

An approximate model on three-dimensional groundwater infiltration in sewer systems

Shuai Guo, Yulong Yang and Yiping Zhang

ABSTRACT

Groundwater infiltration through cracked sewer pipes has caused significant economic losses. This paper presents a three-dimensional analytical expression for calculating the steady-state groundwater infiltration rate into sewer systems. As an extension of the previously developed two-dimensional model by the author, this new model can be used to simulate the infiltration through an orifice defect. The derived analytical expression has been validated with experimental results. The new model incorporates all the related resistances during the infiltration process, including the soil head loss and the orifice loss. The soil head loss is assessed with Ergun equation, which involves an inertial loss term in addition to the viscous loss. This has significantly extended the application range of the new expression. A new OS number (the ratio of orifice loss to soil loss) expression is also presented. The order analysis of the OS number expression has demonstrated that in most real cases, the head loss through the soil layer dominates the whole infiltration process.

Key words | groundwater, infiltration, orifice, sewer systems, steady-state

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INTRODUCTION

As sewer pipes age, a number of defects or cracks emerge on the pipe wall due to various factors (Davies *et al.* 2001; Kuliczowska 2016). These defects or cracks may serve as the entrance for groundwater infiltration into the sewer pipe when the groundwater table is greatly raised above the pipeline during the rainy season. As a common problem existing in sewer systems worldwide, groundwater infiltration has caused great economic losses. This part of 'unwanted water' (Wittenberg & Aksoy 2010) from infiltration increases the cost of wastewater treatment and the energy consumption and operating costs of pumping stations. Therefore, in the past few decades, significant attention has been drawn on the infiltration control and management (De Benedittis & Bertrand-Krajewski 2005).

Many methods have been proposed for the catchment or sub-catchment scale groundwater infiltration estimation, including the traditional water balance method (Brombach *et al.* 2002; Weiss *et al.* 2002), the pollutant loads analysis method (Kracht & Gujer 2005), the tracer method (Kracht *et al.* 2007; Prigiobbe & Giulianelli 2009) and numerical modelling (Aksoy & Wittenberg 2011; Karpf & Krebs 2013; Thorndahl *et al.* 2016). The pollutant loads analysis and the traditional method are relatively expensive and their

precisions are difficult to ensure. The tracer method has good precision, but it can only be applicable to urban catchments, where the suitable isotopic separation between drinking water and potential infiltration sources exists. Although numerical models have been reported with good accuracy, the computational costs for long term modelling will be prohibitive. On the other hand, municipal agencies worldwide are conducting structural assessment on their sewer systems by using advanced inspection technologies (Wirahadikusumah *et al.* 1998; Costello *et al.* 2007). Therefore, a method connecting the routine expenditure on inspection with the infiltration estimation would be attractive and beneficial. Guo *et al.* (2013a) proposed a two-dimensional model for calculating the infiltration rate of the individual sewer pipe segment with a line defect on the pipe wall. The method aimed to connect the routine expenditure on sewer inspection with infiltration estimation to save the budget allocation for the sub-catchments scale infiltration investigation.

As an extensional work of the previous model, this study presents a three-dimension infiltration model. Therefore, the following reasonable assumptions used in the previous study are also kept in this study: the groundwater table above the

embedded sewer pipe is horizontal and the surrounding soil is homogeneous and isotropic. Experiments were conducted to verify the derived analytical model. The computed results showed a good agreement with experimental data. From the parametric analysis, the OS number, which is defined as the ratio of orifice loss to soil loss, has been demonstrated to be less than unity in most real cases; this indicates the head loss through the soil dominate the infiltration process.

METHODS

Flow field and governing equation

We consider a circular sewer pipe with a diameter of D which is embedded in a homogeneous isotropic saturated porous aquifer. There is a circular orifice defect with a diameter of D_0 on the top of the pipe wall, and the depth of the orifice below the groundwater table is h . The external flow field around the orifice is generally three dimensional, as shown in Figure 1(a). The conservation equation for the steady-state flow of groundwater through porous media is given as (Terzaghi *et al.* 1996):

$$\nabla \cdot \mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad (1)$$

where V is the specific velocity, and V_x , V_y , V_z are velocities in three directions separately.

