Estimation of hydraulic jump characteristics of channels with sudden diverging side walls via SVM

Kiyoumars Roushangar, Reyhaneh Valizadeh and Roghayeh Ghasempour

ABSTRACT

Sudden diverging channels are one of the energy dissipaters which can dissipate most of the kinetic energy of the flow through a hydraulic jump. An accurate prediction of hydraulic jump characteristics is an important step in designing hydraulic structures. This paper focuses on the capability of the support vector machine (SVM) as a meta-model approach for predicting hydraulic jump characteristics in different sudden diverging stilling basins (i.e. basins with and without appurtenances). In this regard, different models were developed and tested using 1,018 experimental data. The obtained results proved the capability of the SVM technique in predicting hydraulic jump characteristics and it was found that the developed models for a channel with a central block performed more successfully than models for channels without appurtenances or with a negative step. The superior performance for the length of hydraulic jump was obtained for the model with parameters F_1 (Froude number) and $(h_2\_h_1)/h_1$ (h_1 and h_2 are sequent depth of upstream and downstream respectively). Concerning the relative energy dissipation and sequent depth ratio, the model with parameters F_1 and h_1/B (B is expansion ratio) led to the best results. According to the outcome of sensitivity analysis, Froude number had the most significant effect on the modeling. Also comparison between SVM and empirical equations indicated the great performance of the SVM.

Key words | central block, empirical equations, hydraulic jump, stilling basin, sudden diverging side walls, SVM

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INTRODUCTION

The purpose of the design of energy dissipators is to dissipate part of the kinetic energy of the inflowing flow in order to return safely the flow to the downstream channel or river and prevent scour below overflow spillways, chutes and sluices. Based on the energy dissipating action of hydraulic jumps, stilling basins are one of the possible solutions which may be adopted. The performance or efficiency of any stilling basin is usually assessed in terms of characteristics of the jump allocated (Negm 2000). The length of the hydraulic jump is mostly taken as a design parameter or as an indicator of the length of the paved downstream section, i.e. the stilling basin. The hydraulic jump is a natural phenomenon that occurs when supercritical flow is forced to change to subcritical flow by an obstruction to the flow. Depending on the geometry of the channel and tailwater conditions, the hydraulic jump can assume several distinct forms. The hydraulic jump is a useful means of dissipating the excess energy of supercritical flow so that scour in the downstream is minimized. It has also been used to raise the water level on the downstream to provide the requisite head for diversion into canals. However modeling hydraulic jump characteristics has great importance since it plays an important role in designing hydraulic structures. So far, various studies have been carried out to explain the complex phenomenon of the hydraulic jump and to estimate its characteristics. Hughes & Flack (1983) studied the effect of various roughness designs on the hydraulic jump characteristics in a stilling basin. Hager & Bremen (1989) investigated the influence of wall friction on the sequent depths ratio. Bhutto *et al.* (1989) developed analytical solutions for computing sequent depth and relative energy loss for free hydraulic jump in sloping and horizontal rectangular channels. Finnemore & Franzini (2002) stated that the characteristics of the hydraulic jump depend on the Froude number. Negm (2000) studied the hydraulic performance of rectangular and radial stilling basins, where the latter stand for the diverging channels.

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Ayanlar (2004) studied the effect of corrugated beds on hydraulic jump properties with altering flow characteristics. Bilgin (2005) studied the correlation and distribution of shear stress for turbulent flow in a smooth rectangular basin. However, due to the complexity and uncertainty of the hydraulic jump phenomenon, the results of the classical models are not general and do not show the same results under variable conditions. Therefore, it is essential to use methods which are capable of predicting hydraulic characteristics within the basins.

In recent years the artificial intelligence tools (e.g. artificial neural networks, neuro-fuzzy models, genetic programming and support vector machine (SVM)) have been used for the assessment of the accuracy of hydraulic and hydrologic complex phenomena, such as prediction of total bedload (Chang *et al.* 2012), prediction of suspended sediment concentration (Kisi & Shiri 2012), modeling the rainfall-runoff process (Nourani *et al.* 2011; Kisi *et al.* 2013), precipitation forecasting (Kisi & Shiri 2011), predicting of total bed material load (Roushangar *et al.* 2014a), developing stage-discharge curves (Azamathulla *et al.* 2011) and predicting of energy dissipation over spillways (Roushangar *et al.* 2014b).

Among others, the SVM technique has been applied in modeling various components of water resources systems. Khan & Coulibaly (2006) and Lan (2014) applied SVM for long-term prediction of lake water levels. Sivapragasam & Muttil (2005) suggested the use of SVM in the extrapolation of rating curves. Roushangar & Koosheh (2015) evaluated the genetic algorithm–support vector regression (GA-SVR) method for modeling bedload transport in gravel-bed rivers. Kisi (2012) applied least square SVM for modeling the discharge–suspended sediment relationship.

In the present study the capability of the SVM approach was assessed for modeling hydraulic jump characteristics in three different sudden diverging stilling basins. Various input combinations based on hydraulic characteristics and geometry of the appurtenances were used in order to find the most appropriate combination for modeling hydraulic jump characteristics. Finally, the accuracy of the best SVM model was compared with the existing classic approaches.

MATERIALS AND METHODS

The data sets

In this study, the experimental data presented by Bremen (1990) and Gandhi (2014) were employed for prediction goals. The experiments of Bremen (1990) were carried out at the Laboratoire de Constructions Hydrauliques of the Ecole Polytechnique Federale de Lausanne and were intended for sudden diverging basins without appurtenances, with a central sill and with a negative step (Figure 1). During experiments the 17 m^3 upstream basin was supplied by two conduits of 0.30 m diameter (maximum total discharge 375 L/s). A 0.5 m wide and 10.8 m long prismatic rectangular and horizontal channel was connected to the basin. At the upstream channel extremity a standard-shaped 0.50 m wide and 0.70 m high spillway of design head $H_0 = 0.2$ m controlled the channel inflow. According to Figure 2 two states of asyrnmetric and symmetric abrupt diverging were considered in experiments.

The experiments of Gandhi (2014) were carried out at a rectangular channel made up of Perspex sheets and the setup consisted of a constant head tank with volume of $3.6 \times 3.6 \times 3$ m^3. A symmetric-shaped channel, sized $2.1 \times 0.445 \times 1.2$ m and flow Froude number ($2 < F_1 < 9$) and the expansion ratio ($0.4 < B < 0.8$), was used for experiments. The expansion ratio ($B = b_1/b_2$) shows the expansion geometry in which b_1 and b_2 are approach channel width and expanded channel width, respectively. The ranges of experimental data used in experiments are given in Table 1.

Support vector machine

The foundations of SVMs were developed by Vapnik (1995). The SVM is essentially used in information categorization and data set classification. This approach is known as structural risk minimization, which minimizes an upper bound on the expected risk, as opposed to the traditional empirical

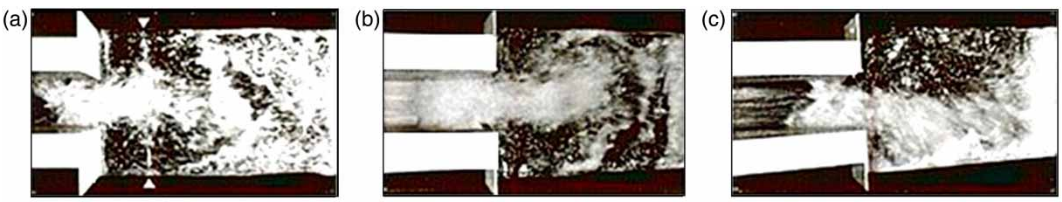

Figure 1 | Plane view of hydraulic jump in sudden diverging basin; (a) normal, (b) with negative step and (c) with central sill (Bremen (1990), reproduced with kind permission of the author).

