Optimization of hydrometric monitoring network in urban drainage systems using information theory

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ABSTRACT

Regular and continuous monitoring of urban runoff in both quality and quantity aspects is of great importance for controlling and managing surface runoff. Due to the considerable costs of establishing new gauges, optimization of the monitoring network is essential. This research proposes an approach for site selection of new discharge stations in urban areas, based on entropy theory in conjunction with multi-objective optimization tools and numerical models. The modeling framework provides an optimal trade-off between the maximum possible information content and the minimum shared information among stations. This approach was applied to the main surface-water collection system in Tehran to determine new optimal monitoring points under the cost considerations. Experimental results on this drainage network show that the obtained cost-effective designs noticeably outperform the consulting engineers' proposal in terms of both information contents and shared information. The research also determined the highly frequent sites at the Pareto front which might be important for decision makers to give a priority for gauge installation on those locations of the network.

Key words | entropy, information theory, monitoring, optimization, urban drainage

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INTRODUCTION

Entropy theory is usually applied in hydrology for optimality evaluation of information resulting from the measurements in monitoring stations of water resources. In these applications, economic efficiency of the monitoring system can be evaluated by comparison of the costs related to the monitoring system and the information obtained. In other words, entropy theory is one of the methods for evaluation and quantification of redundant information in water resources and environmental systems using limited available data. Entropy theory, unlike statistical methods, does not need assumptions such as system linearity or normality of assumed probability distribution for uncertain variables. This method is also beneficial in situations with non-homogeneous data and spatial–temporal multivariate features. This theory has been extensively applied to design of monitoring networks in water resources and environmental engineering systems. Some examples are briefly reviewed in the rest of this section.

Alfonso (2010) optimized a monitoring network of polders in the Netherlands using information theory and obtained the best choice of gauge locations by simultaneously considering the joint entropy and total correlation. Lee *et al.* (2014) determined the optimal places for monitoring stations in the lake of Yang Dam, South Korea, based on information theory and weighting method taking into account several water quality variables. The results showed that the optimal network provides all the information content of existing stations in the lake with fewer monitoring stations. However, the method had high computational burdens. Hirshleifer & Riley (1979) studied whether more investments to receive more information is economic or not and they proposed the value of information theory. Their method gives the opportunity to decision makers to measure the value of information under uncertain conditions and to make the correct decision. Masoumi & Kerachian (2008) developed a new entropy-based approach for assessing the location of salinity monitoring stations in Tehran Aquifer, Iran. They proposed a transinformation–distance (T–D) curve and used it in a multi-objective genetic algorithm-based optimization model, which provided the best locations for monitoring stations. The results showed the applicability and the efficiency of the model in assessing

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the groundwater monitoring system. Karamouz *et al.* (2009) presented an entropy-based approach for design of an online water quality monitoring network in rivers, in which the number and location of sampling sites as well as the sampling frequencies were determined by minimizing the redundant information. Bashi-Azghadi & Kerachian (2010) developed a new methodology based on information theory for optimally locating monitoring wells in groundwater systems, identifying an unknown pollution source using monitoring data. Mahjouri & Kerachian (2011) evaluated and revised the spatial and temporal sampling frequencies of the water quality monitoring system of the Jajrood River in the northern part of Tehran by developing a discrete entropy-based optimization approach. The results showed that the existing monitoring system of the river should be partially strengthened and in some cases the sampling frequencies should be increased. Li *et al.* (2012) introduced the maximum information minimum redundancy (MIMR) criterion for the design (or evaluation) of hydrometric networks. The MIMR selection was compared with another entropy-based approach and showed better performance at finding stations with high information content, and locating independent stations. Yoo *et al.* (2012) applied the entropy theory with the EPANET hydraulic solver to determine the best sites to install the pressure loggers in water distribution systems. Samuel *et al.* (2013) proposed a combined regionalization and dual entropy-multi-objective optimization (CRDEMO) method for determining the minimum river monitoring network that meets the World Meteorological Organization (WMO) standards. The credibility of the method was investigated on the monitoring network of two watersheds in Canada. Su & You (2014) proposed a spatial information estimation model for the analysis of precipitation gauge networks to improve previous methods based on information theory. The proposed model employed a two-dimensional T–D relationship in conjunction with multivariate information approximation to estimate transinformation to ungauged locations from existing stations, while taking into consideration the influence of multiple stations and anisotropy. Xu *et al.* (2015) used an entropy theory-based multi-criteria method which simultaneously considered the information derived from rainfall series, minimized the bias of areal mean rainfall and minimized the information overlapped by different gauges to resample the rain gauge networks with different gauge densities. The results revealed that the method provides an optimal design of rain gauge networks which is of vital importance in regional hydrological studies and water resources management.

As mentioned above, entropy theory has been widely used in the design of various monitoring networks of water resources system. While several studies have been done at national and international level to determine the optimal monitoring stations in water bodies such as rivers and groundwater resources, in urban systems studies are very limited. This paper proposes a methodology for finding the optimal layout of monitoring hydrometric gauges on urban drainage systems based on developing an entropy-based multi-objective optimization technique integrated with rainfall-runoff simulation models. Details of this modeling approach are also described.

