Sediment deposit thickness and its effect on critical velocity for incipient motion

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ABSTRACT

The understanding of how the sediment deposit thickness influences the incipient motion characteristic is still lacking in the literature. Hence, the current study aims to determine the effect of sediment deposition thickness on the critical velocity for incipient motion. An incipient motion experiment was conducted in a rigid boundary rectangular flume of 0.6 m width with varying sediment deposition thickness. Findings from the experiment revealed that the densimetric Froude number has a logarithmic relationship with both the thickness ratios $t_s/d$ and $t_s/y_0$ ($t_s$: sediment deposit thickness; $d$: grain size; $y_0$: normal flow depth). Multiple linear regression analysis was performed using the data from the current study to develop a new critical velocity equation by incorporating thickness ratios into the equation. The new equation can be used to predict critical velocity for incipient motion for both loose and rigid boundary conditions. The new critical velocity equation is an attempt toward unifying the equations for both rigid and loose boundary conditions.

Key words | critical velocity, densimetric Froude number, incipient motion, loose boundary, rigid boundary, sediment deposit thickness

INTRODUCTION

Incipient motion is defined as the critical condition that is adequate to initiate sediment particles motion (Dey & Papa-nicolaou 2008). Currently, the majority of the literature on incipient motion covers the loose boundary condition with unlimited sediment depth and supply as compared to rigid boundary condition with limited sediment depth and supply (Novak & Nalluri 1984; Mohammadi 2005). Novak & Nalluri (1975) found that the incipient motion value is substantially lower for a rigid boundary condition as compared to loose boundary condition. For design purpose, the Shields diagram (Shields 1936), which was developed for a loose boundary channel such as alluvial channel, was widely used to predict incipient motion of granular particles. For a rigid boundary channel such as sewer system, despite the very different boundary condition (Ashley et al. 2004), the Shields diagram has been applied in a number of studies on sewer and storm sewer (Verbanck et al. 1994).

The Shields diagram was developed using a relationship (see Equation (1)) based on the balance between particle weight and boundary shear stress:

$$\theta_c = \frac{\tau_c}{gd(\rho_s - \rho)} = f\left(\frac{u^2d}{\nu}\right) = f(Re^*)$$

(1)

where $\theta_c$ is the dimensionless Shields stress; $\tau_c$ is the critical shear stress (N/m²); $g$ is the gravitational acceleration (m/s²); $\rho_s$ is the sediment density (kg/m³); $\rho$ is the fluid density (kg/m³); $d$ is the grain size (nearly $d = d_{50}$ for uniform sediment) (m); $u = \sqrt{\tau_c/\rho}$ and $\nu$ is the kinematic viscosity of fluid (m²/s) and $Re^*$ is the dimensionless grain Reynolds number. A recent study has shown that the sediment pickup rate as well as incipient motion was better correlated with the densimetric Froude number than the Shields number (Cheng & Emadzadeh 2016). Rewriting Equation (1) in terms of densimetric Froude number $F_d$ by assuming the Shields criterion $\tau_c/gd(\rho_s - \rho) = F_d^2 = 0.056$ and Manning’s $n = 0.04d^{1/6}$ (d in metres) for...
a wide channel, Equation (2) was obtained (Novak & Nalluri 1984):

\[
F_d = \frac{V_c}{\sqrt{gd(S_s - 1)}} = 1.92 \left(\frac{d}{y_0}\right)^{-0.167}
\]

where \(V_c\) is the critical velocity (m/s); \(S_s\) is the specific gravity of the sediment and \(y_0\) is the normal flow depth of flow in the channel.