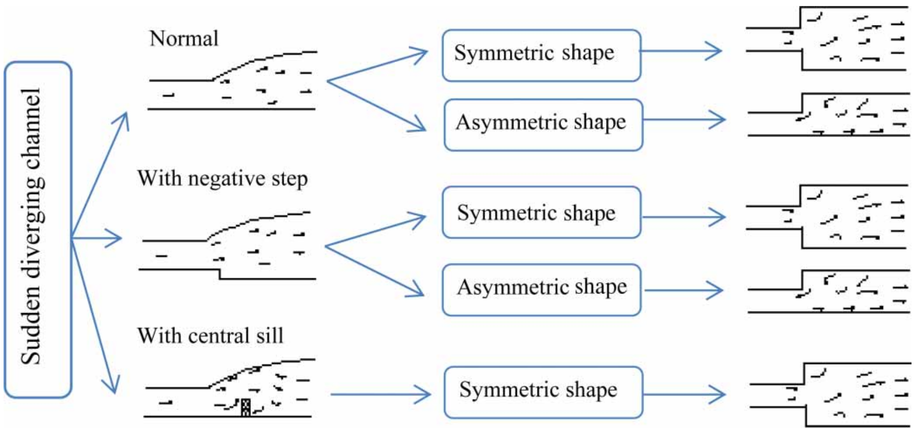

Figure 2 | Schematic view of the experiments based on Bremen (1990).

Table 1 | The range of experimental data

Researcher	Series	F_1	B (b_2/b_1)	Q (l/s)	Y (h_2/h_1)	Xs (m) from	Xs (m) to	S/h_1 from	S/h_1 to	Channel shape	No. of data
Normal											
Bremen (1990)	1	2.63–8.12	5	2–18.1	2.02–11.26	–	–	–	–	Asym	162
	2	2.63–8.05	3	5.1–26.9	2.18–10.28	–	–	–	–	Sym	178
	3	2.65–8.13	1.5	10–40	3.46–12.05	–	–	–	–	Asym	91
Gandhi (2014)	1	2–9	0.4	6.44–15	2–8	–	–	–	–	Sym	8
	2	2–9	0.5	12.3–12.8	2–9	–	–	–	–	Sym	8
	3	2–9	0.6	6.44–10.6	2–9.5	–	–	–	–	Sym	8
	4	2–9	0.8	6.3–9.8	2–10	–	–	–	–	Sym	8
Negative step											
Bremen (1990)	1	3.2–8.43	1.5	11–39.5	4.15–12.59	–	–	0.61	1.64	Asym	105
	2	2.68–8.12	2	11–39.5	4.15–12.59	–	–	0.59	1.61	Asym	112
	3	2.05–7.37	3	4.3–24	1.45–11.05	–	–	0.49	1.66	Sym	129
Central sill											
Bremen (1990)	1	3,5,7,9	3	31–225	2.89–8.81	0.2	0.8	0.6	3	Sym	78
	2	3,5,7,9	2	31–225	3.07–9.88	0.2	0.8	0.6	3	Sym	75
	3	3,5,7,9	1.5	31–225	3.15–10.63	0.2	0.8	0.6	3	Sym	60

risk which minimizes the error on the training data. It is this difference that equips SVM with a greater ability to generalize. The main goal in statistical learning is ability to generalize (Gunn 1998). The SVM method is based on the concept of optimal hyper plane that separates samples of two classes by considering the widest gap between two classes (see Figure 3). SVR is an extension of SVM regression. The aim of SVR is to characterize a kind of function that has at most ε deviation from the actually obtained objectives for all training data y_i and at the same time it would be as flat as possible. SVR formulation is as follows:

$$f(x) = w\varphi(x) + b \quad (1)$$

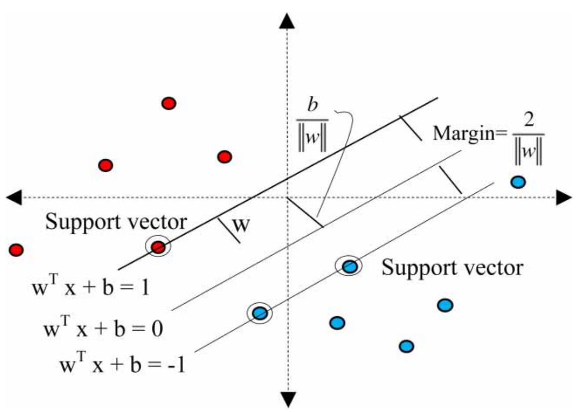

Figure 3 | Data classification and support vectors.

where $\varphi(x)$ denotes a nonlinear function in feature of input x, b is called the bias and the vector w is known as the weight factor and expressed as Equation (2) in which α_i is Lagrange multiplier, y_i is forecasted value and x_i is input value.

$$w = \sum_{i=1}^{n} \alpha_i y_i x_i \tag{2}$$

The coefficients of Equation (1) are predicted by minimizing the regularized risk function as expressed below:

$$R_{\min} = C\frac{1}{N}\sum_{i=1}^{n} L_\varepsilon(t_i, y_i) + \frac{1}{2}\|w\|^2 \tag{3}$$

where

$$L_\varepsilon(t_i, y_i) = \begin{cases} 0 & |t_i, y_i| \le \varepsilon \\ |t_i, y_i| - \varepsilon & \text{Otherwise} \end{cases} \tag{4}$$

The constant C is the cost factor and represents the trade-off between the weight factor and approximation error. ε is the radius of the tube within which the regression function must lie. The $L_\varepsilon(t_i, y_i)$ represents the loss function in which y_i is forecasted value and t_i is desired value in period i. The $\|w\|$ is the norm of w vector and the term $\|w\|^2$ can be expressed in the form $w^T \cdot w$ in which w^T is the transpose form of w vector. According to Equation (4), if the forecasted value is out of ε -tube then the loss will be the absolute value, which is the difference between forecasted value and ε. Since some data may not lie inside the ε-tube, the slack variables (ξ, ξ^*) must be introduced. These variables represent the distance from actual values to the corresponding boundary values of ε-tube. Therefore, it is possible to transform Equation (3) into:

$$R_{\min} = C\sum_{i=1}^{n} \left(\xi_i, \xi_i^*\right) + \frac{1}{2}\|w\|^2 \tag{5}$$

subject to: $t_i - w_i\varphi(x_i) - b \le \varepsilon + \xi_i, w_i\varphi(x_i) + b - t_i \le \xi_i^*, \xi_i + \xi_i^* \ge 0$