METHODS AND MATERIALS

Entropy theory

Herein, discrete entropy theory is used to define a quantified criterion for evaluation of information content and assessment of duplicate information measured at the monitoring stations. According to Shannon's information theory, information R is the reduction in uncertainty $H(X)$; the latter is known as entropy. The definition of uncertainty indicates how surprising it is, on average, to get a value x from a random variable X that can take the possible values x_1, x_2, …, x_n each with probability $p(x)$:

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i) \tag{1}$$

When there are two variables, some of the information is shared between them. In this case, the joint entropy is defined as:

$$H(X_1, X_2) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(x_{1i}, x_{2j}) \log p(x_{1i}, x_{2j}) \tag{2}$$

where $p(x_{1i}, x_{2j})$ is the joint probability of variables X_1 and X_2 with n and m events, respectively.

In the case of n variables $X_1, X_2, \ldots, X_n$ and $n_1, n_2, \ldots, n_n$ events, the multivariate joint entropy can be generalized as:

$$H(X_1, X_2, \ldots, X_n) = -\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\cdots\sum_{k=1}^{n_n} p(x_{1i}, x_{2j}, \ldots x_{nk}) \times \log p(x_{1i}, x_{2j}, \ldots, x_{nk}) \tag{3}$$

To obtain the amount of information shared by all variables, the concept of total correlation (Alfonso 2010) can be used, which quantifies the dependencies among the set of variables:

$$C(X_1, X_2, \ \ldots, \ X_n) = \sum_{i=1}^{N} H(X_i) - H(X_1, X_2, \ \ldots, \ X_n) \quad (4)$$

The main difficulty in using the above equations, especially when the number of variables is considerably high, is estimating the joint entropy of multiple variables $H(X_1, X_2, \ \ldots, \ X_n)$, which in fact requires the estimation of the joint distribution of $p(x_{1i}, x_{2j}, \ \ldots x_{nk})$. One solution for this task is using the grouping property of mutual information (Kraskov *et al.* 2005; Alfonso 2010). In this method, one variable is agglomerated by another in such a way that the entropy of the new variable is the equivalent of the joint entropy of the original pair. This agglomeration can be done by putting the records of two original variables together, constituting a set of new records for the new variable. Then, the new variable is agglomerated by the next variable and so on. The last step involves summing up the partial total correlations for each built variable. To clarify this approach, a numerical example taken from Alfonso (2010) is illustrated here.

As shown in Table 1, two time series of variables x_1 and x_2 are considered. Firstly, records of variables are transformed into a discrete space (Alfonso 2010):

$$X = a\left[\frac{2x + a}{2a}\right] \quad (5)$$

Equation (5) rounds the original variable x into its nearest lowest integer multiple of a, called X. In this paper, the recorded discharges are rounded to their integer value (by $a = 1$) in order to apply the grouping property of mutual information. This choice makes the information retained by stations more precisely quantified, and provides good estimations for the entropy of original variables (Samuel *et al.* 2013). Therefore, the time series in Table 1 are transformed to the discrete values through rounding the digits (columns 3 and 4). Table 1 shows the mechanism of calculating the entropy of variables X_1 and X_2, using Equation (1). To calculate the joint entropy $H(X_1, X_2)$, two approaches can be followed. The first one is using Equation (2), illustrated in columns 1 to 3 of Table 2. This approach, however, is not straightforward for more than two variables. The second approach is agglomerating the variables. Agglomeration for two variables X_1 and X_2 is done by $A = 10X_1 + X_2$. This creates a unique value for new variable A corresponding to unique pairs of (X_1, X_2). Then, the probability of samples of new variable A can be calculated as shown in column 5 of Table 2. Subsequently, the entropy of A (i.e. $H(A)$) is calculated using Equation (1) which is exactly the same as $H(X_1, X_2)$ obtained from Equation (2) by the first approach.

Similarly, variable A can be agglomerated with variable X_3 to create a new variable B using $B = 10A + X_3$ and so on. When there are n variables, they can be hierarchically agglomerated to estimate $H(X_1, X_2, \ \ldots, \ X_n)$ by the same approach.

Case study

The study area here is the main drainage system of the capital city, Tehran, in the central part of Iran. As shown in Figure 1, the main drainage system in Tehran is divided

Table 1 | Process of calculating the entropy (Equation (1)) for separate variables (Alfonso 2010)

x_1	x_2	X_1	X_2	$p(X_1)$	$p(X_2)$	$\log[p(X_1)]$	$-p(X_1)^*\log[p(X_1)]$	$\log[p(X_2)]$	$-p(X_2)^*\log[p(X_2)]$
3.24	2.2	3	2	0.1	0.3	−1	0.1	−0.52288	0.157
4.25	2.08	4	2	0.1	–	−1	0.1	–	0
5.3	1.15	5	1	0.7	0.1	−0.155	0.108	−1	0.1
5.33	4.81	5	4	–	0.3	–	0	−0.523	0.157
5.45	5.4	5	5	–	0.2	–	0	−0.699	0.140
5.7	4.36	5	4	–	–	–	0	–	0
6.55	4.6	6	4	0.1	–	−1	0.1	–	0
5.42	5.21	5	5	–	–	–	0	–	0
5.4	3.13	5	3	–	0.1	–	0	−1	0.1
5.25	2.91	5	2	–	–	–	0	–	0
							$H(X_1) = 0.408$		$H(X_2) = 0.654$

Table 2 | Process of agglomerating two variables and calculating joint entropy (Alfonso 2010)

