Multi-objective evolutionary polynomial regression-based prediction of energy consumption probing
Hossein Bonakdari, Isa Ebtehaj and Azam Akhbari

ABSTRACT
Electrocoagulation (EC) is employed to investigate the energy consumption (EnC) of synthetic wastewater. In order to find the best process conditions, the influence of various parameters including initial pH, initial dye concentration, applied voltage, initial electrolyte concentration, and treatment time are investigated in this study. EnC is considered the main criterion of process evaluation in investigating the effect of the independent variables on the EC process and determining the optimum condition. Evolutionary polynomial regression is combined with a multi-objective genetic algorithm (EPR-MOGA) to present a new, simple and accurate equation for estimating EnC to overcome existing method weaknesses. To survey the influence of the effective variables, six different input combinations are considered. According to the results, EPR-MOGA Model 1 is the most accurate compared to other models, as it has the lowest error indices in predicting EnC(289,657),(780,691)(289,691),(780,725)(289,726),(780,760) (MARE = 0.35, RMSE = 2.33, SI = 0.23 and $R^2 = 0.98$). A comparison of EPR-MOGA with reduced quadratic multiple regression methods in terms of feasibility confirms that EPR-MOGA is an effective alternative method. Moreover, the partial derivative sensitivity analysis method is employed to analyze the EnC variation trend according to input variables.

Key words | dye removal, electrocoagulation, energy consumption, multiple regression

INTRODUCTION
The large quantity of aqueous waste generated by dyeing industries has become a significant environmental problem. Several industries, such as leather, textile, dyestuffs, tanning, plastics, rubber, and cosmetics, use huge amounts of synthetic dyes and pigments (Baldikova et al. 2015). The disposal of colored textile wastewater into the environment without efficient treatment poses a threat of serious damage to aquatic life. Dye wastewater can be treated by biological means, chemical coagulation, activated carbon adsorption, ultrafiltration, ozonation, and electrocoagulation (EC) with electroflotation (Do & Chen 1994; Ghanbari et al. 1997; Ghanbari & Moradi 2015).

One of the methods developed to overcome the drawbacks of conventional water and wastewater treatment technologies is EC. EC is a potentially effective method for treating different kinds of wastewater with high removal efficiency (Golder et al. 2007). The EC process is a simple, reliable and cost-effective method of treating wastewater without the need for any additional chemicals and consequently, without causing secondary pollution. It also reduces the amount of sludge that requires disposal. The EC process has successfully been utilized for the treatment of wastewater, including electroplating wastewater (Ge et al. 2004; Escobar et al. 2006), laundry wastewater (Drouiche et al. 2009), and photovoltaic wastewater (Daneshvar et al. 2006). Meanwhile, the EC process has been widely used to decolorize various textile wastewaters (Giustolisi & Savic 2006; Kabdasli et al. 2009; Parama Kalyani et al. 2009).

Pollution parameters can be effectively removed via EC provided the operating conditions are carefully optimized and feasibility studies are conducted. Considering that the main operating costs associated with the EC treatment process are electrical energy requirements and sludge handling, it is very important to assess these two economic parameters before proposing EC as an alternative treatment option for dyehouse effluents. Until now, EC electrical energy requirements have been addressed in only a few studies (Arslan-Alaton et al. 2008). Investigating the influence of operating parameters is at the center of attention; hence the role of these parameters should be considered. Several researchers