The complex boundary condition at the defect prevents the absolute analytical solution for the groundwater infiltration. To obtain any tractable analytical result, some simplification has to be taken. By computational fluid dynamics modeling, Collins *et al.* (2010) found that if the diameter of the pipe is small compared to the size of the flow field, then the flow field will increasingly appear to be a point sink. Therefore, in their following study on contaminant intrusion through apertures in distribution pipes (Collins & Boxall 2013), the external flow field around the pipe is modeled as spherically symmetrical with the intrusion orifice as a sink at the center, as shown in Figure 1(b). Guo *et al.* (2013a) used the conformal mapping technique to simplify the boundary conditions and the flow field was then transferred to be spherically symmetrical. On the other hand, study from tunneling field has demonstrated analytically that the dominant factor for the shape of the flow field belongs to the ratio of groundwater table and pipe size (El Tani 2003; Kolymbas & Wagner 2007; Park *et al.* 2008). Only if $h \gg D$, the groundwater will infiltrate into the pipe from all directions and the flow field will approach spherically symmetrical; otherwise, the percolating flow will mainly come from the soil layer above the pipe due to the shape effect from the waterproof part of the pipe wall.

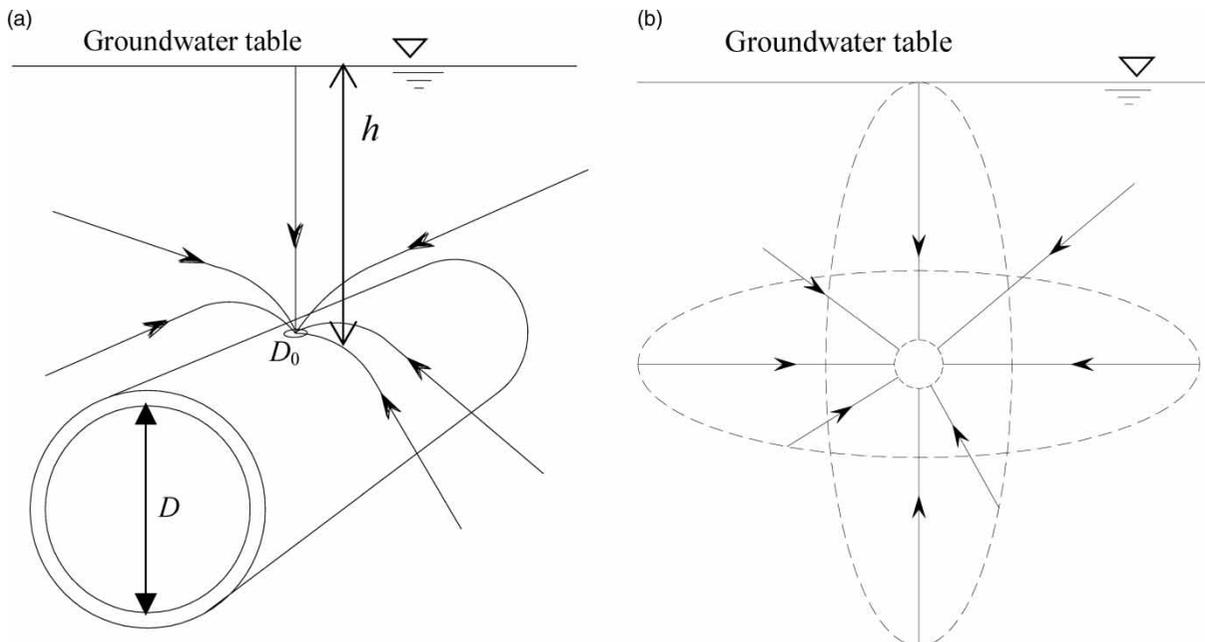


Figure 1 | Schematic of a cracked sewer pipe and flow condition: (a) real flow field; (b) transferred flow field.

Since the sewer pipe is usually buried several meters below the ground surface, the groundwater table is expected to be comparable with the pipe size. Therefore, an assumed semi-spherical flow field assumption around the minor crack, as shown in Figure 2, is more reasonable and was used in this study.