Using Lagrangian multipliers in Equation (5) thus yields the dual Lagrangian form:

$$Maxl(\alpha_i, \alpha_i^*) = -\varepsilon\sum_{i=1}^{n}\left(\alpha_i + \alpha_i^*\right) + t_i\sum_{i=1}^{n}\left(\alpha_i - \alpha_i^*\right) - \frac{1}{2} \times \sum_{i=1}^{n} \times \sum_{j=1}^{n}\left(\alpha_i - \alpha_i^*\right) - \left(\alpha_j - \alpha_j^*\right) - K(x_i, x_j) \tag{6}$$

subject to: $\sum_{i=1}^{n}\left(\alpha_i - \alpha_i^*\right) = 0, 0 \le \alpha_i, \alpha_i^* \le C, \mathrm{i} = 1, 2, \ldots \mathrm{N}$

where α_i and α_i^* are Lagrange multipliers and $l(\alpha_i, \alpha_i^*)$ represents the Lagrange function. $K(x_i, x_j)$ is a kernel function to yield the inner products in the feature space $\varphi(x_i)$ and $\varphi(x_j)$ and expressed as the following:

$$K\left(x_i, x_j\right) = \varphi(x_i) \times \varphi\left(x_j\right) \tag{7}$$

The appropriate selection of kernel type is the most important step in the SVM due to its direct impact on the training and classification precision. In general, there are several types of kernel function, namely linear, polynomial, radial basis function (RBF) and sigmoid functions.

Classical models

So far, a variety of formulas have been developed to predict hydraulic jump properties, ranging from simple regressions to complex multi-parameter formulations. There are different concepts and approaches that are used in the derivation and extraction process of these formulas (i.e. different assumptions, statistical correlations, and experimental information). In the current study the applicability of several existing equations for hydraulic characteristics was assessed in order to indicate the capability of the applied approach in this study. The used equations are given in Table 2.

Performance criteria

In the current study, the model's performance was assessed using three statistical criteria, namely, correlation coefficient (R), determination coefficient (DC), and root mean square error ($RMSE$), expressions for which are as follows:

$$DC = 1 - \frac{\sum_{i=1}^{N}\left(l_o - l_p\right)^2}{\sum_{i=1}^{N}\left(l_o - \overline{l_p}\right)^2},$$

$$R = \frac{\sum_{i=1}^{N}\left(l_o - \overline{l_o}\right) \times \left(l_p - \overline{l_p}\right)}{\sqrt{\sum_{i=1}^{N}\left(l_o - \overline{l_o}\right)^2 \times \left(l_p - \overline{l_p}\right)^2}}, \tag{16}$$

$$RMSE = \sqrt{\sum_{i=1}^{N}\frac{\left(l_o - l_p\right)^2}{\mathrm{N}}}$$

where l_o, l_p, $\overline{l_o}$, $\overline{l_p}$, and N respectively represent the measured values, predicted values, mean measured

Table 2 | Utilized equations in the study

Parameter	Researcher	Equation	Consideration	Equation number
Sequent depth ratio	Kusnetzow (1958)	$Y = \frac{0.5}{B} K_K\left[\sqrt{1+8BFr_1^2}-1\right]$	$K_K = 0.8 - \left(0.9 - \frac{1}{B}\right)\times 0.15$	(8)
	Hager (1985)	$\frac{Y^*-Y}{Y^*-1} = \left(1-\frac{1}{\sqrt{B}}\right)\times[1-th(1.9X_1)]$	$Y^* = 0.5\left[\sqrt{1+8Fr_1^2}-1\right]$	(9)
	Herbrand (1973)	$Y = Fr_1\sqrt{\frac{2}{B}-\frac{1}{2B}}$	$Y = \frac{h_2}{h_1}$	(10)
Hydraulic jump length	Smetana (1934)	$Lj = 6\times Hj$	$Hj = h_1 - h_2$ For free jump in horizontal smooth bed	(11)
	Safranez (1929)	$Lj = 6h_1\times Fr_1$	For free jump in horizontal smooth bed	(12)
	Bakhmeteff & Matezke (1936)	$Lj = 5\times Hj$	$2.5 < F_1 < 4.5$ For free jump in horizontal smooth bed	(13)
	Hager (1985)	$Lj = \left\{1+\left(1-\frac{1}{\sqrt{B}}\right)\times[1-th(1.9X_1)]\right\}Lj^*$	$Lj^* = h_1\times 220\times th\left(\frac{Fr_1-1}{22}\right)$ $2.5 < F_1 < 12,\ 1 < B < 10,\ X_1 > 0.1$	(14)
	Silvester (1964)	$Lj = 6.02\times Hj$	For free jump in horizontal smooth bed	(15)

values, mean predicted values and number of data samples. The *RMSE* describes the average difference between predicted values and measured values, *R* provides information for the linear dependence between observation and corresponding predicted values and *DC* is the coefficient used to determine the relative assessment of the model performance in dimensionless measures. The smaller the *RMSE* and the greater the *DC* and *R*, the higher the accuracy of the model will be. It should be noted that predicting the intended parameter using non-normalized data may lead to undesirable results; therefore, in this study all input variables were scaled to fall in the range 0.1–1 to eliminate the influence of the variables with different absolute magnitudes. This will bring about the uniformity of the data values for the network and increase the training speed and the capability of the model (Dawson & Wilby 1998). The following equation is used to normalize the data.

$$n_n = 0.1 + 0.9\times\left(\frac{n-n_{\min}}{n_{\max}-n_{\min}}\right) \tag{17}$$

which n_n, n, $n_{\max}$, and $n_{\min}$ respectively are the normalized value, the original value, and the maximum and minimum of variable n. The normalization used only rescales the data to the range of 0.1 and 1, which after modeling can be de-normalized to real values.

Simulation and model development

Input variables

In a data-driven model, selection of appropriate parameters as input variables is a crucial step during the modeling process. In Figure 4 the quantities measured for jumps in sudden expansion basins with different appurtenances are shown. The important variables which affect the jump pattern and energy dissipation can be expressed as a function (Rajaratnam & Subramanya 1968; Hager & Bremen 1989; Gandhi 2014):

$$f(h_1,\ h_2,\ V_1,\ L_j,\ E_L,\ \mu,\ g,\ \rho,\ b_1,\ b_2,\ s) = 0 \tag{18}$$

where h_1 and h_2: sequent depth of upstream and downstream, V_1: upstream flow velocity, μ: dynamic viscosity of water, g: acceleration due to gravity, L_j: length of jump, ρ: density of water, b_1 and b_2: approach channel, and expanded channel width, E_L: E_1–E_2 in which E_1 and E_2 are energy per unit weight before and after the jump and s: height of the step or sill.