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have investigated the effect of pH on the EC process using various types of wastewater. By taking into account the necessity for pH adjustment for usually extremely alkaline dyehouse effluents, it seems to make more sense to apply the EC process to high-strength, low-volume and slightly acidic effluents from polyamide dyeing operations with acid dyes and their respective dye auxiliaries (wetting agents, pH buffers). It has been established (Gao et al. 2005) that pH has a considerable effect on the efficiency of the EC process. Moreover, as observed by other investigators, the pH of the medium changes during the process. This change depends on the type of electrode material and initial pH and alkalinity. Also, it is known that electrical current density is the major operating variable directly affecting EC performance and operating costs. It is found that by increasing the applied current from appreciably improved decolorization, the addition of an electrolyte (Na2SO4 or NaCl) can enhance wastewater conductivity, decrease cell voltage and reduce electrical energy as well. In general, electrolytes are used to obtain conductivity in the EC process. Solution conductivity affects current efficiency, cell voltage and electrical energy consumption (EnC) in electrolytic cells. Also, current density is the most important parameter in the entire EC process. The current density determines not only the coagulant dosage rate but also the bubble production rate and size, and the floc growth, and it strongly influences the treatment efficiency of the EC process. The EnC increases with increasing current density due to an increase in ion production on the anode and cathode (Edris et al. 2008; Sridhar et al. 2014).

The operating cost is a very important economic parameter in the EC process. Operating costs involve the costs of chemicals, electrodes and EnC as well as labor, maintenance, sludge disposal and fixed costs. Energy, electrode and chemical costs are taken into account as major cost items in calculating operating costs (Bayramoglu et al. 2004; Sridhar et al. 2014). Therefore, a technically effective process must be economically feasible with regard to its electrical EnC and practical applicability to environmental problems. Therefore, EnC under optimum conditions is of great importance.

Evolutionary polynomial regression (EPR) is a data-driven hybrid technique that benefits from a combination of the effectiveness of genetic programming and numerical regression for developing simple and easily interpretable mathematical model expressions (Fonseca & Fleming 1993). The EPR approach overcomes some drawbacks of other modeling approaches, such as physical-based models and black-box data-driven models. The former can be difficult to construct due to underlying mechanisms that may not always be fully understood or the need for much data that is sometimes difficult to measure in the field. The latter, for example artificial neural networks, are very effective in reproducing any database related to a certain observed phenomenon but also carry overwhelming problems, like model structure identification, overfitting to training data, and the inability to exploit physical insight about the phenomenon at stake. EPR can overcome these problems by means of an explicit model expression for the system under observation (Deb et al. 2002; Maleki et al. 2014). Real-world optimization problems usually involve several conflicting objectives. The main aim in multi-objective optimization is to select the best tradeoffs among the conflicting objectives. Multi-objective evolutionary algorithms are regarded as promising methods for dealing with multi-objective optimization problems (MOPs) owing to their ability to generate populations of solutions in a more efficiently approximating a diverse set of optimal solutions in a single run (Deb et al. 2002). Therefore, developing an efficient optimization method to deal with expensive simulation-based MOPs by providing good accuracy and fewer function evaluations simultaneously is a crucial contemporary challenge.

The main goal of this study is to investigate the evolutionary polynomial regression–multi-objective genetic algorithm (EPR-MOGA) method in predicting EnC probing. For this purpose, the effective variables for predicting EnC probing are identified. Subsequently, to investigate the effect of each variable, six different input combinations are examined. All input combinations are modeled with EPR-MOGA and a simple equation is presented for each one. After selecting the best input combination, the results of EPR-MOGA are compared with reduced quadratic multiple regression. A partial derivative sensitivity analysis (PDSA) is also done to analyze the EnC variation trend of the proposed equation.

**EXPERIMENTAL DATA**

To estimate the EnC of EC, experimental data of Maleki et al. (2014) were utilized. Maleki et al. (2014) conducted experiments in an EC system consisting of a cubic (12 × 12 × 21 cm) glass reactor, 400 rpm mixer, DC power supply (high stability and reliability, and low-noise DC adjustable power supply RXN-303D-II, Zhaoxin Electronic Tech. Co.), and two aluminum electrodes (see Maleki et al. [2014] for more details). Table 1 represents the range of parameters that affect EnC in the Maleki et al. study.
Table 1 | Range of data in Maleki et al. (2014) study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pH (pH0)</td>
<td>2–9</td>
</tr>
<tr>
<td>Initial dye concentration (C0), mg/l</td>
<td>8–100</td>
</tr>
<tr>
<td>Applied voltage (V_{EC}), V</td>
<td>10–30</td>
</tr>
<tr>
<td>Initial electrolyte concentration (C0), mg/l</td>
<td>0–3</td>
</tr>
<tr>
<td>Treatment time (t_{EC}), min</td>
<td>0.5–50</td>
</tr>
<tr>
<td>Energy consumption (EnC), Wh</td>
<td>0.001–76</td>
</tr>
</tbody>
</table>

