 2013; Sood & Smakhtin 2015). However, because measuring all hydrological properties of a hydrological model is impossible in practice, calibration is the most effective method for the model application (Tang & Reed 2007; Abbaspour & Rouholahnejad 2015; Shafii et al. 2015). Many optimization algorithms have been developed and successfully applied, including gradient methods and global search methods. Evolutionary algorithms have been broadly used in hydrological modelling in the past 20 years; meta-heuristic methods such as genetic algorithm (Melanie 1998; Deb 2000; Giacobbo et al. 2002), particle swarm optimization (PSO) (Yoshida & Kawata 2000; Liang & Qin 2006), tabu search (Zheng & Wang 1996) and ant colony optimization (ACO) (Dorigo & Caro 1991; Dorigo & Blum 2005) are popular types of evolutionary algorithms.

The ACO, proposed by Dorigo & Caro (1991), Dorigo 2001; Dorigo & Blum 2005), is meta-heuristic approach that is efficient for solving hard combinatorial optimization problems. The original ACO achieved encouraging results for the well-known traveling salesman problem (TSP) (Dorigo & Caro 1991; Dorigo & Maniezzo 1996; Dorigo & Stützle 2004). After that, many extensions and improvements were introduced and applied to many optimization problems, such as scheduling problems, vehicle routing problems and more recent applications in the bioinformatics and biomedical fields. Additionally, when the problem is formulated, it is relatively straightforward and common practice to bias the probabilistic transitional rules towards promising regions (López-Ibáñez & Stützle 2010). The ACO has been widely and well used in static and dynamic combinatorial optimization problems, but few studies have evaluated the ACO performance in hydrologic model parameter optimization. This paper aims to explore the feasibility of an ACO in hydrologic model parameter calibration (Wen & Du 2002; Wang & Wu 2005; Blum 2005; Li et al. 2011).
The variable infiltration capacity (VIC) model is a physical based model that can represent spatial heterogeneity of catchments. The model has been applied to a number of river basins (Gao et al. 2009). Nijssen & Lettenmaier (1997) found that it performed well in moist areas. Zhou et al. (2006) concluded that the model was efficient for studying the hydrological cycle. Xie et al. (2007) successfully implemented this model for river basins in China. Park & Markus (2014) also suggested that the model can be used in snowmelt-driven flood peak studies. Because it is based on the soil–vegetation–atmosphere transfer scheme and considers snow melt, it is very suitable for use in mountainous catchments (Liang et al. 1996; Sinha et al. 2010; Oubeidillah & Kao 2014).

In this study, the ACO was used to calibrate the parameters of the VIC model. The Upper Heihe River basin and the Xitiaoxi River basin were chosen as the study area. To better understand the effect of parameter estimation, model performance is compared with the shuffled complex evolution (SCE-UA) algorithm (Duan & Gupta 1993; Duan et al. 1994), which has been successfully applied to the VIC model (Crow 2003; Muttill & Jayawardena 2008; Franz et al. 2010; Sridhar et al. 2013; Liu et al. 2015; Mendoza et al. 2015). The strategies of the ACO are also discussed. The ant number and pheromone evaporation rate of the ACO will have a bigger influence on the searching time rather than the optimization results.

The paper is organized as follows. First a brief introduction of the ACO algorithm and the VIC model is given, followed by an introduction to the study area and data. Then the ant’s behavior, calibration method and comparison results are defined. Finally, the conclusions are presented.

METHODS

ACO algorithm

ACO, a population-based meta-heuristic algorithm, was inspired by the observation of real ant colonies. Ants are social insects that live in colonies and their behaviors emphasize the importance of the colony rather than the individuals. The most interesting and important behavior of ant colonies is their foraging behavior, which prompts them to find the shortest path between food sources and their nest (Dorigo 2001; Duan 2005; Li et al. 2012).

Each ant explores the area around the nest at random initially and will deposit a substance that is called pheromone in the foraging path if they find food. Subsequent ants tend to choose, in probability, the marching direction by sensing the pheromone intensity of the path. When more paths are available from the nest to the food source, subsequent ants may exploit the pheromone trails left by the returning ants to discover a shorter path, which will soon result in all ants following the shorter path. This form of autocatalytic (positive feedback) of the ants enables them to eventually find the shortest path. The success of the ACO lies in its pheromone representation and the pheromone updating rule.

