Hierarchical prediction of industrial water demand based on refined Laspeyres decomposition analysis

Yizi Shang, Shibao Lu, Jiaguo Gong, Ling Shang, Xiaofei Li, Yongping Wei and Hongwang Shi

ABSTRACT

A recent study decomposed the changes in industrial water use into three hierarchies (output, technology, and structure) using a refined Laspeyres decomposition model, and found monotonous and exclusive trends in the output and technology hierarchies. Based on that research, this study proposes a hierarchical prediction approach to forecast future industrial water demand. Three water demand scenarios (high, medium, and low) were then established based on potential future industrial structural adjustments, and used to predict water demand for the structural hierarchy. The predictive results of this approach were compared with results from a grey prediction model (GPM (1, 1)). The comparison shows that the results of the two approaches were basically identical, differing by less than 10%. Taking Tianjin, China, as a case, and using data from 2003–2012, this study predicts that industrial water demand will continuously increase, reaching 580 million m³, 776.4 million m³, and approximately 1.09 billion m³ by the years 2015, 2020 and 2025 respectively. It is concluded that Tianjin will soon face another water crisis if no immediate measures are taken. This study recommends that Tianjin adjust its industrial structure with water savings as the main objective, and actively seek new sources of water to increase its supply.

Key words | grey prediction model, hierarchical prediction, industrial water demand, Tianjin, water saving

INTRODUCTION

Water demand forecasting is an essential component of effective water resource planning and management, as it helps determine the required timing and capacity for developing new water resources (Armstrong & Fildes 2006). For instance, public utilities must be aware of current and predicted future demand in order to operate their treatment plants and wells to meet such demand. Water demand forecasting can simulate future conditions and help identify suitable management options that balance water supply and demand; thus, it is becoming increasingly important in water resource planning (Mohamed & Aysha 2010).

Numerous water demand forecasting methods and models are available (Archibald 1985; Donkor et al. 2014). These range from simple methods to complex models and can be qualitative or quantitative in nature. Quantitative methods include heuristics or rule-based methods that forecast the value of a variable of interest, and include the elastic coefficient method (Renzetti 1992) and regression analysis method (Herrera et al. 2010). Such methods must first identify a numerical relationship between external factors (such as industrial capacity, gross industrial output, and industrial added value) and water demand, and then predict future water demand (Pulido-Calvo et al. 2007; Pulido-Calvo & Gutierrez-Estrada 2009) based on the accurate inference of future external factors. Moreover, these methods consider the factors influencing water demand; thus, the inferred results are more accurate than those of other methods. However, many external factors must be predicted, making the
process rather complex. Prediction errors for external factors will also have an effect on the accuracy of water demand predictions (Lee et al. 2010). Quantitative methods also include another method, often referred to as extrapolation forecasting. These methods make predictions based on historical water demand curve trends but do not consider external factors influencing water demand. Such methods include the trend analysis method (Adamowski & Karapataki 2010), grey prediction model (GPM) (Deng 1989), and models based on neural networks (Firat et al. 2009). These methods are characterized by simple operation and greater accuracy when predicting individual trend data series. In the literature, the use of the GPM for water demand forecasting has primarily entailed comparative assessment of the performance of various demand-forecasting models (Yu et al. 2000; Zhang 2013).

Although forecasting is not a new discipline, its application to demand estimation for the water sector is fraught with many problems, and it is notorious for difficulties posed by the numerous variables hypothesized to affect water demand (Chronis et al. 2016). These challenges have prompted a plethora of studies attempting to improve forecast reliability (Zhou et al. 2000; Gato et al. 2007a, 2007b), and a hybrid approach has been proposed for accurate forecasting (Pulido-Calvo & Gutierrez-Estrada 2009). This approach uses more than one method and/or model to generate a composite forecast. Specifically, it usually uses either simple or weighted averages from a combination of model forecasts (Wang et al. 2009; Caiado 2010) or applies a mix of methods and models to forecast the decomposed components of a time series (Aly & Wanakule 2004; Alvisi et al. 2007).