To present a new model for EnC prediction, 173 samples (Maleki et al. 2014) are used in this study. The samples regard different values of input parameters, namely initial pH, initial dye concentration, applied voltage, initial electrolyte concentration and treatment time.

The EPR-MOGA results presented in this paper are compared with the stepwise multiple linear regression (SMLR) results presented by Maleki et al. (2014). The comparison indicates that EPR-MOGA outperforms SMLR. The weakness of other artificial intelligence-based methods is the lack of a specific equation for use in future studies, whereas EPR-MOGA yields an expression. Therefore, the proposed method is more accurate than other artificial intelligence methods such as artificial neural networks that do not present specific equations (e.g. multiple linear regression), and can thus be used for future studies.

Moreover, the number of data in artificial intelligence methods such as EPR-MOGA is different and a good model should contain less than 100 samples (e.g. Rezania et al. 2008; Ahangar-Asr et al. 2011; Fiore et al. 2012). Therefore, 173 samples are sufficient to find an appropriate model in this study. These data were collected from Maleki et al. (2014).

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**EPR**

EPR is a data-driven hybrid technique, which combines the effectiveness of genetic programming with numerical regression to develop simple and easily interpretable mathematical model expressions (Giustolisi & Savic 2006). The EPR approach overcomes some drawbacks of other modeling approaches, such as physical-based models and black-box data-driven models. The former can be difficult to construct due to underlying mechanisms that may not always be fully understood, or owing to the need for much data that is sometimes difficult to measure in the field. The latter, for example artificial neural networks, are very effective in reproducing any database related to some observed phenomenon but bring some overwhelming problems like model structure identification, overfitting to training data, and the inability to exploit physical insight about the phenomenon at stake. EPR can overcome these problems by means of an explicit model expression for the system under observation. EPR-MOGA can be defined as a nonlinear global stepwise regression for symbolic data modeling.

These models can then be verified on a test set and gaps can be filled in test datasets by using one selected model. Because of the pseudo-polynomial formulations achievable by EPR-MOGA, it requires fewer parameters to be estimated, which in turn requires shorter time series for training. Another advantage of the EPR-MOGA approach is the ability to choose objective functions pertaining to accuracy and parsimony (Barca et al. 2015).

The expressions obtained with the EPR method generally contain different terms that are multiplied by different coefficients (e.g. polynomials) as follows:

\[
\hat{Y} = a_0 + \sum_{j=1}^{m} (X_1)^{ES(j,1)} \cdot \ldots \cdot (X_k)^{ES(j,k)} \\
\quad \cdot f((X_1)^{ES(j,1)}, \ldots, (X_1)^{ES(j,k)}))
\]

where \(m, X_i\) and \(\hat{Y}\) are the maximum amounts of additive terms, input variables, and output variables; the \(f\) function is determined by the user; and \(ES((i,j))\) are chosen from a set of \(EX\) applicants provided by the user. A genetic algorithm is used to select the \(ES(j,i)\) exponents among the values presented in \(EX\). In fact, an integer coding of possible exponents \(ES(j,i)\) is assumed to achieve nonlinear relationships. If \(ES(j,i) = 0\) and the set of exponents includes zero, the related input is deleted from the final expression. Therefore, versatile and flexible structures can be expressed to generate patterns in data similar to Equation (1) (Giustolisi & Savic 2006).