From dimensional analysis and considering h_1, g and μ as repeating variables, these parameters Equation (18) can be expressed as the following:

$$f\left(\frac{h_2}{h_1},\ \frac{E_L}{E_1},\ \frac{L_j}{h_1},\ \frac{b_1}{h_1},\ \frac{b_2}{h_1},\ \frac{v_1^2}{gh_1},\ \frac{\rho v_1 h_1}{\mu},\ \frac{s}{h_1}\right) = 0 \tag{19}$$

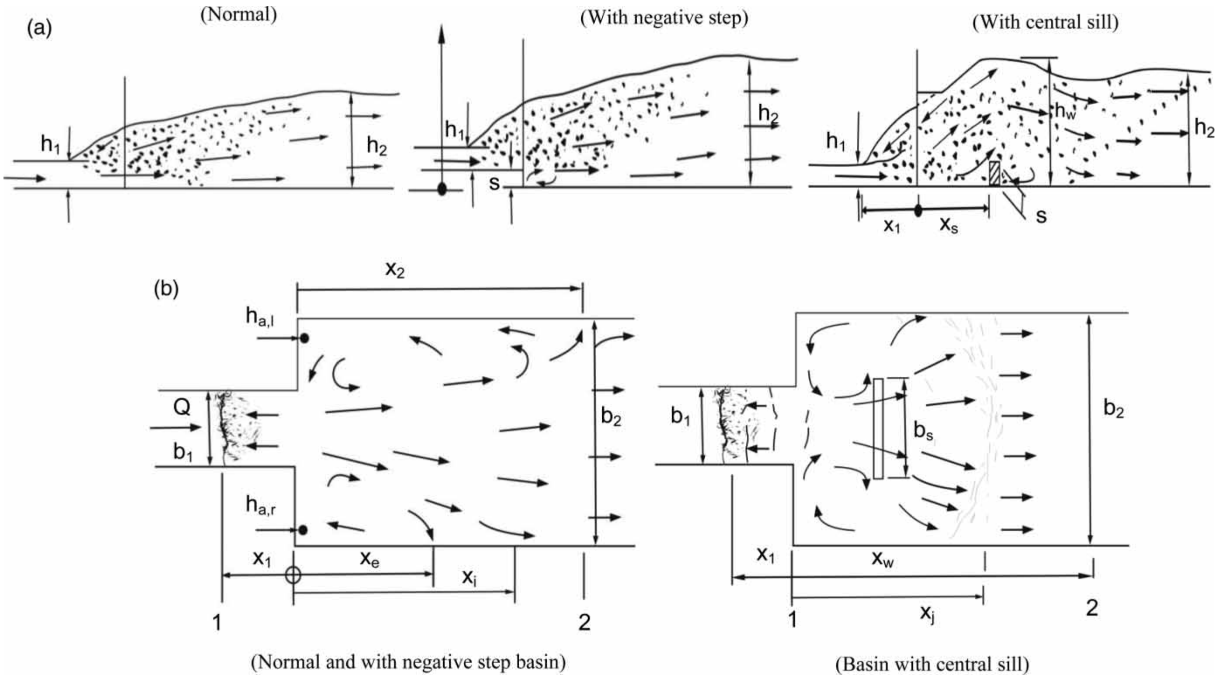

Figure 4 | Quantities measured and notation used for jumps in sudden expanding channel; (a) axial view, (b) plan view (Bremen (1990), reproduced with kind permission of the author).

Equation (19) can be expressed as:

$$f\left(\frac{h_2}{h_1}, \frac{E_L}{E_1}, \frac{L_j}{h_1}, B, F_1, R, \frac{s}{h_1}\right) = 0 \quad (20)$$

where F_1: flow Froude number, R: flow Reynolds number, $B = (b_1/b_2)$: ratio of expansion.

Experimental studies by Elevatorski (2008), Wu & Rajaratnam (1996) and Ranga Raju *et al.* (1980) revealed that hydraulic jump characteristics only depend on Froude number, and Reynolds number has no significant impact on the prediction of hydraulic jump characteristics. Also, Hager (1992) showed that the length of hydraulic jump depends on the height of jump and Froude number. Therefore, in this study, based on upstream hydraulic data and geometric data, the models of Table 3 were considered for modeling hydraulic jump characteristics of sudden diverging basins.

SVM model development

For determining the best performance of SVM and selecting the best kernel function, different models were predicted via SVM using various kernels. Table 4 shows the results of statistical parameters of different kernels for model *S(II)* of a

Table 3 | SVM developed models

Output variable	Model	Input variable(s)
Sequent depth ratio h_2/h_1	S(I)	F_1
	S(II)	F_1, h_1/B
	S(III)	F_1, S/h_1
	S(IV)	F_1, B
Length of hydraulic jump L_j/h_1	L(I)	F_1
	L(II)	F_1, $(h_2-h_1)/h_1$
	L(III)	F_1, h_2/h_1
	L(IV)	F_1, S/h_1
	L(V)	F_1, B
Loss of energy E_L/E_1	E(I)	F_1
	E(II)	F_1, h_1/B
	E(III)	F_1, S/h_1

Table 4 | The statistical parameters of SVM method with different kernel functions; model S(II)

	Train			Test		
Kernel function	*R*	*DC*	*RMSE*	*R*	*DC*	*RMSE*
Linear	0.885	0.818	0.138	0.882	0.722	0.149
Polynomial	0.977	0.899	0.118	0.968	0.748	0.123
RBF	**0.994**	**0.987**	**0.026**	**0.993**	**0.985**	**0.029**
Sigmoid	0.574	0.108	0.203	0.343	0.095	0.273

Bold values correspond to the superior kernel type.

basin with central sill. The results of Table 4 revealed that using the model with the kernel function of RBF led to better prediction accuracy in comparison to the other kernels. According to Noori *et al.* (2011) the SVM model via RBF kernel is very desirable for use in prediction of hydraulic and hydrological features since: (a) unlike the linear kernel, the RBF kernel can handle the case when the relation between class labels and attributes is nonlinear, (b) the RBF kernels tend to give better performance under general smoothness assumptions, (c) it has fewer tuning parameters than the polynomial and the sigmoid kernels. Therefore, RBF kernel was selected as the core tool of SVM and was applied for the rest of the models. Implementation of SVM requires the selection of three parameters, which are constant C, ε and kernel parameter γ, where γ is a constant parameter of the RBF kernel. The coefficient C is a positive constant that influences a trade-off between an approximation error and the regression and must be selected by the user. ε has an effect on the smoothness of the SVM's response and it affects the number of support vectors, so both the complexity and the generalization capability of the network depend on its value. If epsilon is larger than the range of the target values we cannot expect a good result. If epsilon is zero, we can expect overfitting (Smola 1996). The variable parameter used with kernel function (γ) considerably affects the flexibility of function. These parameters should be selected by the user. The selection of appropriate values for the three parameters (C, ε, γ) has been proposed by various researchers. In the current study, according to Cherkassky & Yunqian (2002), optimization of these parameters has been done by a systematic grid search of the parameters using cross-validation on the training set. A normal range of parameter settings are investigated in this grid search. First, optimized values of C and ε for a specified γ were obtained and then γ was changed. Statistical parameters were used to find optimums. The statistics parameters via γ values to find SVM optimums of the testing set for the model *S(II)* of the basin with central sill are shown in Figure 5. In the same way, optimal parameters were obtained for all models.