$p(X_1, X_2)$	$\log[p(X_1, X_2)]$	$-p(X_1, X_2)*\log[p(X_1, X_2)]$	A	$P(A)$	$\log[P(A)]$	$-p(A)*\log[p(A)]$
0.1	−1	0.1	32	0.1	−1	0.1
0.1	−1	0.1	42	0.1	−1	0.1
0.1	−1	0.1	51	0.1	−1	0.1
0.2	−0.699	0.14	54	0.2	−0.699	0.14
0.2	−0.699	0.14	55	0.2	−0.699	0.14
–	–	0	54	–	–	0
0.1	−1	0.1	64	0.1	−1	0.1
–	–	0	55	–	–	0
0.1	−1	0.1	53	0.1	−1	0.1
0.1	−1	0.1	52	0.1	−1	0.1
		$H(X_1, X_2) = 0.88$				$H(A) = 0.88$

into four independent catchments, namely EDC, TCP, WDC and CRB, based on their naturally draining rivers inside the city including Sorkhe Hesar, Kan and Vard-Avard rivers. EDC includes the areas of the east, north-east and a part of the central region (220 km^2); WDC consists of all adjacent areas of the Kan River (80 km^2), north and west of the flood division channel (70 km^2); TCP covers the enclosed area between EDC and WDC (165 km^2) catchments, which is drained by a few large tunnels; and CRB has an area of 85 km^2 in the west where the runoff is

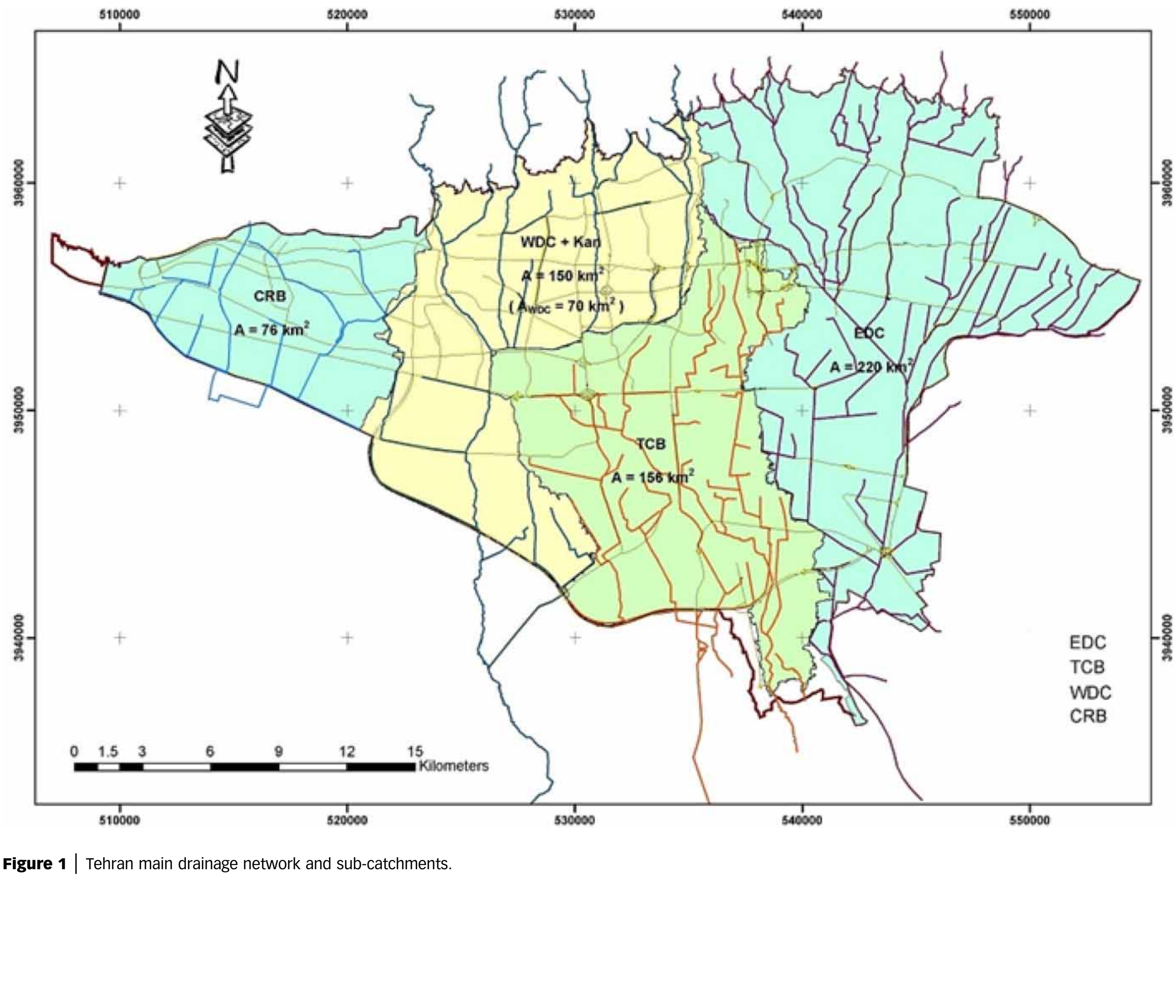

Figure 1 | Tehran main drainage network and sub-catchments.

discharged into the Chitgar River (also called Vard-Avard). The general slope of the land in Tehran decreases from the north to the south. Land slope in the north is nearly 10%, which decreases to around 5% in the south of the city. Mean annual rainfall in Tehran is about 320 mm, but annual rainfall varies at 200 mm in southern parts of the city to 500 mm in the north. In the upstream mountainous area, annual rainfall reaches 700 mm. In addition to four main urban catchments, the Tehran region also has some suburban (mainly mountainous) catchments upstream of the city, the runoff from which moves towards the urban catchments. Total annual runoff of Tehran basin is nearly 250 MCM of which 40% is related to the urban catchments and 60% to the suburban catchments. Total urban and suburban areas are 620 and 670 km^2, respectively.