In the development strategy of the model proposed by EPR, the final expression is linear according to the \(a_i\) coefficient. Parameter estimation is solved as an inverse linear problem to guarantee a unique relationship between each model parameter and its structure. In terms of numerical regression strategy, EPR, similar to the artificial neural network method, makes nonlinear mapping between data although the parameters are estimated with some constants and nonlinear regression (Haykin 1999). These features prevent overfitting, especially when the data number is low. In addition, the initial assumption in the mathematical
The effective EC operational parameters are generally the initial pH ($pH_0$), initial dye concentration ($C_0$), applied voltage ($V_{EC}$), initial electrolyte concentration ($C_S$) and treatment time ($t_{EC}$). Therefore, to consider all these parameters in EnC estimation, a functional relationship is provided as follows:

$$EnC = f(pH_0, C_0, V_{EC}, C_S, t_{EC})$$  \(2\)

The methodology applied in this study entails dividing the dataset into two groups: training and testing. From the combinations employed in this study, Equation (1) was selected and modeled using EPR-MOGA. Then to examine the effect of each parameter, different models were proposed as follows:

- EPR-MOGA (1): $EnC = f(pH_0, C_0, V_{EC}, C_S, t_{EC})$
- EPR-MOGA (2): $EnC = f(pH_0, C_0, V_{EC}, C_S)$
- EPR-MOGA (3): $EnC = f(pH_0, C_0, V_{EC}, t_{EC})$
- EPR-MOGA (4): $EnC = f(pH_0, C_0, C_S, t_{EC})$
- EPR-MOGA (5): $EnC = f(pH_0, V_{EC}, C_S, t_{EC})$
- EPR-MOGA (6): $EnC = f(C_0, V_{EC}, C_S, t_{EC})$

By developing EPR-MOGA, various equations are proposed for all six input combinations as presented in Table 2. Maleki et al. (2014) used the SMLR methodology for EnC after neglecting the non-significant terms (Bhatti et al. 2012).

### Table 2 | Equations obtained by EPR-MOGA for calculating EnC

<table>
<thead>
<tr>
<th>Model no.</th>
<th>Equations</th>
<th>Eq. no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$EnC = 8.5156 \times \left( \frac{V_{EC} t_{EC}}{pH_0^2 C_S^2} \right) + 5.5463E - 4 \times V_{EC}^2 C_S^{2.5} t_{EC} + 4.1254E - 5 \times C_S^{0.5} V_{EC}^2 C_S^{0.5} t_{EC}$</td>
<td>(3)</td>
</tr>
<tr>
<td>2</td>
<td>$EnC = 1.0734 \times \left( \frac{V_{EC}^2 t_{EC}}{pH_0^6 C_S^2} \right) + 4.7369E - 2 \times V_{EC}^{1.5} C_S^{0.5} + 2.2147E - 3 \times pH_0^{0.5} V_{EC}^{0.5} C_S^{0.5}$</td>
<td>(4)</td>
</tr>
<tr>
<td>3</td>
<td>$EnC = 0.24059 \times \left( \frac{V_{EC}^2 t_{EC}}{pH_0^2 C_S^2} \right) + 1.415E - 2 \times V_{EC}^{1.5} t_{EC}^{1.5} + 2.292E - 3 \times pH_0^{0.5} V_{EC}^{0.5} t_{EC}$</td>
<td>(5)</td>
</tr>
<tr>
<td>4</td>
<td>$EnC = 7.6889 \times \left( \frac{V_{EC}^2 t_{EC}}{pH_0^2 C_S^2} \right) + 1.2121 \times C_S^{0.5} t_{EC}^{0.5} + 4.9741E - 4 \times pH_0 \times C_S \times t_{EC}$</td>
<td>(6)</td>
</tr>
<tr>
<td>5</td>
<td>$EnC = 1.5837E - 4 \times \left( \frac{V_{EC}^2 t_{EC}}{pH_0^2 C_S^2} \right) + 2.1034E - 3 \times V_{EC}^{1.5} C_S^{0.5} t_{EC}^{0.5} + 6.9144E - 4 \times V_{EC}^{2} C_S^{0.5} t_{EC}$</td>
<td>(7)</td>
</tr>
<tr>
<td>6</td>
<td>$EnC = 3.9866E - 4 \times V_{EC}^{1.5} t_{EC} + 5.9118E - 4 \times V_{EC}^{2} C_S^{0.5} t_{EC} + 2.0217E - 5 \times C_S^{0.5} V_{EC}^{2} C_S^{0.5} t_{EC}$</td>
<td>(8)</td>
</tr>
</tbody>
</table>
Figure 1 | EPR-MOGA performance evaluation in EnC prediction.
The model is given in Equation (9).