Despite these efforts, the models used in water demand forecasting undertaken by public utilities and their consultants can differ widely. Thus, past research and practices serve as prologues to our study. Billings & Jones (2008) described the method of extrapolation forecast as applied by water utilities, while Jentgen et al. (2007) reported on specific public utilities in the United States (Jacksonville Electric Authority, San Diego Water Department, Colorado Springs Utilities, and Las Vegas Valley Water District) that used heuristics, regression, and neural networks to prepare water demand forecasts for developing utilities.

Planning for decision making forms the basis of forecasting for the water sector. All water demand forecasting exercises can be used for strategic, tactical, or operational decision-making plans (Kiefer & Porter 2000; Rinaudo 2015), which, respectively, concern capacity expansion, investment planning and system operation, and management and optimization decisions (Archibald 1985; Tate 1986; Butler & Memon 2006). In planning for the development of water resources, domestic, industrial, and agricultural water demand must all be predicted (Water Resources and Hydropower Planning and Design General Institute of MWR 2010). For rapidly developing countries such as China, the industrial sector generally shows the most acute changes in the three areas of demand, and corresponding predictions most significantly impact the construction of local water supply infrastructure and the policies governing infrastructure (Jia 2001; Shang et al. 2015). Industrial water demand for such countries and regions has been predicted to grow exponentially over the next 10 to 20 years, thus significantly increasing overall water demand (Varis & Vakkilainen 2001; Alhumoud 2008). Currently, developing countries are experiencing rapid industrial development, with industrial production showing average annual growth of more than 10% for the past few decades (Hussey & Pittock 2012). Industrial development in these countries and regions will continue at medium- to high speed (Li et al. 2008). Thus, industrial water demand will certainly increase significantly, according to the traditional industrial water prediction method of multiplying the gross industrial output value by the water demand per unit output value to predict future total water demand. Even if water demand per unit output value decreases significantly, industrial water demand is still predicted to quadruple (Institute of Geographic Sciences and Natural Resources Research & Key Laboratory of Water Cycle and Related Land Surface Processes 2012).

However, there is reason to doubt the accuracy of this prediction. Firstly, according to the laws of economics, water demand will be restricted by water costs and benefits. Secondly, based on the experiences of developed countries, gross industrial output value and industrial water usage cannot grow rapidly for an infinite period, but will eventually stop growing or even decline (Jia et al. 2006). Taking Tianjin, China, as an example: Tianjin promulgated its comprehensive water resource plan in 2000. Under the plan, industrial water usage was predicted to increase fourfold between 2000 and 2010 (Tianjin Water Authority 2010). However, historical statistical data show that Tianjin’s industrial water usage increased less than twofold over that period (Tianjin Development and Reform Commission 2004–2015); thus, the prediction deviated from reality by more than 200%.

In practice, the use of extrapolation forecasts is driven by a desire for rudimentary forecasts that simplify forecasting. However, changes in industrial water demand are influenced by many factors (Shang et al. 2016), which have rarely been considered in water resource
development planning using extrapolation forecasts. Previously, Shang et al. (2017) used a refined Laspeyres decomposition model to decompose Tianjin’s industrial water usage into usages driven by output, technological, and structural factors which in turn were used to classify industrial water into three corresponding hierarchies. Using previous research as its foundation, the present study made independent predictions of each hierarchy, incorporating all such predictions for Tianjin’s overall industrial water demand, and used the GPM (1, 1) to verify the prediction results based on the hierarchical prediction model. In this study, the extrapolation forecast method was adopted to predict future water demand. Water demand data vary slightly for the structural hierarchy, with a relatively complex trend due to uncertainty in industrial structure changes. Thus, the present study set different conditions for predicting the water demands of the structural hierarchy. The present study aims to provide a guide for water utility managers, to improve forecasting for effective decision making, and to expand upon current knowledge in the field.