Rainfall-runoff modeling

Rainfall-runoff modeling is carried out to simulate and analyze the urban drainage network under different flooding loads. This modeling is used to estimate flood variables such as discharge time series in ungauged points of the network. Model setup and calibration for the study area has already been carried out in previous studies (Mahab Ghods Consultant Engineers 2011a, 2011b) and here the same modeling approach and calibrated parameters have been used for the model preparation. The first step for rainfall-runoff modeling was determination of design rainfall including rainfall depth and duration with its spatial–temporal pattern. Rainfall depth can be obtained from the intensity–duration–frequency curves, extracted from the recorded rainfalls. According to the previous studies (Mahab Ghods Consultant Engineers 2011a), design rainfall with a 25-year return period has been recommended by the authorities and thus here this return period is selected. The design rainfall duration is suggested to be at least equal to 'time of concentration' (Yazdi *et al.* 2013). This parameter for the studied urban area was estimated as 150 minutes. After a sensitivity analysis on the severity of produced runoff from rainfalls with different durations, a 6 hour duration has been recognized as the critical value and has been selected for design rainfall duration.

According to the meteorological data, recorded rainfalls in nine precipitation gauges inside the city have been analyzed and the following formula has been proposed for short-term rainfalls:

$$i = C_{Alt.RP} D^{-0.645} \tag{6}$$

where i is the rainfall intensity, D is rainfall duration (min) and $C_{Alt.RP}$ is a coefficient associated with the return period of design rainfall and the mean height of the sub-catchment. Therefore, the spatial variation of rainfall throughout the catchment is manipulated by the above equation using the average height of sub-catchments. The value of $C_{Alt.RP}$ is determined using a lookup table according to the rainfall return period and the average height of the sub-catchment.

A set of approaches have also been proposed in the literature to determine the temporal pattern of design rainfall. Some of the relevant approaches are local pattern (McCuen 1998), Hershfield method (1961), Huff method (1990), SCS depth–duration curves (Yazdi *et al.* 2015) and some other standard types of patterns such as Euler-II, DVWK and uniform patterns (German ATV Rules and Standards 1999) used in UK and German guidelines. A sensitivity analysis has been carried out on the abovementioned methods and the 'local pattern' method has been found to be able to produce the largest peak runoff at different points of the drainage network, and therefore the temporal pattern obtained by this method has been selected as the relevant temporal pattern of short-term rainfalls in Tehran. Figure 2 shows the temporal pattern of design rainfall in sub-catchments of the study area.

To convert design rainfall into flood hydrographs at sub-catchments, a rainfall-runoff model must be employed. In this study, HEC-HMS version 3.5 has been used for this purpose. HEC-HMS is a free public domain software, developed by the US Hydrologic Engineering Center (HEC), for simulating the rainfall-runoff processes of watershed systems (USACE 2010). Simulation results are stored in the data storage system HEC-DSS and can be used in conjunction with

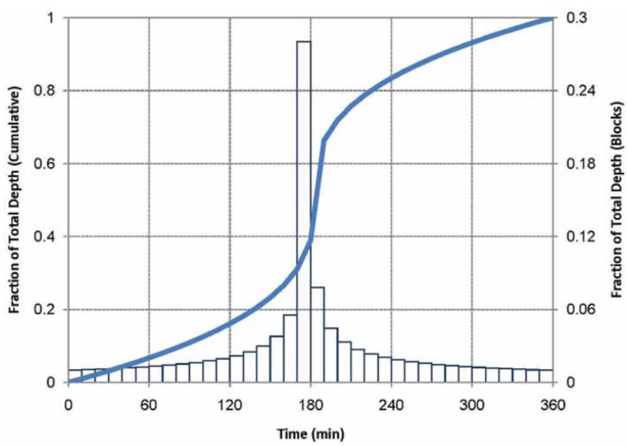

Figure 2 | Temporal pattern of design rainfall in sub-catchments.

other software. The SCS curve number method, introduced by the US Soil Conservation Service (SCS), was set up for estimating the precipitation losses based on the local land use and soil type properties (USACE 2010).

Furthermore, in order to transform excess precipitation into surface runoff, the SCS unit hydrograph was utilized. According to the land uses and soil types, a total of 182 sub-catchments were considered in hydrological modeling including 81, 42, 35 and 24 sub-catchments for EDC, TCP, WDC and CRB catchments, respectively. After calculating sub-catchment surface runoff by model execution, they were introduced to the hydraulic model of the urban drainage network, here EPA's Storm Water Management Model (SWMM), for flood routing in the channels and conduits of the network. SWMM software is generally used for planning, analysis and design related to stormwater runoff, combined and sanitary sewers, and other drainage systems in urban areas. It is a dynamic hydrologic–hydraulic water quality simulation model, used for single event or continuous simulation of runoff quantity and quality from primarily urban areas. The routing portion transports this runoff through a system of pipes, channels, storage/treatment devices, pumps, and regulators. It has recently been extended to model the hydrologic performance of specific types of low impact development controls (EPA 2016).