Inspired by the ant-based optimization principle, there are several algorithms for solving the classical combinational optimization problems. In this study, an ACO algorithm based on the grid partitioning strategy was used to optimize the model parameters. The implementation steps are as follows.

1. Set initial values of $m, N_{\text{max}}, \rho$, and estimate the value range of each parameter $x_{j,\text{lower}} \leq x_j \leq x_{j,\text{upper}} (j = 1, 2, \ldots n)$ based on the available information; $x_{j,\text{lower}}$ and $x_{j,\text{upper}}$ are the lower and upper limit, respectively, of the parameter $x_j$; $m$ is the ant number; $\rho$ ($\rho < 1$) is a coefficient of the pheromone evaporation rate; $N_{\text{max}}$ is the largest number of iterations.

2. Divide each parameter into $N$ equal parts:

$$h_i = \frac{x_{j,\text{upper}} - x_{j,\text{lower}}}{N} (i = 1, 2, \ldots n)$$  \hspace{1cm} (1)

This means each parameter is discretized into a number of strata. Let the middle of each stratum represent that stratum:

$$x_{ij} = x_{j,\text{lower}} + (j - 1) h_i (j = 1, 2, \ldots N)$$  \hspace{1cm} (2)

where $n$ is the number of parameters; $N$ is the number of strata of each parameter; $h_i$ is the interval of parameter $i$; and $x_{ij}$ is the value of parameter $i$ at stratum $j$.

3. Place $m$ ants randomly on the first parameter; each ant will move to the next parameter stratum with a probability that is a function of the pheromone intensity:

$$P_{ij} = \frac{\tau_{ij}}{\sum_{i=1}^{N} \tau_{ij}}$$  \hspace{1cm} (3)

where $\tau_{ij}$ is the intensity of pheromone of the edge $(i, j)$.

4. The pheromone intensity $\tau_{ij}$ is updated according to the equation:

$$\tau_{ij}(t + 1) = (1 - \rho) \tau_{ij}(t) + \Delta \tau_{ij}$$  \hspace{1cm} (4)
where \( \tau_{ij}(t) \) is the intensity of pheromone on the edge \((i, j)\) at time \(t\); \((1 - \rho)\) represents the rest of pheromone rate between iteration \(t\) and \(t + 1\); and \(\Delta\tau_{ij}\) is the quantity of pheromone deposited on the edge \((i, j)\) by the \(k\)th ant between iteration \(t\) and \(t + 1\), typically given by:

\[
\Delta\tau_{ij} = \sum_{k=1}^{n} \Delta\tau_{ij}^{k} 
\]

(5)

\(\Delta\tau_{ij}^{k}\) is represented by

\[
\Delta\tau_{ij}^{k} = \begin{cases} 
\frac{1}{f_k} & \text{if ant } k \text{ travels on value } i \text{ the } j^{th} \text{ parameter} \\
0 & \text{otherwise}
\end{cases}
\]

(6)

where \(f_k\) is the objective function value of the \(k\)th set of parameter, which is a combination of parameters that ant \(k\) chooses. Small values of the objective function receive higher pheromone intensities.

5. After all ants have completed the tour, score the value of the objective function of the best parameter combination of this iteration among the \(m\) ants.

6. Once the scoring is completed, update the pheromone intensity and repeat the process until a desired stopping rule is reached.

The processes of the ACO in this study are shown in Figure 1.

The VIC model

The VIC (Liang et al. 1994) model is a macroscale distributed hydrological model, based on the spatial distribution grid to simulate the physical exchange process between the soil–vegetation–atmosphere, that also considers snow, snow melt and hydrologic characteristics such as soil freezing and thawing at the same time (Sinha et al. 2010). It was jointly developed by the University of Washington, University of California at Berkeley and Princeton University.