METHODS

Hierarchical prediction approach

The industrial water use curve exhibits a complex trend under the influence of multiple factors. In general, it is not appropriate to use extrapolation forecasting for long-term prediction of national or regional industrial water demand. The basic principle of the extrapolation forecast method, also called trend analysis or curve-fitting analysis, is to observe the characteristic changes in historical data over time and establish a regression equation that accurately fits the relationship between the temporal and data variables. This equation must accurately reflect data trends during the study period, and its predictions are highly precise. However, there are strict requirements that trends in the historical data should be monotonous and exclusive.

To obtain a water use curve exhibiting a monotonous and exclusive trend, Shang et al. (2017) decomposed the changes in industrial water use under the exclusive influences of output, technological, and structural factors. For decomposition, these drivers are used to characterize the contributions of expanding industrial scale, water-saving technologies, and industrial restructuring, respectively.

The change in industrial water use for the tth year is written as follows:

$$\Delta Q = \Delta Q_M + \Delta Q_w + \Delta Q_q$$

(1)

where $\Delta Q$ represents the change in industrial water use, and $\Delta Q_M$, $\Delta Q_w$, and $\Delta Q_q$ represent those resulting from output, technology, and industrial structure, respectively.

The formulae for calculating the parameters of Equation (1) using the refined Laspeyres models are given below:

$$\Delta Q_M = \sum_{i=1}^{n} M_i^0 q_i^0 \Delta \mu_i + \frac{1}{2} \Delta \mu_i (q_i^0 \Delta M + M_i^0 \Delta q_i) + \frac{1}{3} \Delta \mu_i \Delta M \Delta q_i$$

(2)

$$\Delta Q_w = \sum_{i=1}^{n} \mu_i^0 q_i^0 \Delta M + \frac{1}{2} \Delta M (q_i^0 \Delta \mu_i + \mu_i^0 \Delta q_i) + \frac{1}{3} \Delta \mu_i \Delta M \Delta q_i$$

(3)

$$\Delta Q_q = \sum_{i=1}^{n} \mu_i^0 M_i^0 \Delta q_i + \frac{1}{2} \Delta q_i (M_i^0 \Delta \mu_i + \mu_i^0 \Delta M) + \frac{1}{3} \Delta \mu_i \Delta M \Delta q_i$$

(4)

where $M_i^0$ is the value of industrial output in the previous year, $q_i^0$ is water use per 10,000 yuan of industrial added value within industrial sector i during the previous year, and $\mu_i^0$ is the proportion of Tianjin’s total industrial output value that derived from industrial sector i in the previous year. $\Delta M$, $\Delta \mu_i$, and $\Delta q_i$ refer to the changes in industrial output value, water use per 10,000 yuan, and the output proportion of industrial sector i, respectively.

Based on refined Laspeyres decomposition analysis, this study proposes a hierarchical approach for forecasting future industrial water demand. The steps involved in the hierarchical prediction are summarized as follows.

Step 1: Conduct attribution analysis of industrial water demand, identify the factors driving industrial water demand changes, and apply a refined Laspeyres decomposition model for the hierarchical decomposition of industrial water demand of a city.

Step 2: Analyze the data decomposition results, and adopt the traditional mature trend analysis method to predict water demand according to trends in the decomposed data. Owing to the uncertainties surrounding changes in water demand, scenarios should be set according to potential future conditions.

Step 3: Superimpose the predictions for the three different hierarchies, define the numerical range of future water
demand, and obtain the prediction results for overall industrial water demand.

GPM: overview

The GPM (1, 1) model is the most commonly used GPM; its first-order differential equation contains only one variable, and the basic modeling process is outlined below.