Existing equations in the literature to predict critical velocity for rigid boundary condition are usually in the form of Equation (3) (El-Zaemey 1991; Novak & Nalluri 1984):

\[
\frac{V_c}{\sqrt{gd(S_s - 1)}} = a \left(\frac{d}{R}\right)^b
\]

where \(R\) is the hydraulic radius of the flow section; \(a\) and \(b\) are coefficients. Novak & Nalluri (1984) defined the \(a\) and \(b\) as 0.5 and -0.4, while El-Zaemey (1991), as 0.75 and -0.34, respectively. Some literature suggested that Equation (3) became less accurate as the sediment deposit thickness increased (Bong et al. 2013; Salem 2013). Bong et al. (2013) observed that as the thickness of sediment deposit increased, the critical velocity required for incipient motion also increased. This could be due to the increased ‘support’ effect from more neighbouring particles with irregular shapes as the sediment deposit thickness increased, resulting in greater friction between sediment particles leading to higher critical velocity to move the particles (Bong et al. 2013). Besides that, thicker sediment deposit tends to cause formation of micro-bed forms leading to additional resistance and friction between sediment particles.

The use of a Shields diagram for self-cleansing design of sewers and storm sewers with limited sediment deposit will produce significant errors. Sediment deposit in combined sewers generally has limited thickness from less than 10 mm to 60 mm (Lange & Wichern 2013) and up to 100 mm (Ashley et al. 1992), while for open storm sewers, the sediment deposit thickness could range from 10 mm to 330 mm (Bong et al. 2014). Conversely, since the existing critical velocity equations for rigid boundary condition did not take into account the effect of sediment deposit thickness, it is inaccurate with the increasing deposition thickness.

To describe a two-phase phenomenon involving fluid and sediment, three components are involved, namely: (i) fluid; (ii) non-cohesive granular medium; and (iii) flow (Yalin 1977). The fluid is defined by its density \(\rho\) (kg/m\(^3\)); the non-cohesive granular medium is defined by its density \(\rho_s\) (kg/m\(^3\)) and size \(d\) (normally \(d = d_{50}\) for uniform sediment) (m); and the flow is defined by the hydraulic radius of flow area \(R\) (m) and gravity acceleration \(g\) (m/s\(^2\)) (Bong et al. 2013). For the current study, the sediment deposit thickness \(t_s\) (m) and the normal flow depth \(y_0\) (m) (see Figure 1) was included in the analysis. Replacing \(g\) with \(\gamma_s = g(\rho_s - \rho)\) where the specific weight for the sediment \(\gamma_s\) can be excluded, as it would be a constant for the current study, the dimensionless terms for the incipient motion function were given by:

\[
\frac{V_c}{\sqrt{gd(S_s - 1)}} = f \left(\frac{d}{R}, \frac{t_s}{d}, \frac{t_s}{y_0}\right)
\]

Previous work by Bong et al. (2013) was limited to sediment depth of 24 mm, while the current work extended the sediment deposition thickness to 100 mm. This study also attempts to establish the trend of the sediment deposition thickness effect on critical velocity during incipient motion, which the previous studies have not recognized. Using the results from the current study, a new critical velocity equation was proposed by incorporating the effect of sediment deposit thickness.

**METHODOLOGY**

The current work was an extension of previous work by Bong et al. (2013) and involved a rectangular flume as
shown in Figure 2. Six sediment deposit thicknesses were used for the incipient motion experiment, namely one layer \( (t_i = d_{50}) \), 5 mm, 10 mm, 24 mm, 48 mm and 100 mm (see Figure 3). The experiment was conducted with two flume slopes (0.001 and 0.002) to reconfirm the results for different slope values. The non-cohesive uniform sediment used had median \( d_{50} \) sizes of 0.81 mm, 1.53 mm and 4.78 mm with specific gravity of 2.54, 2.55 and 2.57, respectively. The selection of flume slopes, sediment size and deposition thickness was based on site observations by Bong et al. (2014) and Ab. Ghani et al. (2000) for an open channel storm sewer. The definition for incipient motion used in the current study was of general movement via visual observation (Kramer 1955) where particles of all sizes are moving at all points on the bed at all times.