The water and energy theory was improved by Liang, and he updated the model to VIC-2L. To accurately express the soil influence on the runoff in the vertical direction, the model was transformed into VIC-3L. The VIC-3L model divided the soil into two categories: surface and subsurface. Considering that different cover types in each grid will have different soil moisture distributions, the surface (canopy) was described by \(N + 1\) vegetation types, where \(n = 1, 2, ..., N + 1\), \(N\) represents \(N\) different types of vegetation and \(N = N + 1\) represents bare soil (Figure 1). (In Figure 2, \(D_1\) and \(D_2\) represent the diffusions between the soil layers, and \(K_1\) and \(K_2\) represent the drainages between the soil layers.) The subsurface was characterized as consisting of...
two soil layers: the upper layer, which contains the top thin layer, and the lower layer. The model can be driven by meteorological data. The VIC-3L model considers the Horton runoff model and it is suitable for runoff yield under excess infiltration in arid regions. All processes in each grid cell will be simulated independently. Using the stand-alone routing method developed by Lohmann & Raschke (1998) for the runoff and baseflow routed to the basin outlet, the whole simulation runoff of the basin will be determined.

The main features of the VIC model are: (1) consideration of the land-atmosphere fluxes and water and energy balances of the land surface; (2) two runoff yield patterns: infiltration excess runoff (Hortonian flow) and saturation excess runoff (Dunne flow); (3) consideration of the hydrological features such as snow melt, freezing and thawing; and 4. sub-grid heterogeneity (e.g., elevation, landcover) handled via statistical distributions (Liu 2012). The VIC model has been widely applied. In this study, the VIC-3L model version 4.0.5 was used.

**CASE STUDY**

The Upper Heihe basin and Xitiaoxi basin, located in the arid climate zone and humid climate zone respectively, were chosen to calibrate the VIC model for testing the ability of the ACO algorithm.

The Heihe River is the second largest inland river in north-western China. The basin is located between 37°-43° N and 97°-102° W and the total area of the Heihe River basin is approximately 142,900 km². The climate is cold and damp with an annual precipitation of approximately 400 mm and annual mean temperature of approximately 2 °C.

The Xitiaoxi River basin lies in the upper reaches of the Tai Lake basin, which supplies about 27.7% of the total water volume of the Taihu basin and plays an important role in the flood management of the downstream area. And the drainage area of the Xitiaoxi River basin at the outlet, which is located in the Heng Tangcun station, is 1,412 km². It has a subtropical monsoon climate, the average annual precipitation is 1,385 mm and the average annual temperature is 15 °C.

The meteorological stations and hydrologic station distribution of these two river basins are shown in Figure 3.

**Data description**

The meteorological forcing data used in this paper, including daily precipitation and daily minimum and maximum temperature of the stations in and around the two basins, were downloaded from the National Meteorological Information Center. Daily runoff data was available for the outlet of the two basins. The river basin characteristics and the calibration and validation period are listed in Table 1. The Thiessen polygon method was used to interpolate the meteorological forcing data for each grid.

The digital elevation model and basin boundary data (Figure 3) were obtained from the website http://www.gscloud.cn/. They were used to divide the computing grid, center coordinates, flow direction and flow accumulation area of each grid. Soil texture classification is based on the global 5 min of soil data provided by the NOAA hydrographic office. Landcover data, which was released by the University of Maryland, had a 1 km resolution.
RESULTS AND DISCUSSION

Model setup

The VIC model was driven by daily meteorological forcing data using the water balance mode. Preparation work includes dividing the spatial computing grid, interpolating the meteorological data and extracting the flow direction, center coordinates and flow concentration ratio of each grid.

There are seven parameters chosen for calibration. The description, units and range of these parameters are listed in Table 2.

The objective function is a numerical measure of the difference between the simulated model and observed results. A popular objective function in the hydrological literature is the Nash–Sutcliffe efficiency function (NSE), which represents the proportion of the variance of the data explained by the model. The closer the NSE is to 1, the better the model simulation results are. Mean absolute error (MAE), root mean square error (RMSE), and relative error (RE) were also used to evaluate the performance of the ACO algorithm. Table 3 shows the mathematical expressions of these evaluation criteria, in which $Q_{obs}$ and $Q_{sim}$ represent the observed and simulated daily runoff from day 1 to $n$, respectively, and $\overline{Q}_{obs}$ and $\overline{Q}_{sim}$ represent the mean of $Q_{obs}$ and $Q_{sim}$.

The performance of the evolutionary algorithms often depends on their parameter settings. The behavioral solutions of the ant number $m$, pheromone evaporation rate $\rho$ and parts of the parameters, $N$, are identified by different strategies. The parameters of the ACO were first determined using a classical method of keeping one parameter constant and varying the others.