Step 1: Establish the original data series, \( x^{(0)} \):

\[
x^{(0)} = [x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)].
\]

Step 2: Generate the first derivative accumulative series, \( x^{(1)} \):

\[
x^{(1)} = [x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)],
\]

where

\[
x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j), \quad k = (1, 2, 3, \ldots, n).
\]

Step 3: Calculate the background value, \( \{z(k)\} \), \( k = 1, 2, \ldots, n \), as follows:

\[
Z(k) = \frac{1}{2} (x^{(1)}(k - 1) + x^{(1)}(k)).
\]

Step 4: Create the differential equation model, as follows:

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = u.
\]

Step 5: Calculate the parameters \( \hat{a} \) and \( \hat{u} \):

\[
\begin{pmatrix} \hat{a} \\ \hat{u} \end{pmatrix} = (B^T B)^{-1} B^T Y_n,
\]

where

\[
Y_n = (x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n))^T, \quad B = \begin{pmatrix} -Z(2) & 1 \\ -Z(3) & 1 \\ \vdots & \vdots \\ -Z(n) & 1 \end{pmatrix}.
\]

Step 6: Calculate the predicted value, \( \hat{x}^{(1)}(k + 1) \), of the accumulative series, as follows:

\[
\hat{x}^{(1)}(k + 1) = \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-ak} + \frac{u}{a}, \quad k = 1, 2, \ldots, n.
\]

Step 7: Convert the predicted value into the original data series form, \( \hat{x}^{(0)}(k + 1) \):

\[
\hat{x}^{(0)}(k + 1) = (1 - e^a) \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-ak}, \quad k = 1, 2, \ldots, n.
\]

The GPM firstly obtains a regular data series through the accumulating or inverse accumulated generating operations; it then conducts a prediction using the regular data series; and, finally, it converts the data into its original format, and the obtained values represent the predicted future changes for the objects. The GPM has three major features: (1) its establishment requires fewer historical data, and future changes can be predicted using four historical datasets; (2) when the model is used for prediction, the distribution rule of the original data series does not need to be mastered in advance, and the original irregular, random, and discrete series can be translated into regular series via several data generation operations; and (3) the prediction can better describe and reflect the actual conditions of the system, thus achieving greater accuracy.

CASE STUDY

Tianjin is an important industrial base, but is also one of the most water-scarce cities in China, with serious water supply and demand contradictions. The industrial sector accounts for a large proportion of total water consumption in the city. Tianjin has fostered 36 major industrial sectors, including ferrous metal smelting, and the manufacture of transportation and communications equipment, which are highly dependent on water resources. Water usage data for the Tianjin case study are sourced from the Tianjin Water Resources Bulletin (Tianjin Water Authority 2004–2013). Statistics on water use by industrial sectors are adopted from the Tianjin Industrial Energy Efficiency Guide (Tianjin Development and Reform Commission 2004–2013) and the Tianjin Municipal Bureau of Statistics, and output statistics are from the Tianjin Statistical Yearbook (2004–2013) (Statistical Bureau of Tianjin, 2004 to 2013). The location of Tianjin in China is shown in Figure 1.
In a previous study, Shang et al. (2017) decomposed Tianjin’s industrial water use from 2003 to 2012, as illustrated in Figure 2. If industrial water demand is only affected by the output driving force, it presents a linearly increasing trend with a growth rate of approximately 60 to 70 million m$^3$ from 2003 to 2012. Overall, industrial water use for the whole of Tianjin will increase sharply, by 0.7 billion m$^3$, by 2012. If industrial water use is only affected by the technological driving force, it presents a rapidly decreasing trend at a rate of between 20 and 140 million m$^3$. The city’s total industrial water demand would decrease by 0.72 billion m$^3$ between 2003 and 2012. For both the output and technological driving forces, the historical industrial water use curve exhibits a simple trend under the influence of a single factor; thus, the trend analysis method is suitable for predictions. However, the structural

**Figure 1** Location of Tianjin in China.

**Figure 2** Hierarchy of Tianjin’s historical industrial water use, 2003–2012.
driving force exerts either accelerative or inhibitive effects according to changes in the industrial structure. The inset in Figure 2 shows that, under the influence of structural drivers, industrial water use fluctuates around 470 million m$^3$ prior to 2008 and then increases slightly.