\[
EnC = 5.7837 - 6.3669(C_s) - 0.7365(t_{EC}) + 0.3223(V_{EC})(C_s) + 0.0287(V_{EC})(t_{EC}) + 0.2689(C_s)(t_{EC}) - 0.0106(V_{EC})^2 + 0.0045(t_{EC})^2
\]

### RESULTS AND DISCUSSION

In this section, the EnC results are predicted using the EPR-MOGA technique. For this purpose, the statistical indices utilized are the determination coefficient \((R^2)\), root mean square error \((RMSE)\), mean absolute relative error \((MARE)\), scatter index \((SI)\) and BIAS, which are calculated as follows:

\[
R^2 = \left( \frac{\sum_{i=1}^{n}(O_i - P_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O}_i)^2} \right) \frac{\sum_{i=1}^{n}(P_i - \bar{P}_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O}_i)^2} \right)^2
\]

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (O_i - P_i)^2}
\]

\[
MARE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{|O_i - P_i|}{O_i} \right)
\]

\[
SI = \frac{RMSE}{\sum_{i=1}^{n} (O_i)}
\]

\[
BIAS = \frac{\sum_{i=1}^{n} (P_i - O_i)}{n}
\]

where \(O_i\) and \(P_i\) are the observed and predicted \(EnC\) respectively, \(\bar{O}_i\) and \(\bar{P}_i\) are the average observed and predicted \(EnC\) values, and \(n\) is the number of parameters.

Figure 1 shows the performance of EPR-MOGA with six different input combinations (Model 1 to Model 6) in \(EnC\) prediction. Model 1 considers all effective parameters in estimating \(EnC\), which are presented as Equation (3). Table 3 presents the statistical indices for \(EnC\) modeling by EPR-MOGA. According to Figure 1, EPR-MOGA is highly capable of estimating \(EnC\), as many estimated values have little difference from the observed values \((R^2 = 0.98; MARE = 0.35; RMSE = 2.33; SI = 0.23)\). It can be seen that this model often estimates with less than 15% relative error, which indicates good model accuracy.

Model 1 makes underestimations and overestimations, while on average and especially for large errors, the \(EnC\) predictions are underestimated \((BIAS = -0.44)\). Not using \(t_{EC}\) as an input parameter in EPR-MOGA \(EnC\) modeling leads to significantly reduced modeling accuracy. The \(EnC\) prediction results by Model 2 indicate low accuracy according to the statistical indices \((R^2 = 0.54; MARE = 0.3.3.7; RMSE = 12.59; SI = 1.22)\). Models 3 and 4 do not use \(C_s\) and \(V_{EC}\) and are weaker than Model 2. Therefore, \(t_{EC}, V_{EC}\) and \(C_s\) should be used in predicting \(EnC\). The most effective variable is the initial electrolyte concentration, with the largest MARE in the test stage \((MARE = 14.65)\).

Unlike \(t_{EC}, C_s\) and \(V_{EC}, C_0\) and \(pH_0\) have low impact on \(EnC\) prediction by EPR-MOGA, whereby not using \(C_0\) and \(pH_0\) increases the relative errors by approximately three and four times (respectively) with Model 1, while the \(MARE\) values of Models 2, 3 and 4 are more than 10 times greater than for Model 1. However, Models 5 and 6 in which \(C_0\) and \(pH_0\) were removed from the input combinations (respectively) do not present an acceptable level of accuracy.