It is noteworthy that the parameters of both models (HEC-HMS and SWMM) were set as those of the calibrated models reported by Mahab Ghods Consultant Engineers (2011a, 2011b). Figure 3 shows the layout of the main urban drainage network of Tehran city in the SWMM-EPA model. The nodes and numbers on the map show the candidate sites for establishing new hydrometric stations, selected according to engineering judgments.

Search algorithm

Another part of the modeling approach is a multi-objective evolutionary algorithm (MOEA) which is coupled with the rainfall-runoff model and entropy criteria to provide a unified framework for design of the monitoring network. The aim of MOEA application in this research is to find optimal sites for establishing new gauges in the urban drainage network considering two or more criteria simultaneously. These criteria can be considered as maximizing the information contents achieved by the new gauges, minimizing the redundancy information or minimizing the costs of new gauge installation. Here, maximizing the joint entropy and minimizing

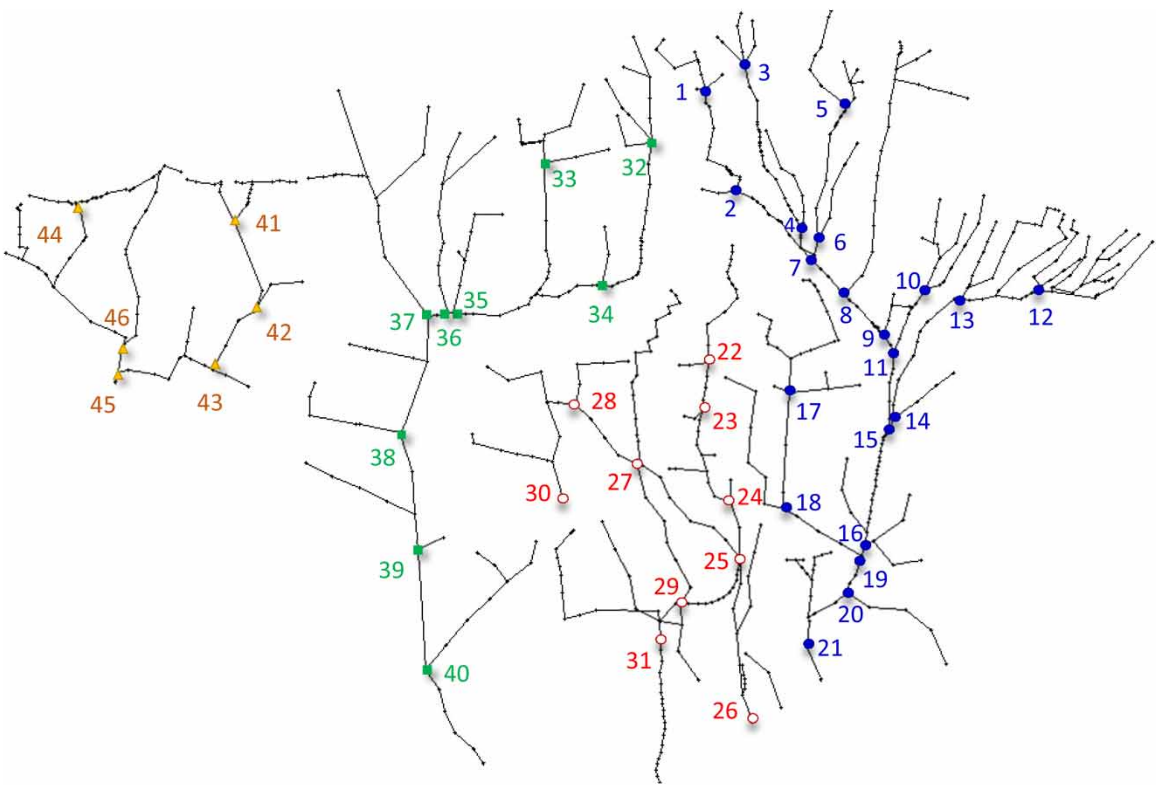

Figure 3 | Main urban drainage network of Tehran city in the SWMM-EPA model.

the total correlation are considered as the objective functions addressing the maximum possible information content and the minimum shared information among stations. The optimization problem can then be formulated as:

$$\begin{aligned} &\min C(X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m) \\ &\max H(X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m) \end{aligned} \tag{7}$$

$$\begin{aligned} &s.t. \\ &m + n = L \end{aligned} \tag{8}$$

where $X_1, X_2, \ldots, X_n$ are decision variables indicating the location of n new hydrometric gauges and $Y_1, Y_2, \ldots, Y_m$ are the geographical location of m existing gauges on the network. Equation (8) shows the total number of gauges subject to a limitation. This limitation can be related to the available funds for the gauge installation or can be considered as recommended values by national standards. To solve the above optimization problem with discrete variables, the widely used NSGA-II algorithm (Deb *et al.* 2002) has been applied. The main steps of this algorithm are briefly described in the appendix (available with the online version of this paper).