RESULTS AND DISCUSSION

Industrial water demand prediction using a hierarchical approach

Prediction of water demand according to output hierarchy

Trend analysis commonly includes linear, logarithmic, exponential, and power equations. This study adopted all four types for trend-fitting analysis, and determined the optimal equation based on the correlation coefficient, $R^2$ (range 0–1), where a $R^2$ value closer to 1 indicates a better fit between the fitting curve and the actual value curve, and greater simulation accuracy. Table 1 shows the correlation coefficient of each equation. During the study period, the data exhibited a linearly increasing trend. Although all equations had $R^2$ values >0.9, the linear equations provided the optimum fitting, with $R^2$ values as high as 0.9982, such that a linear equation was used to simulate the output hierarchy. Table 2 presents the simulated and actual values, as well as the error calculation.

The absolute value of the relative error was less than 2%, and the linear fitting result was ideal. To further verify the accuracy of the simulation, a posterior variance test was applied to the fitting equation, and in Figure 2 shows that, under the influence of structural drivers, industrial water use fluctuates around 470 million m$^3$ prior to 2008 and then increases slightly. The inset in Figure 2 shows that, under the influence of structural drivers, industrial water use fluctuates around 470 million m$^3$ prior to 2008 and then increases slightly.

Finally, the posterior error ratio $C$ and infinitesimal error probability $P$ were determined by the following two equations:

\begin{equation}
C = \frac{\sqrt{S_2^2}}{S_1^2} = 0.037
\end{equation}

and

\begin{equation}
P = P\{|e(k) - \bar{e}| < 0.6745S_1 = 15500\} = 1, \quad k = 1, 2, 3, \ldots, n
\end{equation}

From these equations, the posterior error ratio $C$ was found to be 0.037 (i.e., less than the maximum error of 0.35), and the infinitesimal error probability $P$ was 1 (i.e., greater than 0.95). As seen in the accuracy grade table (Wackerly et al. 2007), the linear mathematical equation had a grade A+ fitting degree, thus passing the test.

Prediction of water demand according to technological hierarchy

The trend analysis method was also used to predict data for the technological hierarchy. However, exponential and power equations were not suitable for the fitting analysis because the real data included negative values. Therefore,

\begin{equation}
\begin{aligned}
\bar{e} &= \frac{1}{n} \sum_{k=1}^{n} e(k) = 0.07, \quad k = 1, 2, 3, \ldots, n
\end{aligned}
\end{equation}

and

\begin{equation}
S_1^2 = \frac{1}{n} \sum_{k=1}^{n} [q(k) - \bar{q}]^2 = 528118, 903, \quad k = 1, 2, 3, \ldots, n;
\end{equation}

\begin{equation}
S_2^2 = \frac{1}{n} \sum_{k=1}^{n} [e(k) - \bar{e}]^2 = 729053, \quad k = 1, 2, 3, \ldots, n
\end{equation}

Table 2 presents the simulated and actual values, as well as the error calculation.

As shown in Table 2, the simulated output hierarchy values followed the sequence $\bar{q} = \{48600.0, 57910.7, 65394.4, \ldots, 11780.3\}$. The residual errors followed the sequence $e = q - \bar{q} = \{0, 421.2, 897.6, \ldots, 807.1\}$, in which $q$ indicates a real data series. The averages of the real data series $q$ and the residual error data series $e$ were determined using the following two equations, respectively:

\begin{equation}
\bar{q} = \frac{1}{n} \sum_{k=1}^{n} q(k) = 83921.0, \quad k = 1, 2, 3, \ldots, n;
\end{equation}

Table 1 | Correlation coefficients for the output hierarchy of industrial water data

<table>
<thead>
<tr>
<th>Fitting equation</th>
<th>Exponential equation</th>
<th>Linear equation</th>
<th>Logarithmic equation</th>
<th>Power equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation coefficient ($R^2$)</td>
<td>0.9815</td>
<td>0.9982</td>
<td>0.9079</td>
<td>0.9684</td>
</tr>
</tbody>
</table>

...
logarithmic and linear equations were used for the fitting analysis for this hierarchy, with the equation correlation coefficients shown in Table 3. During the study period, the data exhibited a monotonous trend. All equations fitted the trend well, but the logarithmic equations achieved an optimal fitting, with $R^2$ as high as 0.9903. Thus, a logarithmic equation was used for simulation with the technological hierarchy. See Table 4 for a comparison of the simulated and actual values, and the error calculation.