Table 1 | List of the river basin characteristics and the calibration and validation period

<table>
<thead>
<tr>
<th>River basin</th>
<th>Basin area (km²)</th>
<th>River length (km)</th>
<th>Annual rainfall (mm)</th>
<th>Annual runoff (mm)</th>
<th>Warming-up period</th>
<th>Calibration period</th>
<th>Validation period</th>
</tr>
</thead>
</table>

Table 2 | VIC model parameter list

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>–</td>
<td>Variable infiltration curve parameter</td>
<td>[0, 0.4]</td>
</tr>
<tr>
<td>Ds</td>
<td>–</td>
<td>Fraction of the maximum velocity of base flow where nonlinear base flow begins</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Dmax</td>
<td>mm/h</td>
<td>Velocity of the base flow</td>
<td>[0, 30]</td>
</tr>
<tr>
<td>Ws</td>
<td>–</td>
<td>Fraction of the maximum soil moisture where nonlinear base flow occurs</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>d1</td>
<td>m</td>
<td>Thickness of soil layer 1</td>
<td>[0.1, 1.5]</td>
</tr>
<tr>
<td>d2</td>
<td>m</td>
<td>Thickness of soil layer 2</td>
<td>[0.1, 1.5]</td>
</tr>
<tr>
<td>d3</td>
<td>m</td>
<td>Thickness of soil layer 3</td>
<td>[0.1, 1.5]</td>
</tr>
</tbody>
</table>

Figure 3 | The position of the study river basin. (a) Upper Heihe River basin. (b) Xitiaoxi River basin.
It is found that the best ratio between $N$ and $m$ is 1.5, which is similar to the empirical settings of the ant-colony algorithm (Ni & Xing 2013). Ant number $m$ and pheromone evaporation rate $\rho$ are two sensitive parameters for the ACO. Increase of $\rho$ will enhance the convergence speed, but reduce the model efficiency. In contrast, the model efficiency is improved as $m$ increases. However, the benefit is not significant once $m$ exceeds a certain threshold. Considering both the precision and convergence speed, the parameter set with $\rho = 0.2$ and $m = N/1.5$ was adopted in the model setting.

The parameters of SCE-UA were set based on the recommendation of Duan & Gupta (1993) and the former research on the two basins (He et al. 2014; Zhang et al. 2014). The ACO is compared with the SCE-UA method to further investigate the calibration effect. The optimum parameter sets of VIC models in the two basins by ACO and SCE-UA are listed in Table 4. It is observed that the optimum parameter sets of ACO and SCE-UA are different except for $d_1$, which is 0.15 and 0.42 for ACO, and 0.16 and 0.42 for SCE-UA respectively.

The simulation results of the ACO and SCE-UA for calibration and validation are presented in Table 5. The results show that the model efficiency of the ACO is similar to SCE-UA in both the calibration and validation periods of the two basins.

| Table 4 | VIC model parameter optimization results |
|---|---|---|---|---|---|---|---|---|---|---|
| River basin | Parameter | B | Ds | Dmax | Ws | d1 | d2 | d3 |
| Upper Heihe | ACO | 0.28 | 0.13 | 21.84 | 0.41 | 0.15 | 0.31 | 0.75 |
| | SCE-UA | 0.24 | 0.55 | 19.55 | 0.53 | 0.16 | 0.83 | 1.04 |
| Xitiaoxi | ACO | 0.15 | 0.10 | 23.10 | 0.56 | 0.42 | 0.37 | 1.47 |
| | SCE-UA | 0.20 | 0.06 | 30.00 | 0.50 | 0.42 | 0.38 | 0.35 |

| Table 5 | Simulation results for model calibration and validation |
|---|---|---|---|---|---|---|
| River basin | Period | Method | NSE | RMSE (m$^3$/s) | MAE (m$^3$/s) | RE |
| Upper Heihe | Calibration | ACO | 0.62 | 31.09 | 19.04 | −0.129 |
| | | SCE-UA | 0.61 | 31.38 | 21.29 | −0.241 |
| | Validation | ACO | 0.65 | 34.2 | 22.74 | −0.034 |
| | | SCE-UA | 0.64 | 34.7 | 22.93 | −0.089 |
| Xitiaoxi | Calibration | ACO | 0.81 | 26.86 | 14.37 | 0.11 |
| | | SCE-UA | 0.81 | 26.52 | 14.43 | 0.13 |
| | Validation | ACO | 0.86 | 28.81 | 17.85 | 0.07 |
| | | SCE-UA | 0.86 | 28.2 | 17.98 | 0.094 |
period and 0.5 (m$^3$/s), 0.19 (m$^3$/s) and 0.055 in the validation period, respectively.