Therefore, EPR-MOGA Model 1 with the lowest error index among all models is the most accurate in predicting the \(EnC \) \((MARE = 0.41 and 0.35; RMSE = 1.62 and 2.33; SI = 0.23 and 0.23 and \(R^2 = 0.98 and 0.98 in training and testing, respectively). It can be seen that Model 1 has the highest compliance with the experimental values and is slightly different from the experimental model.
Sensitivity analysis

To analyze the trend changes in the proposed EPR-MOGA equation according to the input variables considered for each equation, PDSA is applied in this study (Ebtehaj et al. 2015). With this method, the partial derivative of the equation presented for each input variable is calculated. Then the trend changes are studied for different input
variable values, from which the derivative of each equation has been calculated. It is clear that the extent of the partial derivative calculated is directly related to its effect on the predicted result. The positive and negative values of a partial derivative show that increasing the input parameter value leads to a decrease or increase in the results, respectively.

Figure 2 presents the partial derivative results of Equation (3) for the parameters provided in this relationship. The partial derivatives for the $V_{EC}$, $I_{EC}$ and $C_3$ parameter values are positive. The increase (or decrease) in these three parameters leads to an increase (or decrease) in the $EnC$ estimated using Equation (3). It is clear that the derivative of Equation (3) has a negative $pH_0$ value. Therefore, the $pH_0$ trend changes do not result in similar $EnC$ estimation trends. Due to the positive partial derivative value of Equation (3) for variables $C_S$, $I_{EC}$ and $V_{EC}$, the change in these parameters is directly related to the $EnC$ value attained from Equation (3). The partial derivative results concerning $EnC$ compared to parameter $C_0$ are positive and negative. Therefore, by keeping all parameters fixed in this relationship and increasing or decreasing one of these two parameters, no clear trend in the results is observed. The maximum partial derivative value is related to the result obtained for $C_3$. Therefore, it is concluded that Equation (3) displays the greatest sensitivity to this parameter, and changing this parameter leads to significant changes in $EnC$ results.

The EPR-MOGA model results are compared with the SMLR results for $EnC$ prediction presented by Maleki et al. (2014). The EPR-MOGA and SMLR model results are plotted in Figure 3. Here, the EPR-MOGA $EnC$ results are mostly close to the exact line, while SMLR made over- and underestimations with over 15% relative error and MARE of 658.54. Unlike SMLR, the EPR-MOGA model predicted most $EnC$ values with less than 15% relative error and MARE of 0.41 and 0.35 in testing and training respectively. Consequently, according to Figure 3, the EPR-MOGA model outperformed SMLR in $EnC$ prediction. In line with the explanations given, the equations proposed in this study perform better than the equations suggested in previous studies. In addition, a comparison between the EPR-MOGA and SMLR models demonstrates the superior performance of EPR-MOGA in predicting $EnC$.

CONCLUSION

In this study, multi-objective evolutionary polynomial regression was combined with a genetic algorithm (EPR-MOGA) for $EnC$ and new model expressions were derived. The EPR-MOGA and SMLR soft computing models were compared in terms of predicting the $EnC$ of EC. In order to find the best process condition, the influence of various parameters was investigated. It is evident from the results that the data predicted by EPR-MOGA are in good compliance with the experimental data. In training and testing modes, the RMSE values obtained for the superior model were 1.62 and 2.33 respectively, while the SI and BIAS values calculated were 0.23 and 0.23, and $-0.1$ and $-0.44$ respectively. The superior model's sensitivity to the input parameters was evaluated using PDSA. According to the sensitivity analysis results, with increasing $C_S$ the sensitivity increased. Additionally, it was found that the EPR-MOGA model outperformed SMLR in making predictions, suggesting the inherent sensitivity and robustness of the model. A comparison of the EPR-MOGA and SMLR methods in terms of feasibility confirmed that EPR-MOGA is an effective alternative method.

REFERENCES


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