The logarithmic equation achieved a relatively ideal fit, with relative error within 15% (absolute value). The same posterior variance test found a posterior error ratio, $C$, of 0.064 ($<0.35$) and an infinitesimal error probability, $P$, of 1 ($>0.95$). As seen in the accuracy grade table (Wackerly et al. 2007), the fitting degree was a grade A+, and the established logarithmic fitting equation passed the test.

**Prediction of water demand according to structural hierarchy**

The structural hierarchy contains water data with a smaller range and a relatively complex trend. Linear, logarithmic, exponential, and power equations were used for fitting analysis, with the correlation coefficients, $R^2$, of each shown in Table 5. None of the equations achieved ideal fitting results, with most $R^2$ values less than 0.5. Thus, for recent years, the changes in the structural hierarchy data do not fit a specific regression equation. As industrial structure is an artificial and uncontrollable factor, including randomness subject to factors such as national economic development and future infrastructure-related policies, this study predicted the future industrial water demand of the structural hierarchy using the scenario assumption method. The following three water demand plans were set up for the structural hierarchy, according to different water demand scenarios.

1. **High water demand**: It was assumed that the government will not implement any structural adjustment measures targeting water savings, and that the industrial structure will continue to develop with high water usage, as in 2008. Thus, the water demand of the structural hierarchy will maintain an average annual growth rate of 4.1% and is expected to reach 590.45 million m$^3$, 718.37 million m$^3$, and 874.01 million m$^3$ by 2015, 2020, and 2025, respectively.

2. **Medium water demand**: If the government implements effective measures to deter the development of industrial structures with high water demand, the water demand of the structural hierarchy will drop to the historical average of 4,748.9 billion m$^3$ by 2025. In proportion to this decline, demand will be 513.37 million m$^3$ and 494.13 million m$^3$ by 2015 and 2020, respectively.

3. **Low water demand**: If the government implements effective water-saving measures, then the industrial water demand of the structural hierarchy will continue to decline at the annual average rate of 4.0%, as it did prior to 2008. Thus, demand will be 464.41 million m$^3$, 378.66 million m$^3$, and 308.75 million m$^3$ by 2015, 2020, and 2025, respectively.

See Table 6 for predicted structural hierarchy water demand according to these three scenarios.

**Prediction of industrial water demand**

We superimposed the predicted water demands of the output, technological, and structural hierarchies to

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**Table 3** Correlation coefficients for the technological hierarchy of industrial water data

<table>
<thead>
<tr>
<th>Fitting equation</th>
<th>Logarithmic equation</th>
<th>Linear equation</th>
<th>Exponential equation</th>
<th>Power equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation coefficient ($R^2$)</td>
<td>0.9903</td>
<td>0.9891</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
obtain predictions of Tianjin’s overall industrial water demand, as shown in Table 7. The results predict that Tianjin’s industrial water demand will continue rising in the future, and that the growth rate will gradually accelerate due to the rapid growth rate of industrial production. Tianjin’s industrial water demand is expected to reach approximately 580 million m$^3$, 780 million m$^3$, and more than 1 billion m$^3$ by 2015, 2020, and 2025, respectively. Industrial structural adjustment is a very effective means of conserving water, with more pronounced results in years with greater amounts of water consumed. The predictions also suggest that industrial water demand will be reduced by 8.4%, 14.9%, and 15.2% by 2015, 2020, and 2025, respectively, if the enhanced water-saving measures are implemented.