Analytical derivation

For the infiltrating groundwater in the flow field, two different media must be penetrated to enter into the internal of the sewer pipe: the soil layer and the pipe wall. Therefore, the total head loss Δh of the infiltration process consists of two parts: the frictional loss from the porous soil media Δh_s and the loss from the orifice Δh_o ; the orifice loss can be further divided into two parts: the local loss over the orifice Δh_{o1} and the frictional loss from the wall passage Δh_{o2} . The total head loss can then be expressed as:

$$\Delta h = \Delta h_s + \Delta h_o = \Delta h_s + \Delta h_{o1} + \Delta h_{o2} \quad (2)$$

For assessing the frictional loss from the soil, Ergun's equation (1952) that involves an inertial loss term is more suitable and has been proposed in recent studies (Collins & Boxall 2013). The Ergun equation is expressed as:

$$\frac{dh_s}{dr} = AV_{in} + BV_{in}^2 \quad (3)$$

$$A = \frac{1}{K} = \frac{150\mu}{\varphi_p^2 d_p \rho_w g} \frac{(1-\varepsilon)^2}{\varepsilon^3}; B = \frac{1.75}{\varphi_p d_p g} \frac{1-\varepsilon}{\varepsilon^3} \quad (4)$$

where dr is the groundwater moving distance along a streamline, dh_s is the head loss through dr , V_{in} is the infiltration velocity, A is the viscous resistance of the porous media

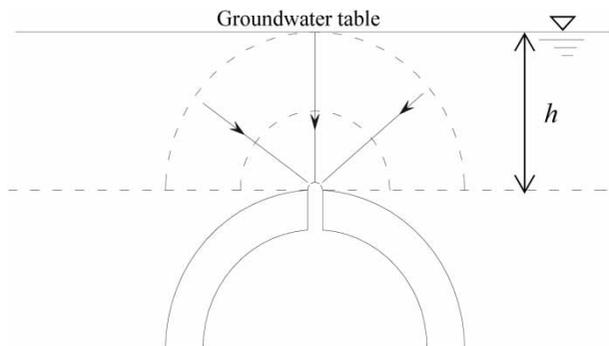


Figure 2 | Schematic of assumed groundwater flow field.

and equals the reciprocal of the hydraulic conductivity K , B is the inertial resistance of the porous media, d_p is the mean particle diameter, φ_p is the particle shape factor and equals 1 for spherical particles, ε is the porosity of the media, μ is the dynamic fluid viscosity, ρ_w is the groundwater density, and g is the acceleration due to gravity.

In the flow field, as shown in Figure 2, the conservation of mass requires that:

$$V_{in} = \frac{Q}{2\pi r^2} \quad (5)$$

Substitute Equation (5) into Equation (3) and integrate over the flow field, then the friction loss from the soil can be obtained as:

$$\begin{aligned} \Delta h_s &= \int_{D_0/2}^h (AV_{in} + BV_{in}^2) dr \\ &= \frac{AQ}{2\pi} \left(\frac{2}{D_0} - \frac{1}{h} \right) + \frac{BQ^2}{12\pi^2} \left(\frac{8}{D_0^3} - \frac{1}{h^3} \right) \end{aligned} \quad (6)$$

Typically, $D_0 \ll h$ and Equation (6) can be simplified to be:

$$\Delta h_s = \frac{A}{\pi D_0} Q + \frac{2B}{3\pi^2 D_0^3} Q^2 \quad (7)$$

The head loss in the orifice originates from two mechanisms: the local head loss due to the inertia of the water, and the friction head loss due to the pipe wall thickness. The first kind of loss can be expressed by the standard orifice equation and the other one can be assessed by Darcy-Weisbach equation. The total orifice loss can be obtained by adding the two parts:

$$\Delta h_{o1} = \frac{1}{C_d^2} \frac{V_{in}^2}{2g} = \frac{1}{C_d^2} \frac{8Q^2}{\pi^2 g D_0^4} \quad (8)$$

$$\Delta h_{o2} = f \frac{t}{D_0} \frac{V_{in}^2}{2g} = f \frac{t}{D_0} \frac{8Q^2}{\pi^2 g D_0^4} \quad (9)$$

$$\Delta h_o = \left(\frac{1}{C_d^2} + f \frac{t}{D_0} \right) \frac{V_{in}^2}{2g} = k \frac{V_{in}^2}{2g} = \frac{8kQ^2}{\pi^2 g D_0^4} \quad (10)$$

where C_d is the discharge coefficient, f is the roughness coefficient, t is the pipe wall thickness, k is introduced as the integrated orifice loss coefficient.