RESULTS AND DISCUSSION

The sequent depth ratio

For evaluating hydraulic jump characteristics in sudden expanding channels with and without appurtenances, several models, according to the flow condition and geometry of the applied appurtenances, were developed. All of the SVM models were trained and tested to carry out the sequent depth ratio prediction in sudden diverging basins. The results of SVM models are shown in Table 5 and Figure 6. From the obtained results of statistical parameters (*RMSE*, *R* and *DC*) it can be stated that between the three types of basins, developed models for the case of the basin with a central sill in modeling of sequent depth ratio performed more successfully than the two other cases. In this state, the model *S(II)* with input parameters of F_1 and h_1/B led to more accurate outcome than the other models. This model also presented higher accuracy for basins without appurtenances and with negative step. A comparison between the results of the models *S(I)* and *S(III)* showed that for predicting the sequent depth ratio in diverging basins with appurtenances, using parameter S/h_1 as input parameter improved the efficiency of the model. This parameter confirms the importance of the relative height of applied step and sill in sequent depth ratio estimating process in channels with appurtenances. Considering the results of the models *S(I)*, *S(II)* and *S(III)*, it could be

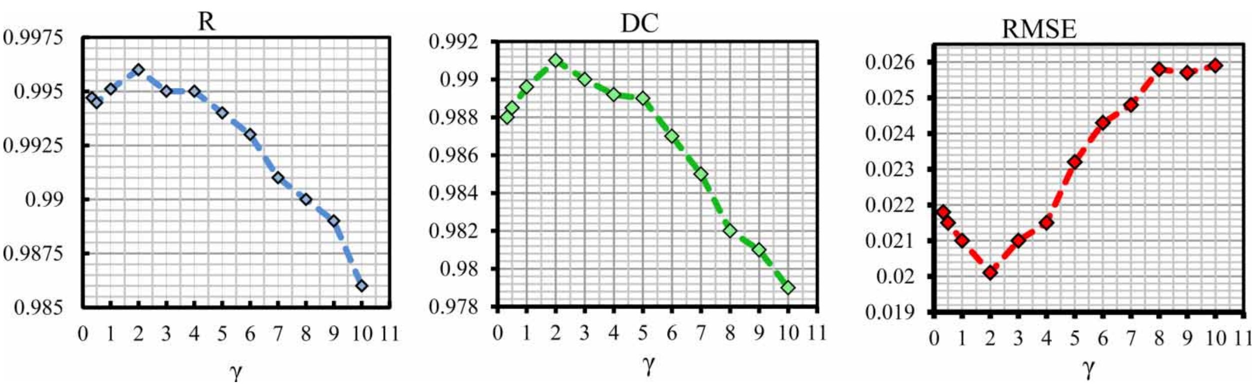

Figure 5 | Statistics parameters via γ values to find SVM optimums of the testing set for model *S(II)* of channel with central sill.

Table 5 | Statistical parameters of the SVM models for sequent depth ratio

		Optimal parameters			Performance criteria					
					Train			Test		
Condition	SVM models	c	ε	γ	R	DC	RMSE	R	DC	RMSE
Basin without appurtenances										
Sym channel	*S(I)*	5.0	0.10	5	0.958	0.916	0.044	0.955	0.911	0.043
	S(II)	**6.0**	**0.10**	**4**	**0.977**	**0.952**	**0.034**	**0.945**	**0.882**	**0.049**
Asym channel	*S(I)*	5.0	0.10	5	0.880	0.771	0.068	0.874	0.754	0.097
	S(II)	**5.0**	**0.10**	**5**	**0.959**	**0.918**	**0.041**	**0.933**	**0.854**	**0.075**
Basin with negative step										
Sym channel	*S(I)*	8.0	0.02	4	0.900	0.805	0.063	0.893	0.744	0.075
	S(II)	**10**	**0.10**	**5**	**0.993**	**0.986**	**0.018**	**0.959**	**0.911**	**0.045**
	S(III)	10	0.10	8	0.947	0.896	0.048	0.931	0.864	0.054
Asym channel	*S(I)*	10	0.01	5	0.963	0.922	0.052	0.962	0.908	0.056
	S(II)	**8.0**	**0.10**	**4**	**0.993**	**0.985**	**0.029**	**0.989**	**0.977**	**0.028**
	S(III)	8.0	0.02	4	0.981	0.961	0.036	0.980	0.959	0.037
	S(IV)	10	0.01	5	0.960	0.920	0.054	0.951	0.910	0.056
Basin with central sill										
Sym channel	*S(I)*	4.0	0.01	2	0.984	0.967	0.039	0.975	0.944	0.057
	S(II)	**5.0**	**0.01**	**2**	**0.994**	**0.987**	**0.026**	**0.993**	**0.985**	**0.029**
	S(III)	5.0	0.01	4	0.985	0.967	0.039	0.979	0.957	0.048
	S(IV)	2.0	0.01	2	0.983	0.965	0.041	0.975	0.940	0.058

Bold values correspond to the superior model for each condition.

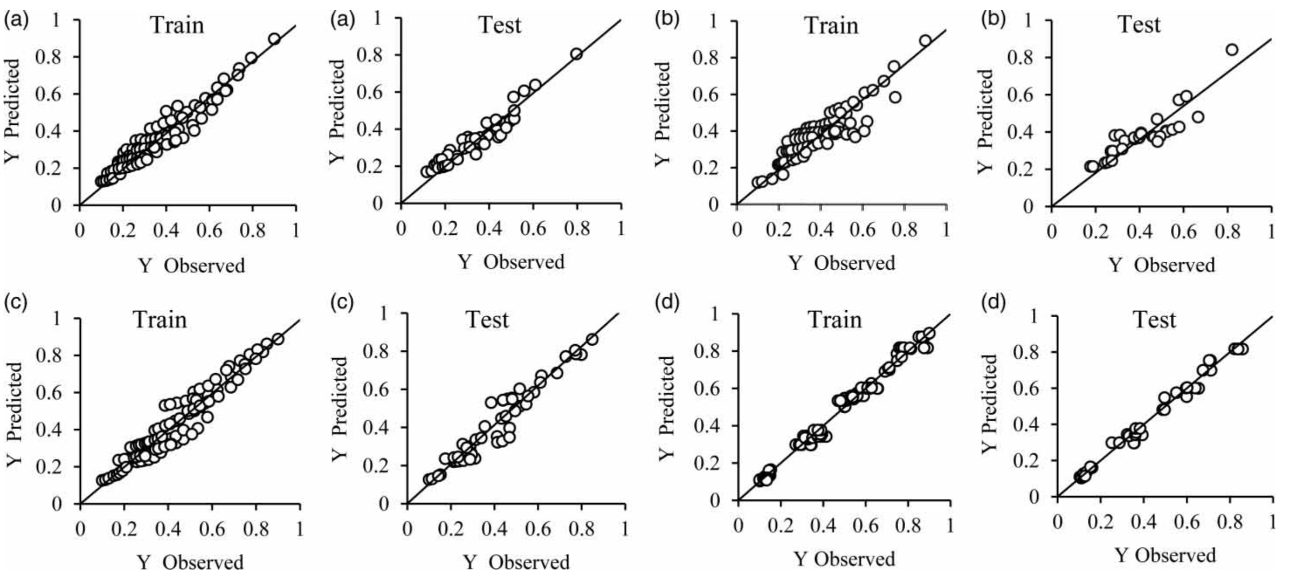

Figure 6 | Comparison of observed and predicted sequent depth ratio for superior model; (a) symmetric basin without appurtenances, (b) symmetric basin with negative step, (c) asymmetric basin with negative step, (d) symmetric basin with central sill.

inferred that the impact of parameter h_1/B in increasing the accuracy of the model is more than that of parameter S/h_1. Also, the model *S(I)* with only input parameter F_1 showed desired accuracy. It could be stated that the applied method can successfully predict the sequence depth ratio using only the upstream flow characteristic as input data. Based on the results of Table 5, for basin with negative step, the models of the channel with asymmetric shape presented better results than symmetric basin, while for basin without appurtenances, the developed models for symmetric channel showed more accuracy. Figure 6 shows the verification between measured and estimated values for the best proposed model for all states.