RESULTS AND DISCUSSION

In highly developed regions such as urban areas, WMO (2008) suggests that the area covered by a hydrometric gauge can be up to 10 km^2. According to this criterion, Tehran city needs around 30 hydrometric gauges. But, the current state of the monitoring network includes only six active hydrometric gauges inside the city. Therefore, one of the modeling scenarios in this research is deemed as installing 24 new gauges throughout the network. Two other scenarios are also considered in this research, 10 and 15 new gauges, respectively, the results of which might be useful when there is a limitation in available funds for gauge establishment. Therefore, three scenarios are considered in total. It is worth mentioning that the three considered scenarios are not in sequence; i.e. each one is not implemented after the other one. They are three separate scenarios for extending the current monitoring network and, thus, they have their own search space and optimal solutions which are not necessarily subsets of each other.

NSGA-II parameters were set as the best known values in the literature (e.g. see Yazdi 2016), presented in Table A1 (appendix, available with the online version of this paper) and the model was run in an Intel® Core™2 Duo CPU system with 4 GB RAM.

Figure 4 shows the Pareto optimal set of solutions found by NSGA-II for each of the scenarios. Among all the feasible solutions, as shown in the figure, 17, nine and nine solutions have been recognized as the non-dominated solutions for scenarios with 10, 15 and 24 new gauges, respectively. The location of the monitoring network suggested by Mahab Ghods Consultant Engineers (2011a) has also been plotted in the objective function space of Figure 4(c). It can be seen

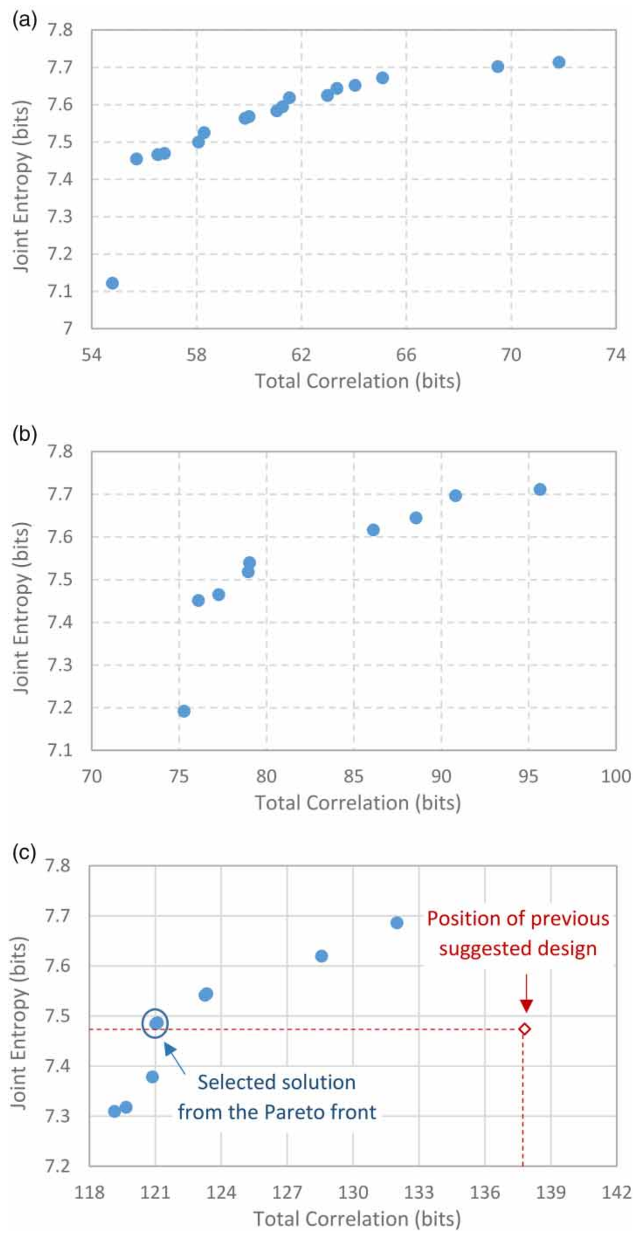

Figure 4 | Pareto front solutions found by NSGA-II, (a) 10 monitoring points, (b) 15 monitoring points, and (c) 24 monitoring points.

that the joint entropy and total correlation of the previously suggested network is far from those of the proposed networks by the optimization algorithm, particularly for the total correlation values where all optimal solutions provide less 'total correlation'; that is, lower shared information or redundancy compared with the previous design.

More specifically, a solution in the Pareto front (shown in Figure 4(c)) with almost the same joint entropy of the previous proposal decreases the overlapping information nearly 13%. Overall, this confirms the high performance of the entropy-based optimization method in finding the effective sites for gauge installation. It also should be mentioned that all solutions above the dashed horizontal line in Figure 4(c) dominate the previously suggested design and are superior to that proposal. Among them, the optimal design with almost the same level of joint entropy with the previously suggested design was selected here for more discussion because it provides the maximum reduction in shared information with respect to the previously suggested design. Generally, each of the solutions above the horizontal line can be selected for implementing on the network according to the criteria such as social aspects and urban aesthetic.