By combining Equations (7) and (10), the total head loss can be expressed as:

$$\Delta h = \left(\frac{2B}{3\pi^2 D_0^3} + \frac{8k}{\pi^2 g D_0^4} \right) Q^2 + \frac{A}{\pi D_0} Q \quad (11)$$

By solving the above quadratic equation, the analytical solution for the groundwater infiltration rate can be obtained:

$$Q_{in} = \frac{-A/\pi D_0}{4B/3\pi^2 D_0^3 + 16k/\pi^2 g D_0^4} + \sqrt{\left(\frac{A/\pi D_0}{4B/3\pi^2 D_0^3 + 16k/\pi^2 g D_0^4} \right)^2 + \frac{\Delta h}{2B/3\pi^2 D_0^3 + 8k/\pi^2 g D_0^4}} \quad (12)$$

RESULTS AND DISCUSSION

Experimental verification

To examine the derived analytical expression, a series of experiments were conducted to simulate the groundwater infiltration into a buried pipeline under steady-state conditions. The schematic of the setup is shown in Figure 3: a rectangular tank ($600 \times 600 \times 1,000 \text{ mm}^3$) with a section of circular Plexiglas pipe going through it. The diameter of the pipe with a thickness of 5 mm is 200 mm. An orifice with a diameter of 10 mm was located on the top of pipe, and a piece of wire was covered over the orifice to prevent

sand particles leaking into the pipe. The pipe was laid with a slope of 2%, so that the height difference between the two ends of the pipe would force the infiltration water to flow out from the lower end. A movable plate set at one side of the container was used as the overflow weir to adjust the water head.

A kind of uniformly screened sand was used as the bed (bed thickness = 100 mm) and backfill soil material (backfill thickness = 200 mm). The particle size distribution of the sand is shown in Figure 4. The mean particle size $d_p = 0.47 \text{ mm}$ and the particle density was measured to be $2,650 \text{ kg/m}^3$. A similar procedure used in our previous study on soil and water erosion was followed for preparing the experiment in this study (Guo et al. 2013b). In each experiment, the sand and water were added and compacted layer by layer, then the movable plate was adjusted to a required water head; the initial porosity ϵ (void fraction) was controlled to be 0.36 for all experiments; during the experiment, the volumetric discharge of the infiltration water from the orifice was measured using beakers, and the measurement time was then determined from a recording video camera.

Before the infiltration experiment, tests with no soil media (only water) were conducted. By fitting the results to Equation (10), the integrated orifice loss coefficient k is found to be 2.8. It should be noted that the tests were conducted with the wire covering over the orifice. Therefore, the possible effect from the wire has been incorporated into the measured value of k .

In order to obtain the particle's shape effect on the infiltration flow and avoid the direct measurement of a great deal of individual particle's shape factor, constant-head tests were conducted according to the standard soil test method. The hydraulic conductivity K of the sand material