Length of hydraulic jump

The obtained results from SVM models for predicting the length of hydraulic jump in three different sudden diverging stilling basins are indicated in Table 6 and Figure 7. The superior performance was obtained for the model *L(II)* in which the basin with a central sill was used and input parameters were F_1, $(h_2 - h_1)/h_1$. Based on the results of the symmetric basin with negative step demonstrated in Table 6, the model *L(IV)* with input parameters F_1, S/h_1 was more accurate than the other models, while for the case of the asymmetric basin the model *L(II)* was superior. According to the obtained results from the models *L(I)*, *L(III)* and *L(IV)*, it could be inferred that for modeling the length of hydraulic jump in diverging basins, adding parameters h_2/h_1 and S/h_1 as an input parameter caused an increment in model efficiency. However, the results of the model *L(II)*, as the superior model, indicated that parameter $(h_2 - h_1)/h_1$ was more effective than parameters h_2/h_1 and S/h_1 in improving the model accuracy. It can be stated that the developed models for a basin with central sill, in predicting of hydraulic jump length, performed more successfully than the two other cases. The scatter plots of observed and predicted relative length of hydraulic jump for the superior model of each case are shown in Figure 7.

Energy dissipation

The results of SVM models for relative energy dissipation ratio are given in Table 7. According to Table 7 it can be seen that for the three cases of basins, without appurtenances, with negative step, and with central sill, the model *E(II)* with input parameters F_1 and h_1/B represented higher performance in comparison with the other models. The results of developed models for the basin with negative step and central sill revealed that using relative height of sill or step as input variables caused an increment in model efficiency. Comparison between the obtained results from the developed models for basins without appurtenances and with negative step indicated that for symmetric channels, the basin without any appurtenances presented higher accuracy. For

Table 6 | Statistical parameters of the SVM models for relative length of hydraulic jump

					Performance criteria					
		Optimal parameters			Train			Test		
Condition	SVM models	c	ε	γ	R	DC	RMSE	R	DC	RMSE
Basin without appurtenances										
Sym channel	*L(I)*	10	0.100	4	0.903	0.812	0.072	0.855	0.720	0.090
	L(II)	**10**	**0.010**	**5**	**0.908**	**0.819**	**0.069**	**0.856**	**0.722**	**0.088**
	L(III)	10	0.010	5	0.907	0.818	0.071	0.855	0.721	0.089
Basin with negative step										
Sym channel	*L(I)*	10	0.001	3	0.895	0.798	0.079	0.884	0.763	0.091
	L(II)	10	0.001	5	0.929	0.857	0.069	0.912	0.827	0.077
	L(III)	8.0	0.001	3	0.925	0.850	0.070	0.910	0.827	0.077
	L(IV)	**10**	**0.100**	**6**	**0.935**	**0.877**	**0.065**	**0.930**	**0.848**	**0.065**
Asym channel	*L(I)*	8.0	0.100	6	0.930	0.861	0.057	0.920	0.845	0.066
	L(II)	**10**	**0.100**	**4**	**0.956**	**0.912**	**0.048**	**0.921**	**0.846**	**0.061**
	L(III)	8.0	0.001	3	0.922	0.849	0.065	0.892	0.820	0.078
	L(IV)	10	0.001	3	0.930	0.859	0.058	0.920	0.845	0.065
	L(V)	8.0	0.100	6	0.907	0.820	0.067	0.857	0.734	0.091
Basin with central sill										
Sym channel	*L(I)*	10	0.100	3	0.935	0.872	0.055	0.898	0.804	0.083
	L(II)	**10**	**0.001**	**2**	**0.958**	**0.915**	**0.035**	**0.930**	**0.860**	**0.048**
	L(III)	10	0.100	3	0.936	0.879	0.052	0.921	0.833	0.073
	L(IV)	10	0.001	1	0.932	0.867	0.056	0.923	0.848	0.064
	L(V)	10	0.100	3	0.921	0.840	0.061	0.882	0.763	0.089

Bold values correspond to the superior model for each condition.

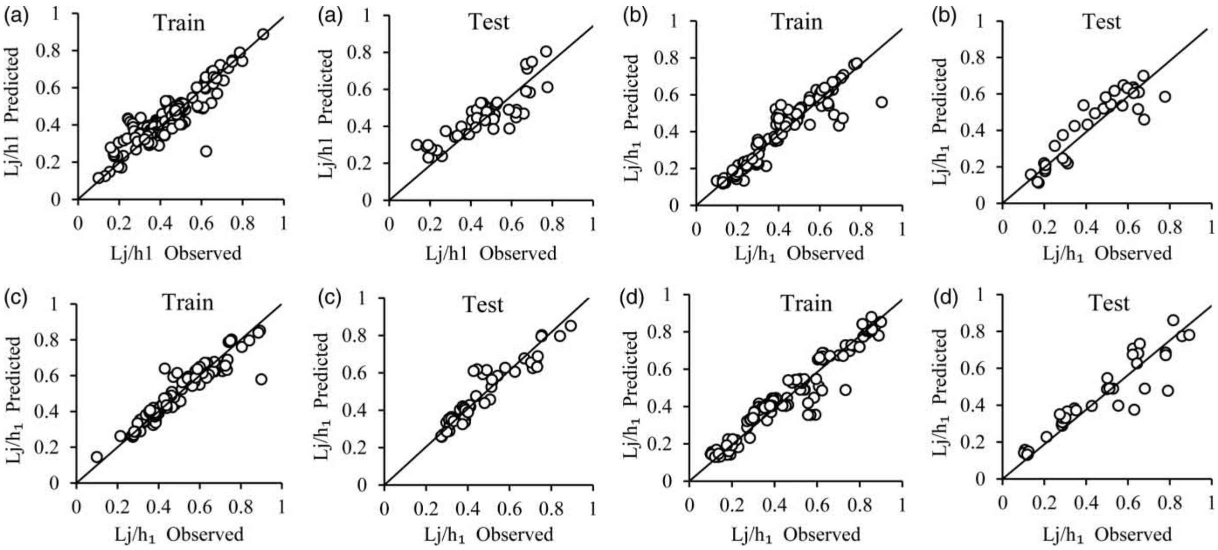

Figure 7 | Comparison of observed and predicted relative length of hydraulic jump for superior model; (a) symmetric basin without appurtenances, (b) symmetric basin with negative step, (c) asymmetric basin with negative step, (d) symmetric basin with central sill.