Prior to the experiment, preliminary clear water experiments were conducted. Findings from the clear water experiments have shown that the flow was fully developed after 3.0 m from the upstream corrugated sheets. Hence, it was decided to locate the observation section for the sediment bed at 3.5 m from the upstream corrugated sheets. During the experiment, water level and discharge were slowly increased by controlling the pumps that supplied water into the flume. At each increment of the discharge via the pumps valves, the flow was maintained uniform before observation was made at the observation section for any sediment particle movement. This procedure was repeated until the incipient motion condition was observed. Once incipient motion was observed, the velocity and discharge values were obtained using a velocity meter with low flow probe and recorder (Nixon StreamFlo 430). The accuracy of the velocity meter was \( \pm 2\% \) of true velocity value. The range of the experimental parameters for the current study is shown in Table 1. To check the repeatability of the experiment, random sets of the experiment were chosen and repeated twice. Furthermore, the trend of the effect of sediment deposition thickness on the critical velocity can be confirmed by comparing the data from experiments for the two different slopes.

![Figure 2](image-url) Schematic diagram of the experimental set-up for the current study (not to scale).

![Figure 3](image-url) Setting up of sediment bed: (a) levelling the sediment bed to the required thickness using trowel; (b) sediment deposition thickness of 24 mm for \( d_{50} = 0.81 \) mm used in the experiment.
Improvement to the existing critical velocity equation was done by performing multiple linear regressions on the dimensionless terms in Equation (4) for the data from the current study. The fitting of all possible regression equation methods was used in this study. A total number of 36 data values obtained from the current study were used for the multiple linear regression analysis with seven possible test cases based on the dimensionless groups in Equation (4). The best regression model was selected based on four criteria: (a) coefficient of determination, \( R^2 \); (b) adjusted \( R^2 \), \( R^2_{\text{adj}} \); (c) mean square error, MSE; and (d) Mallow’s \( C_p \) statistic (Sinnakaudan et al. 2006). For the first criterion, the higher and closer to unity the \( R^2 \) value, the better the model fits the data. The second criterion was based on \( R^2_{\text{adj}} \) that accounts for the number of variables in the model. While adding the predictor variables will increase the value of coefficient of determination; the adjusted coefficient of determination may fall if the added predictor variables have little explanatory power and are statistically insignificant (Hair et al. 1995). The third criterion was based on mean square error where the model with minimum MSE was the best. The fourth criterion was Mallow’s \( C_p \) statistic, which is a measure of the quality of fit for a model, and it tends to find the best subset that includes only the important predictors of the respective dependent variable. The best model has \( C_p \) value approximately equal to the number of terms in the model.

A performance test on the new equation was done by calculating the discrepancy ratio using Equation (5) with the acceptable range of 0.5 to 2.0, which is normally used for the study of sediment transport and incipient motion (Gaucher et al. 2010):

\[
\text{Discrepancy ratio} = \frac{V_c \text{ predicted (m/s)}}{V_c \text{ observed (m/s)}}
\]

RESULTS AND DISCUSSION

Effects of sediment deposit thickness

The effect of sediment thickness on incipient motion can be seen by plotting the densimetric Froude number \( F_d \) calculated from Equation (2) against \( t_s/d \) and \( t_s/y_0 \) as shown in Figure 4 for flume slope of 0.001. It was observed that \( F_d \) increased at diminishing rate as both \( t_s/d \) and \( t_s/y_0 \) increased. This showed that sediment deposit thickness has an effect on the incipient motion of the sediment particle in terms of increasing critical velocity for thin sediment deposit thickness and the effect diminished with the increased thickness of the sediment deposit. The equations in both graphs in Figure 4 show that the relationship between \( F_d \) and both \( t_s/d \) and \( t_s/y_0 \) was best fitted (\( R^2 \) value close to unity) with logarithmic relationships. A similar trend was also observed for the flume slope of 0.002. Friction between the sediment particles, which increase with the increment in deposit thickness, could be the reason behind this behavior of incipient motion. At thicker sediment deposit, the increment of friction between the sediment particles could have achieved the optimum level and is negligible with further increase in deposit thickness. Consequently, this behavior causes the existing equations for incipient motion in rigid boundary condition to be less accurate as the sediment deposit thickness increases.