For the Xitiaoxi River basin, the NSE values of the ACO are approximately equal to the SCE-UA, which are 0.81 in calibration period, and 0.86 in validation. Comparing with other evaluation criteria, the performances of the two methods are also similar. The RMSE in calibration and validation for the ACO is 26.86 m$^3$/s and 28.81 m$^3$/s while the SCE-UA is 26.52 m$^3$/s and 28.20 m$^3$/s. Comparing the MAE and RE, the ACO is superior to SCE-UA; in calibration the reduction of the ACO is 0.06 (m$^3$/s) and 0.02, and in validation the reduction is 0.15 (m$^3$/s) and 0.02.

Figures 4 and 5 showed the observed and simulated runoff hydrographs of the VIC model using the ACO in the two river basins; the simulated hydrographs showed considerable agreement with the observed hydrograph. But for the Upper Heihe River basin, the simulated runoff hydrograph has a slight lag compared with the actual flow process. It is observed that some simulated peak values do not coincide with the observed peaks. This may be the effect of the reservoirs, which is not considered in the model structure.

![Figure 4](https://iwaponline.com/wst/article-pdf/74/4/985/459394/wst074040985.pdf)

**Figure 4** The observed and simulated daily runoff hydrographs of the Upper Heihe River basin: (a) calibration period (2003–2006) and (b) validation period (2007–2008). Q: flow rate; P: precipitation.

![Figure 5](https://iwaponline.com/wst/article-pdf/74/4/985/459394/wst074040985.pdf)

**Figure 5** The observed and simulated daily runoff hydrographs of the Xitiaoxi River basin: (a) calibration period (1995–1998) and (b) validation period (1999–2000). Q: flow rate; P: precipitation.
CONCLUSIONS

The objective of this study is to investigate the applicability of the ACO for parameter calibration. The ACO was used in the parameter calibration of the VIC model in the Upper Heihe River basin and the Xitiaoxi River basin. The results were also compared with those of the SCE-UA to test the model efficiency. Discussions and conclusions are summarized as follows.

A framework for applying the ACO algorithm to hydrological model parameter calibration is established. The results show that the ACO can obtain similar results to the SCE-UA, which has been successfully applied to the VIC model. The results indicate that the ACO is capable of model calibration of the VIC model.

The optimum parameter sets of the two methods are also compared. The results show that most of the parameters are different except for d1, although the model performances are similar. This might be due to the equifinality of the parameter sets (Beven 1993, 2006, 2009; Beven & Freer 2001). The results are also consistent with the former studies on parameter sensitive analysis of these basins (He et al. 2014; Zhang et al. 2014), in which d1 is the most sensitive parameter of VIC models.

This is a preliminary evaluation of the ACO for hydrological models’ parameter calibration. The goal of this paper is mainly to establish a framework for the algorithm application; therefore, we will pursue further research in this area by improving the computational efficiency of the ACO to enlarge its applicability to more distributed hydrological models.

ACKNOWLEDGEMENTS

This research is funded by the National Natural Science Foundation (NSFC) of China, Key project ‘Evolution and coupling mechanism of ecological-hydrological processes in the middle reaches of the Heihe River basin’ (91125015), and ‘Study of a distributed hydrological model based on the complex urban underlying surface characteristics’ (51330909). The authors gratefully acknowledge Miss Zhang L. Y. and Miss He R. for assistance with the original data and computational method, and Mr Zhao G. for model development and proofreading.

REFERENCES


Giacobbo, F., Marseguerra, M. & Zio, E. 2002 Solving the inverse problem of parameter estimation by genetic algorithms: the


First received 7 January 2016; accepted in revised form 26 May 2016. Available online 10 June 2016