In the first scenario, out of a total of 46 candidate locations, 24 sites have been considered in the optimal solutions and the remaining 22 sites are not selected in any solutions. For the two other scenarios, the number of sites included in the Pareto front were 31 and 45 locations, respectively. The layout of current active gauges and the previous proposal for the monitoring network as well as the selected optimal monitoring network design is shown in Figure 5. Among all suggested sites, site no. 8, 14 and 34 are the same in both the previous and proposed designs, but other suggested places in the two designs are different. As shown in Figure 5, the locations of the new stations of the optimal monitoring network are generally more spatially varied than those of the current network and the previous design of the monitoring network. Moreover, according to the selected optimal monitoring network, the drainage network in the east part of the city (EDC in Figure 5) needs more new hydrometric gauges while the west and central networks (see Figure 5) are less sensitive to gauge

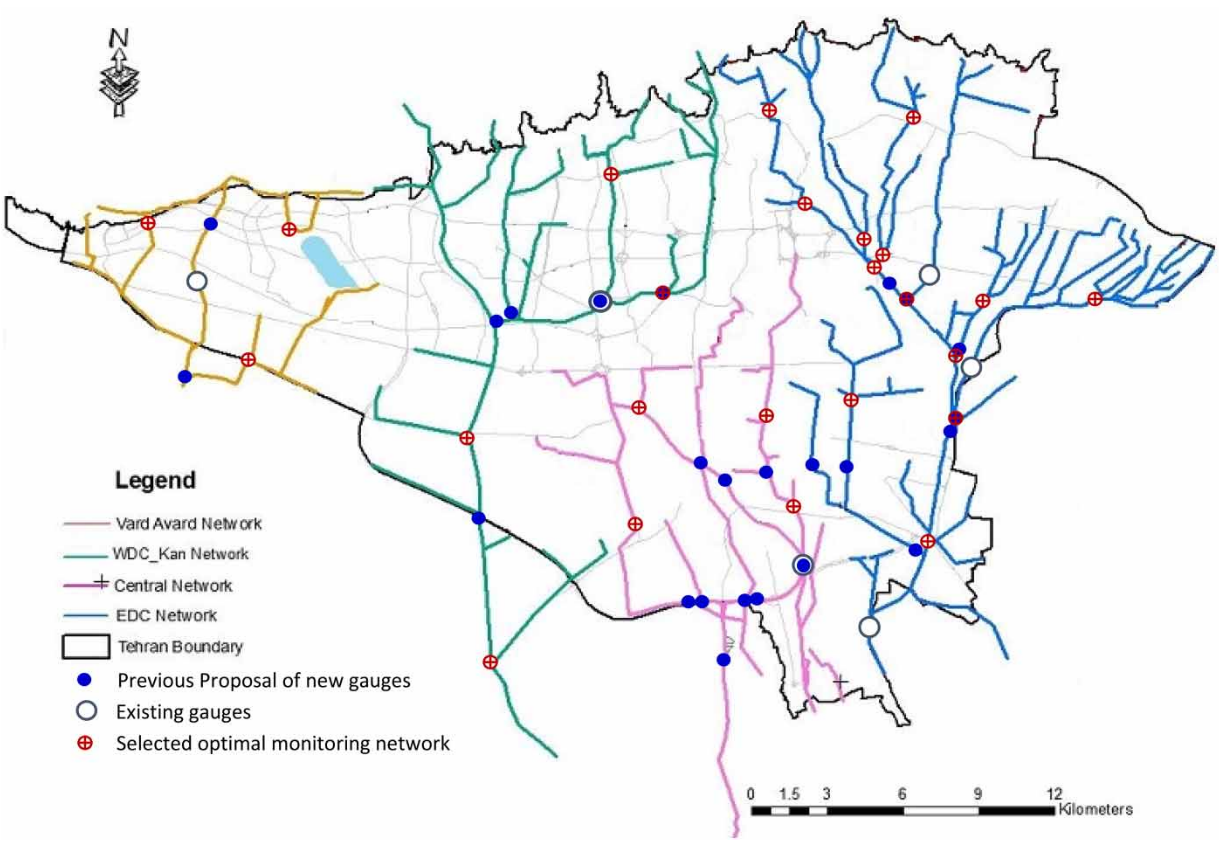

Figure 5 | Layout of monitoring gauges throughout the urban drainage network.

installation. The proposed model has chosen at least three new stations for each of the (sub)networks while the maximum of 13 new stations can be found in EDC. In Figure 5, a cluster of stations is also seen in the left-center part of EDC. The reason is that the main portion of the runoff in EDC comes from the channels in the left-center part and the information content related to the stormwater time series at those canals is relatively high. This increases the entropy of variables related to that part of the network and consequently leads to improvement of the 'joint entropy' as one of the objective functions. Therefore, more stations are suggested for the left-center part of EDC in the selected optimal design. Locating the stations in that area provides a large amount of information related to stormwater flows compared with other parts of EDC.

The locations of new gauges which appear more frequently (dominant stations) in the trade-off for the three considered scenarios were further evaluated in Figure 6. In fact, this figure shows the frequency of candidate sites in the optimal trade-off solutions. It is interesting to know that for the first scenario, out of 24 effective sites (i.e. all sites appear on the Pareto-approximate set of solutions as shown in Figure 4(a)), three sites exist in more than 80% of optimal solutions (site no. 1, 40 and 44). Likewise, among 31 active sites in the second scenario, site no. 1, 2, 6, and 40 appeared with more than 80% frequency and finally, for the third scenario, seven sites out of 45 are found in the trade-off solutions with more than 80% frequencies (site no. 1, 6, 17, 23, 33, 43 and 44). This obviously highlights the effectiveness and the degree of importance of these locations for hydrometric gauge installation. The other sites have appeared in the solutions based on their effectiveness in terms of providing information content with least redundancy. These figures may provide the decision makers with priority of candidate locations for the gauge installation in the network. Indeed, the extent of the monitoring network may be planned for implementation over a period of time longer than a year, so the question is which site is better to start with gauge establishment and then what are the next priorities. Figure 6 can provide guidance on such a problem.