Using Equation (2) which expressed the Shields criterion in terms of \( F_d \), the graph in Figure 5, which shows the relationship between \( F_d \) and \( d/y_0 \), was plotted. The Shields criterion which was calculated as \( F_d \) was plotted as a line in Figure 5. It was observed that as the sediment deposit thickness increases, \( F_d \) from the current study became closer to the \( F_d \) for Shields criterion. It was
also observed that most of the points for \( F_d \) from the current study for sediment deposit thickness of 48 mm and 100 mm either touched or were above the Shields criterion line. This trend was observed for both the flume slopes used in the current study. This showed that the sediment deposit started to behave like a loose boundary for thickness above 48 mm since the points were nearer to the Shields criterion line for loose boundary as compared to the points for sediment deposition thickness below 48 mm.

**Pearson correlation analysis for the dimensionless terms**

The effect of sediment deposit thickness can be included into existing critical velocity equations for rigid boundary
condition by incorporating the dimensionless terms \( t_s/d \) and \( t_s/y_0 \) as shown in Equation (4). According to Evans (1996), the strength of the Pearson correlation coefficient can be considered as strong for values between 0.60 and 0.79 and very strong for values between 0.80 and 1.00. From Pearson correlation analysis, the densimetric Froude number \( F_d \) has a very strong positive correlation with \( t_s/d \) (correlation coefficient value of 0.813) and is statistically significant \((p\text{-value} = 0.000)\). The densimetric Froude number \( F_d \) also has a strong positive correlation with \( t_s/y_0 \) (correlation coefficient value of 0.749) and is statistically significant \((p\text{-value} = 0.000)\).

**Improvement to critical velocity equation**

Table 2 shows the results of the three best multiple linear regression models for the dimensionless terms in Equation (4) for the data from the current study. Since the three dimensionless terms on the right hand side of Equation (4) were linearly dependent when combined all together, only a combination of two of the dimensionless terms was used for each of the regression models. Equation (7) has \( R^2 \) and \( R_{adj}^2 \) values closest to unity among the regression models in the current study and the smallest \( MSE \) value, together with \( C_p \) value that is the same as the number of terms in the model. Hence, Equation (7) was selected for further testing.

Using the data from the current study, a performance test in terms of various sediment deposit thicknesses was conducted on Equation (7) together with existing equations by Novak & Nalluri (1984) and El-Zaemey (1993). From Table 3, it can be observed that as the sediment deposit becomes thicker, equations by Novak & Nalluri (1984) and El-Zaemey (1993) become less accurate, with the discrepancy ratio value further from unity. Equation (7) from the current study appears to be consistent and was not affected by the sediment deposit thickness.

**Validation test with other authors’ data**

To validate the new Equation (7) developed from the current study, data for incipient motion from three existing studies were used, namely Yalin & Karahan (1979), Kuhnle (1993) and Shvidchenko (2000). The existing equations by Novak & Nalluri (1984) and El-Zaemey (1993) were also tested using the data from these researchers. Table 4 shows the range of the experimental data from Yalin & Karahan (1979), Kuhnle (1993) and Shvidchenko (2000) used in the current study for the validation test.