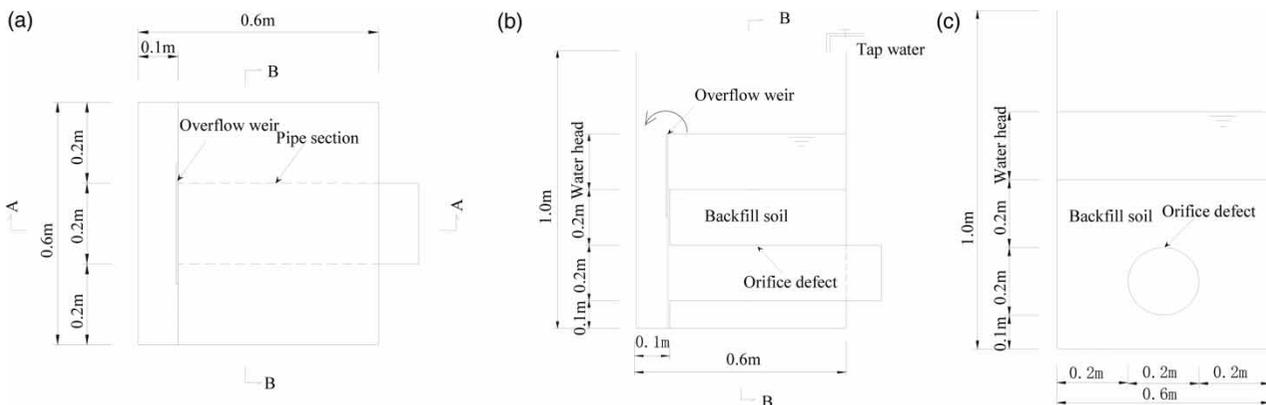


Figure 3 | Schematic of experiment setup: (a) plan view; (b) section A-A; (c) section B-B.

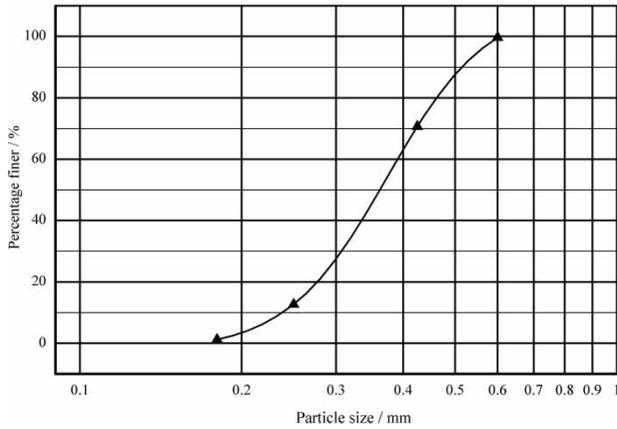


Figure 4 | Particle size distribution of experimental sand.

is measured to be 1.13×10^{-5} m/s. Using Equation (4), the values of A , B , and $\varphi_p d_p$ are calculated to be 885.47 s/m, 6,256.81 m²/s² and 0.39 mm, respectively. Table 1 summarizes all the related experimental parameters.

For the infiltration investigation, six different water tables were added above the sand surface (30, 50, 70, 90, 110, and 130 mm). Since the soil layer height was 200 mm and the internal pressure of the pipe was the atmospheric pressure, therefore, the tested piezometric pressure head difference Δh were 230, 250, 270, 290, 310, and 330 mm. The obtained steady-state infiltration rates under different water tables were compared with the newly derived analytical expression and Collins and Boxall’s analytical model, as shown in Figure 5. Collins and Roxall’s model included a geometry factor, which was defined as G , and calculated by $G = 1/(D_0)^{0.25}$; other parameters’ values are also taken from Table 1. The presented model can reproduce the measured results within an error of 10%, while Collins and Boxall’s model significantly underestimated the infiltration rate.

Parametric analysis

Walski et al. (2006) proposed an OS number that is the ratio of the orifice head loss and soil head loss to model the leakage from water distribution pipes, and concluded that the OS number would be larger than 1 in most cases. The OS

Table 1 | Parameters’ values based on experimental results

d_p/φ_p (mm)	ε	t (mm)	D_0 (mm)	k	K (m/s)	A (s/m)	B (m ² /s ²)
0.39	0.36	5	10	2.8	0.00113	885.47	6,256.81

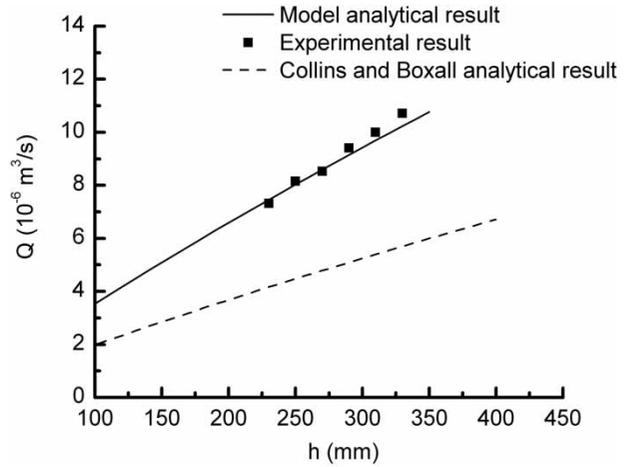


Figure 5 | Comparison of the results by analytical expressions and experimental data.

and Reynolds numbers in this study were carefully measured, as shown in Table 2.