CONCLUSION

This research presented a methodology for evaluating and redesigning the monitoring system of the main surface-water

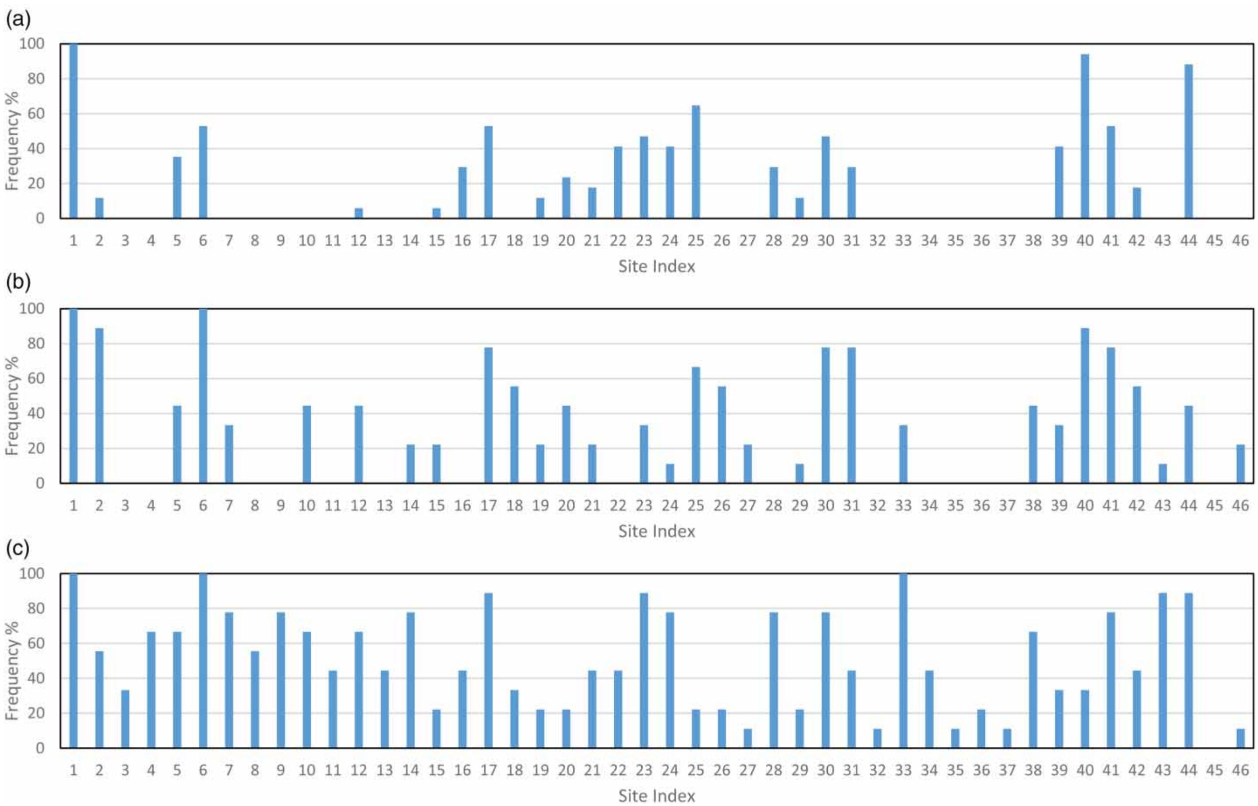

Figure 6 | Frequency of the candidate sites in the Pareto front solutions, in the case of (a) 10 new gauge points, (b) 15 new gauge points, and (c) 24 new gauge points.

drainage network of Tehran city, capital of Iran. Applying the entropy theory, the location of new hydrometric stations were found using the NSGA-II optimization algorithm coupled with the SWMM-EPA model. The obtained optimal monitoring network was compared with the previous design recently proposed by the consultant engineers of the municipality. This comparison showed that the obtained Pareto optimal set outperforms the previous design so that all optimal trade-off solutions provide less total correlation. Particularly, the optimal solution with almost the same level of joint entropy of the previous design yields nearly 13% less total correlation (i.e. shared information by the gauges), which means the proposed model chooses the gauge sites efficiently with minimum redundancy, avoiding waste of investment for installing extra gauges. The relative importance of the locations in the drainage network of Tehran was obtained through frequency analysis of the optimal solutions in the Pareto front. While the sites which appear more frequently at the Pareto front need more priority for gauge installation in the hydrometric network, none of the high frequency solutions was suggested in the previous design. Moreover, it was shown that the east network of the city is more critical for gauge installation than other parts of the drainage network. The final monitoring system design can be selected by decision makers from the Pareto optimal set based on the relative importance of joint entropy and total correlation, social aspects, urban aesthetic, etc.

Based on the results, the proposed approach can be used as an effective tool for evaluating, revising, or redesigning the urban drainage network monitoring systems.

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