For the data from Yalin & Karahan (1979), only five data values from the experiment using sediment in turbulent flow were selected out of a total of 22 (16 laminar flows and six turbulent flows). The rest of the data were not selected because the flow medium used in the laminar flow condition was not purely water and one data value from the turbulent flow was using glass beads to replace sediments. As for Kuhnle (1993), only eight data values for uniform sediment with unimodal characteristics were chosen out of a total of 30. The eight were made up of four values for uniform sand \((d_{50} = 0.476 \text{ mm})\) and four values for uniform gravel \((d_{50} = 5.579 \text{ mm})\). For data from Shvidchenko (2000), 83 out of 312 data values were chosen. The data were chosen

<table>
<thead>
<tr>
<th>Equation</th>
<th>( R^2 )</th>
<th>( R_{adj}^2 )</th>
<th>( MSE )</th>
<th>( C_p )</th>
<th>Equation number</th>
</tr>
</thead>
</table>
| \[
\frac{V_c}{\sqrt{g d (S_t - 1)}} = 0.85 \left( \frac{d}{R} \right)^{-0.18} \left( \frac{t_s}{d} \right)^{0.21}
\] | 0.733 | 0.717 | 0.00518 | 2.98 | (6) |
| \[
\frac{V_c}{\sqrt{g d (S_t - 1)}} = 0.87 \left( \frac{d}{R} \right)^{-0.38} \left( \frac{t_s}{y_0} \right)^{0.20}
\] | 0.779 | 0.765 | 0.00429 | 3.00 | (7) |
| \[
\frac{V_c}{\sqrt{g d (S_t - 1)}} = 0.99 \left( \frac{t_s}{d} \right)^{0.32} \left( \frac{t_s}{y_0} \right)^{-0.12}
\] | 0.694 | 0.675 | 0.00594 | 2.98 | (8) |
only for the experiment with transport intensities within the two critical values of $I = 10^{-4}$ s$^{-1}$ and $I = 10^{-2}$ s$^{-1}$ as defined by Shvidchenko (2000) and also for gravel size not more than 5.65 mm. Out of the 83, seven values are from uniform coarse sand ($d_{50} = 1.5$ mm), eight are from fine gravel ($d_{50} = 2.4$ mm) and the rest from ‘pea’ gravel with $d_{50}$ ranging from 3.4 mm to 5.65 mm. The specific gravity for the sediment was between 2.6 and 2.65.

Table 3 shows the results for the validation test. Equation (7) from the current study performed well by having the highest total number of data fall within the acceptable discrepancy ratio as compared to the existing equations for the three sets of data from the literature. Existing equations by Novak & Nalluri (1984) and El-Zaemey (1991) also performed well for the two sets of data for rigid boundary channel, namely Yalin & Karahan (1979) and Shvidchenko (2000). However for data from thicker sediment thickness, namely Kuhnle (1993), it was observed that Equation (7) performed better by having 100% of the data predicted falling within the acceptable discrepancy ratio range as compared to the existing equations. Figure 6 shows the comparison between observed and predicted critical velocity using the equations of Novak & Nalluri (1984), El-Zaemey (1991) and Equation (7) respectively for the data by Yalin & Karahan (1979), Kuhnle (1993) and Shvidchenko (2000). This validation test shows that by incorporating sediment deposit thickness into Equation (7), the equation is able to perform reasonably well and is comparable with existing equations. Equation (7) also performed better at higher sediment deposit thickness (for data from Kuhnle (1993)) as compared to the existing equations, which were developed for rigid boundary channel with limited sediment thickness.

The new Equation (7) proposed in the current study was observed to be more consistent in terms of prediction performance as compared to existing equations for varying sediment deposit thickness. However, it is yet to be determined whether Equation (7) will be satisfactory in predicting the critical velocity for unlimited sediment deposit thickness.
CONCLUSION

The current study aims to determine the effect of sediment deposit thickness on the critical velocity for incipient motion. Results from the current study showed that the densimetric Froude number was less affected by the deposition thickness at high sediment depth. A new equation was proposed and it appears to be more consistent in predicting the critical velocity at varying sediment deposit thickness as compared to the existing rigid boundary equations in the literature. However, the new equation, which is subject to sediment thickness, might not be suitable to be used for channels with unlimited sediment thickness. Further works can be done to determine the sediment thickness and flow depth ratio where the boundary condition changes from rigid to loose, and on theoretical development of the incipient motion equation that is independent of the sediment deposit thickness.

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