Table 7 | Statistical parameters of the SVM models for relative energy dissipation

		Optimal parameters			Performance criteria					
					Train			Test		
Condition	SVM models	c	ε	γ	R	DC	RMSE	R	DC	RMSE
Basin without appurtenances										
Sym channel	*E(I)*	8.0	0.120	5	0.957	0.909	0.046	0.950	0.898	0.047
	E(II)	**10**	**0.100**	**4**	**0.984**	**0.982**	**0.030**	**0.989**	**0.978**	**0.034**
Asym channel	*E(I)*	8.0	0.001	4	0.873	0.756	0.072	0.867	0.741	0.100
	E(II)	10	0.001	5	**0.959**	**0.917**	**0.042**	**0.935**	**0.848**	**0.077**
Basin with negative step										
Sym channel	*E(I)*	10	0.100	4	0.890	0.790	0.069	0.869	0.710	0.081
	E(II)	**8.0**	**0.001**	**3**	**0.982**	**0.982**	**0.031**	**0.957**	**0.908**	**0.045**
	E(III)	8.0	0.001	2	0.974	0.972	0.035	0.945	0.945	0.049
Asym channel	*E(I)*	10	0.001	5	0.961	0.924	0.052	0.959	0.918	0.054
	E(II)	**10**	**0.120**	**8**	**0.989**	**0.984**	**0.027**	**0.976**	**0.981**	**0.032**
	E(III)	5.0	0.100	4	0.799	0.977	0.029	0.956	0.908	0.055
Basin with central sill										
Sym channel	*E(I)*	8.0	0.100	6	0.980	0.961	0.045	0.976	0.953	0.047
	E(II)	**10**	**0.100**	**4**	**0.993**	**0.985**	**0.027**	**0.992**	**0.984**	**0.030**
	E(III)	10	0.001	3	0.981	0.965	0.042	0.980	0.964	0.044

Bold values correspond to the superior model for each condition.

asymmetric channels, the superior performance was obtained for the basin with negative step. However, according to Table 7, the basin with central sill, in predicting relative energy dissipation, performed more successfully than the other cases. Figure 8 shows the verification between measured and estimated values for the best SVM models for expanding channels.

Sensitivity analysis

To investigate the impacts of different employed parameters from the best proposed models on hydraulic jump characteristics prediction via SVMs, sensitivity analysis was performed. In order to evaluate the effect of each independent parameter, the model was run with

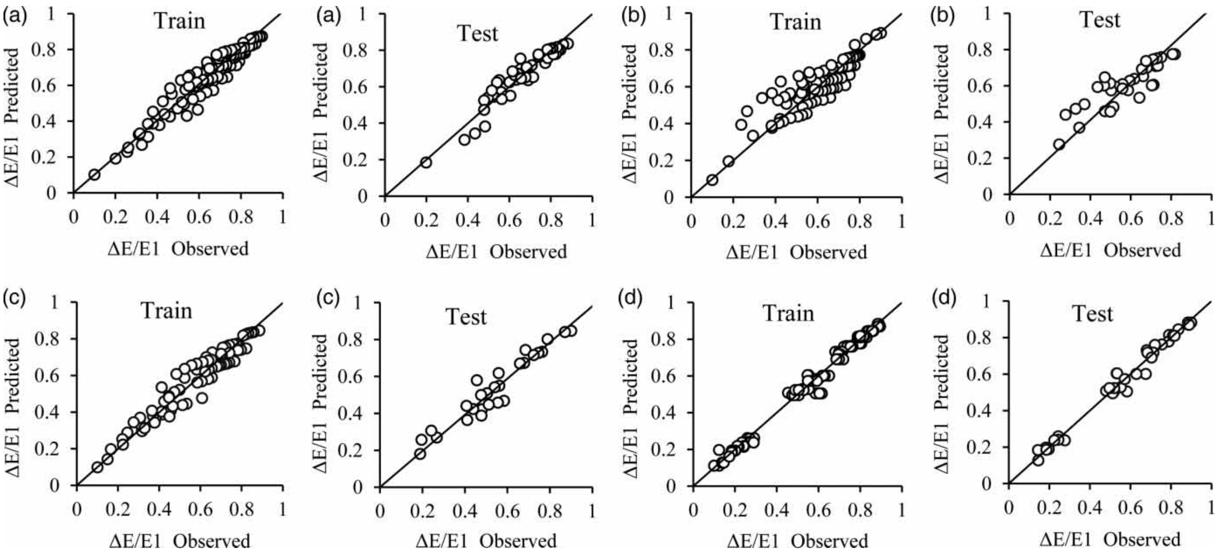

Figure 8 | Comparison of observed and predicted relative energy dissipation for superior model; (a) symmetric basin without appurtenances, (b) symmetric basin with negative step, (c) asymmetric basin with negative step, (d) symmetric basin with central sill.

all input parameters and then one of the input parameters was eliminated and the SVM model was re-run. The *RMSE* criterion was used as indication of the significance of each parameter. The results are listed in Table 8. In this table Δ*RMSE* represents the percentages of the added values to the error criteria for each eliminated parameter. Based on the results from Table 8, it could be inferred that variable F_1 has the most significant effect on the characteristics of a hydraulic jump. Also, Table 8 shows the order of the impact of each input parameter.

Table 8 | Relative significance of each of input parameters of the best models (basin with central sill)

Hydraulic jump characteristics	Eliminated variable	Performance criteria for test series: *RMSE*	Δ*RMSE*%	The order of the impact of parameters
Sequent depth ratio				
Y	F_1, S/h_1, B, h_1/B	0.021	–	–
	$\boldsymbol{F_1}$	**0.184**	**766**	**1**
	B	0.023	9.5	4
	S/h_1	0.025	19	3
	h_1/B	0.028	34	2
Length of hydraulic jump				
L_j/h_1	F_1, $(h_2-h_1)/h_1$, h_2/h_1, S/h_1,	0.037	–	–
	$\boldsymbol{F_1}$	**0.144**	**289**	**1**
	$(h_2-h_1)/h_1$	0.046	24	2
	h_2/h_1	0.043	14	3
	S/h_1	0.041	11	4
	B	0.039	5	5
Loss of energy				
$\Delta E_L/E_1$	F_1, S/h_1, h_1/B	0.025	–	–
	$\boldsymbol{F_1}$	**0.203**	**712**	**1**
	S/h_1	0.030	20	3
	h_1/B	0.044	76	2

Bold values show the most significant parameter.

Combined data

We tried to evaluate the applicability of applied methods for a wider range of data. In other words, all data series were combined, then for predicting the hydraulic jump characteristics, as the dependent variable, the superior model of any characteristics and the model with only F_1 as input variable were reanalyzed for the combined data state. The results of SVM models are given in Table 9 and Figure 9. According to the results of the combined data, it could be stated that adding h_1/B and h_2/h_1 to input parameters caused an increment in model efficiency. However comparison between Tables 5–7 and 9 indicated that SVM models for the combined data set did not show the desired accuracy, and analyzing data sets separately led to more accurate results. It should be noted that for the combined date state, data series with different conditions (i.e. different hydraulic data range and different appurtenances) were used altogether. Therefore, the results are not so accurate. Figure 9 shows the verification between measured and estimated values of test series for the best proposed model for any characteristics.