As shown in Table 2, the OS numbers were very small and greatly less than unity. The disagreement between the two studies was mainly due to the Reynolds number. Walski et al. (2006) used the Darcy equation to assess the head loss through the soil, however, this is only valid for $Re < 10$ when the groundwater flow is laminar. The Reynolds number in the vicinity of the orifice in this study was larger than 40 in all cases, as indicated in Table 2. The disadvantages of Walski et al. (2006) analysis have also been noticed by Van Zyl & Clayton (2007) and Collins & Boxall (2013).

Actually, by combining Equations (7) and (10), we can get a new OS number expression as:

$$OS = \frac{\Delta h_o}{\Delta h_s} = \frac{k}{gD_0(\pi D_0^2 A/8Q + B/12)}$$

$$= \frac{2k}{(gD_0/V_o)A + (gD_0/6)B} \tag{13}$$

where V_o represents the velocity at the orifice and is defined to differ from the specific velocity. The square of the term $(gD_0)^{0.5}$ represents a characteristic velocity at the orifice, then we can deduce that gD_0 or $(gD_0)^{0.5}$ should be of the same order with V_o . The hydraulic conductivity K of the backfill sandy soil is usually 10^{-2} – 10^{-5} m/s (Terzaghi et al. 1996), then $A = 1/K$ will be of the order of 10^2 – 10^5 . The denominator term in Equation (13) should be of the order of 10^2 or larger, while the numerator can be easily assessed according to the Equation (10) and should be generally less than 10^2 .

Table 2 | OS and Reynolds number based on experimental results

Water table (mm)	230	250	270	290	310	330
OS number = $\Delta h_o/\Delta h_s$	0.0054	0.006	0.0062	0.0068	0.0071	0.0076
$Re = \rho_w V d_p/\mu$	44	49	51	56	60	64

Therefore, the OS number would be less than 1, and we can conclude that in most cases, the head loss through the soil dominates the infiltration process.

CONCLUSIONS

Based on some reasonable assumptions, including a homogeneous and isotropic aquifer, this paper presents an approximate model for calculating groundwater infiltration rate into sewer systems through orifice or hole damages. Although the damage is assumed to be on top of the sewer pipe in deriving the expression, the proposed model is still applicable if the orifice locates on other places. In order to validate the proposed model, experiments were conducted and a good agreement between predicted results and experimental results was found.

The derived analytical expression combines the head loss through the orifice and the soil. By introducing Ergun equation instead of the Darcy equation to assess the resistance from the soil layer to the groundwater flow, the application range of the new expression has been significantly extended. A new OS number expression is also presented. The order analysis of the OS number has cleared the fact that the head loss through the soil layer dominates the whole infiltration process in most real cases. At present, advanced inspection technologies capture not only the internal structural condition of the sewer pipe, such as the defect type, size, but also the external groundwater table, soil type, etc. Data collected by these technologies provide the probabilities for using the proposed model in the future.