Comparison of SVM and classic equations

Accuracy of the best proposed models of the basin with central sill developed in this study, and some semi-theoretical formulae available in literature, were compared to evaluate the performance of the applied approach. The results of comparison are represented in Figure 10. According to three evaluated criteria (*R*, *DC*, and *RMSE*) which are shown in Figure 10, it can be seen that the estimated values via SVM models had more accurate results than equations. It can be observed that the formula used for estimating the sequent depth ratio provided a reasonable fit to the experimental data. In comparison with the sequent depth ratio formulas, hydraulic jump length equations did not show desired agreement between the estimated values and the observed sets, while for both hydraulic jump characteristics (*Y* and L_j/h_1) the obtained results by the best SVM models were close to

Table 9 | Statistical parameters of the SVM models for combined data

					Performance criteria					
		Optimal parameters			Train			Test		
Output variable	Input variable(s)	*c*	ε	γ	*R*	*DC*	*RMSE*	*R*	*DC*	*RMSE*
Sequent depth ratio										
h_2/h_1	F_1	10	0.10	8	0.828	0.680	0.092	0.774	0.580	0.098
	F_1, h_1/B	10	0.01	5	0.873	0.760	0.079	0.816	0.660	0.088
Length of hydraulic jump										
Lj/h_1	F_1	8.0	0.01	6	0.435	0.272	0.158	0.401	0.251	0.172
	F_1, h_2/h_1	10	0.10	6	0.564	0.306	0.113	0.554	0.272	0.132
Loss of energy										
E_L/E_1	F_1	10	0.02	5	0.740	0.572	0.188	0.667	0.419	0.196
	F_1, h_1/B	10	0.01	5	0.885	0.781	0.101	0.882	0.647	0.118

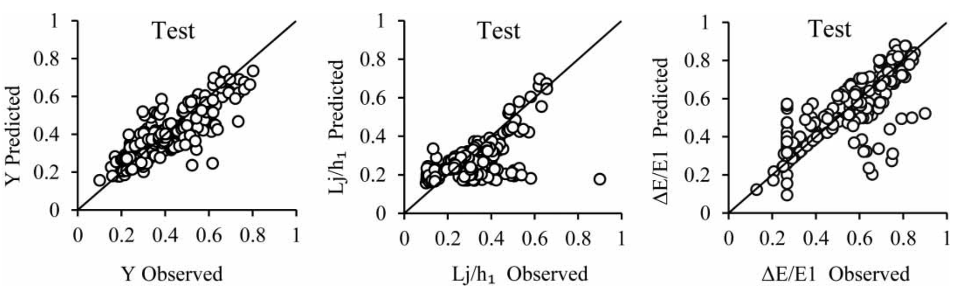

Figure 9 | Comparison of observed and predicted hydraulic jump characteristics; combined data.

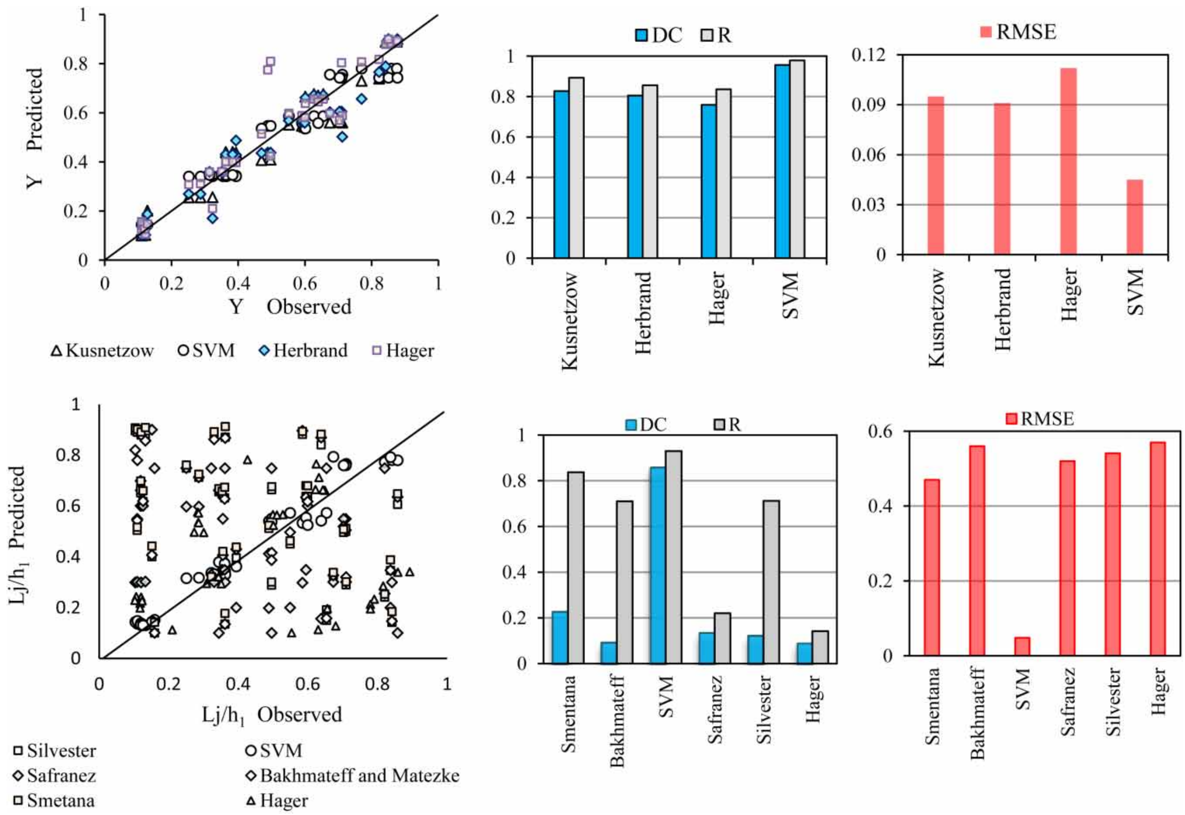

Figure 10 | Comparison of statistical parameters between formula and best SVM model.

the measured data. It should be noted that the existing equations rely on a limited database, untested model assumptions, and a general lack of field data, and do not show the same results under variable flow conditions. However, despite the complexity and uncertainty of the hydraulic jump phenomenon the obtained results confirm the workability of SVM as an efficient machine learning approach in modeling of hydraulic jump characteristics in sudden diverging basins.

CONCLUSION

In the present study, the capability of the SVM approach was verified for predicting hydraulic jump characteristics (i.e. the sequent depth ratio, jump length and energy loss) in sudden diverging basins. The SVM was applied for different data sets in basins without appurtenances, with a negative step or with a central sill. The obtained results indicated that in prediction of the sequent depth ratio and relative energy dissipation the model with parameters F_1 and h_1/B as input variables performed more successfully than the other models. The superior performance for length of hydraulic jump was obtained for the model $L(II)$, in which the inputs were F_1 and $(h_2 - h_1)/h_1$. Comparison between the results of three basins with sudden diverging side walls revealed that the developed models in the case of the basin with a central sill led to a more accurate outcome. It was also observed that the accuracy of the models for the basin with negative step, in a channel with asymmetric shape, was higher than for a symmetric shape channel, while for the basin without appurtenances, a symmetric channel presented better results. It showed that adding S/h_1 as an input parameter improved the efficiency of the models. This issue confirmed the importance of the geometry of applied appurtenances in the hydraulic jump characteristics estimation process in basins with appurtenances. Based on the obtained results from sensitivity analysis, it was found that F_1 had the most effective role in estimation of hydraulic characteristics compared with other parameters (see Table 8). From the results, it was observed that analyzing data sets separately led to a more accurate outcome. A comparison was also made between the applied technique and some classical equations. The applied technique was found to be able to predict hydraulic jump characteristics in sudden diverging basins successfully and in comparison with available classic equations is more reliable.

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