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REFERENCES

- Aksoy, H. & Wittenberg, H. 2011 [Nonlinear baseflow recession analysis in watersheds with intermittent streamflow](#). *Hydrological Sciences Journal* **56** (2), 226–237.
- Brombach, H., Weiss, G. & Lucas, S. 2002 Temporal variation of infiltration inflow in combined sewer systems. In: *9th International Conference on Urban Drainage*, 8–13 September 2002, Portland, OR, USA.
- Collins, R. & Boxall, J. 2013 [Influence of ground conditions on intrusion flows through apertures in distribution pipes](#). *Journal of Hydraulic Engineering* **139** (10), 1052–1061.
- Collins, R., Boxall, J., Besner, M., Beck, S. & Karney, B. 2010 Intrusion modelling and the effect of groundwater conditions. In: *Proceedings of Water Distribution Analysis*, ASCE, Reston, VA, USA, pp. 585–594.
- Costello, S. B., Chapman, D. N., Rogers, C. D. F. & Metje, N. 2007 [Underground asset location and condition assessment technologies](#). *Tunnelling and Underground Space Technology* **22**, 524–542.
- Davies, J. P., Clarke, B. A., Whiter, J. T. & Cunningham, R. J. 2001 [Factors influencing the structural deterioration and collapse of rigid sewer pipes](#). *Urban Water* **3** (1–2), 73–89.
- De Benedittis, J. & Bertrand-Krajewski, J.-L. 2005 Infiltration in sewer systems: comparison of measurement methods. *Water Science & Technology* **52** (3), 219–227.
- El Tani, M. 2003 [Circular tunnel in a semi-infinite aquifer](#). *Tunnelling and Underground Space Technology* **18** (1), 49–55.
- Ergun, S. 1952 Fluid flow through packed columns. *Chemical Engineering Progress* **48**, 89–94.
- Guo, S., Zhang, T., Zhang, Y. & Zhu, D. Z. 2013a [An approximate solution for two dimensional groundwater infiltration in sewer systems](#). *Water Science & Technology* **67** (2), 347–352.
- Guo, S., Shao, Y., Zhang, T., Zhu, D. Z. & Zhang, Y. 2013b [Physical modeling on soil erosion around defective sewer pipes](#). *Journal of Hydraulic Engineering* **139** (12), 1247–1257.
- Karpf, C. & Krebs, P. 2013 [Modelling of groundwater infiltration into sewer systems](#). *Urban Water Journal* **10** (4), 221–229.
- Kolymbas, D. & Wagner, P. 2007 [Groundwater ingress into tunnel—the exact analytical solution](#). *Tunnelling and Underground Space Technology* **22**, 23–27.
- Kracht, O. & Gujer, W. 2005 Quantification of infiltration into sewers based on time series of pollutant loads. *Water Science & Technology* **52** (3), 209–218.
- Kracht, O., Gresch, M. & Gujer, W. 2007 [A stable isotope approach for the quantification of sewer infiltration](#). *Environmental Science & Technology* **41** (16), 5839–5845.

- Kuliczowska, E. 2016 [Risk of structural failure in concrete sewers due to internal corrosion](#). *Engineering Failure Analysis* **66**, 110–119.
- Park, K. H., Owatsiriwong, A. & Lee, J. G. 2008 [Analytical solution for steady-state groundwater inflow into a drained circular tunnel in a semi-infinite aquifer: a revisit](#). *Tunnelling and Underground Space Technology* **23** (2), 206–209.
- Prigiobbe, V. & Giulianelli, M. 2009 [Quantification of sewer system infiltration using \$\delta^{18}\text{O}\$ hydrograph separation](#). *Water Science & Technology* **60** (3), 727–735.
- Terzaghi, K., Peck, R. B. & Mesri, G. 1996 *Soil Mechanics in Engineering Practice*, 3rd edn. John Wiley & Sons, Inc., New York, NY, USA.
- Thorndahl, S., Balling, J. D. & Larsen, U. B. B. 2016 [Analysis and integrated modelling of groundwater infiltration to sewer networks](#). *Hydrological Processes* **30** (18), 3228–3238.
- Van Zyl, J. E. & Clayton, C. R. I. 2007 The effect of pressure on leakage in water distribution systems. *Water Management* **160** (2), 109–114.
- Walski, T., Bezts, W., Posluszny, E. T., Weir, M. & Whitman, B. E. 2006 Modeling leakage reduction through pressure control. *Journal American Water Works Association* **98** (4), 147–155.
- Weiss, G., Brombach, H. & Haller, B. 2002 Infiltration and inflow in combined sewer systems: long-term analysis. *Water Science & Technology* **45** (7), 11–19.
- Wirahadikusumah, R., Abraham, D. M., Iseley, T. & Prasanth, R. K. 1998 [Assessment technologies for sewer system rehabilitation](#). *Automation in Construction* **7** (4), 259–270.
- Wittenberg, H. & Aksoy, H. 2010 [Groundwater intrusion into leaky sewer systems](#). *Water Science & Technology* **62** (1), 92–98.

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