### Industrial water demand prediction using GPM

The GPM (1, 1), achieving greater predictive accuracy with a small number of samples, was used to predict water demand, using Tianjin’s industrial water usage (2008–2012) as model inputs to obtain the corresponding data matrix, as follows:

$$B = \begin{pmatrix} -\frac{1}{2} [x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2} [x^{(1)}(2) + x^{(1)}(3)] & 1 \\ -\frac{1}{2} [x^{(1)}(3) + x^{(1)}(4)] & 1 \\ -\frac{1}{2} [x^{(1)}(4) + x^{(1)}(5)] & 1 \end{pmatrix} = \begin{pmatrix} -59881 & 1 \\ -105424 & 1 \\ -154266 & 1 \\ -204760 & 1 \end{pmatrix};$$

$$Y_n = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ x^{(0)}(5) \end{pmatrix} = \begin{pmatrix} 43,502 \\ 47,584 \\ 50,100 \\ 50,887 \end{pmatrix}.$$  

The model parameters $a$ and $\hat{a}$ were then determined using the formula $\begin{pmatrix} \hat{a} \\ \hat{u} \end{pmatrix} = (B^TB)^{-1}B^TY_n$ to obtain the following series:

$$\begin{pmatrix} \hat{a} \\ \hat{u} \end{pmatrix} = \begin{pmatrix} -59881 \\ -105424 \\ -154266 \\ -204760 \end{pmatrix}^T \begin{pmatrix} -59881 & 1 \\ -105424 & 1 \\ -154266 & 1 \\ -204760 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 43,502 \\ 47,584 \\ 50,100 \\ 50,887 \end{pmatrix} \times \begin{pmatrix} -59881 \\ -105424 \\ -154266 \\ -204760 \end{pmatrix}^T \begin{pmatrix} 43,502 \\ 47,584 \\ 50,100 \\ 50,887 \end{pmatrix} \times \begin{pmatrix} -0.0507 \\ 41378.5 \end{pmatrix}.$$

We substituted the obtained parameters into Equation (13), and obtained an albinism differential equation. The prediction equation of the original series is as follows:

$$\hat{x}^{(0)}(k + 1) = (1 - e^a) \left( x^{(0)}(1) - \frac{U}{a} \right) e^{-ak} = 42231.28 e^{0.0507k}.$$  

See Table 8 for the GPM (1, 1) results for Tianjin’s industrial water demand, which was predicted to reach

<table>
<thead>
<tr>
<th>Fitting equation</th>
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<th>Logarithmic equation</th>
<th>Exponential equation</th>
<th>Power equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation coefficient ($R^2$)</td>
<td>0.4954</td>
<td>0.2713</td>
<td>0.4867</td>
<td>0.2643</td>
</tr>
</tbody>
</table>
600 million m³, 780 million m³, and nearly 1 billion m³ by 2015, 2020, and 2025, respectively.

Table 8 shows that the GPM (1, 1) achieved highly ideal simulation results, with accuracy controlled within 3% for predicting Tianjin’s industrial water demand. From the posterior variance test, the posterior error ratio, $C$, was 0.168 ($<0.35$), and the infinitesimal error probability, $P$, was 1 ($>0.95$). As seen in the accuracy grade table (Wackerly et al. 2007), the fitting degree was a grade A+, thus, the model passed the test.

**Comparing results between the two approaches**

We used actual water consumption data to verify the accuracy of the hierarchical prediction approach, and further compare its performance with that of GPM (1, 1) (Tien 2009). Figure 5 compares the prediction results of the two approaches with actual industrial water data, where the industrial water demand of the hierarchical prediction approach is forecasted under a medium water demand scenario. As can be seen from Figure 5, the predictions of the two models are nearly identical, varying by less than 10%, indicating the applicability of the hierarchical prediction model to the field of industrial water prediction.

Figure 4 shows the future industrial water demand prediction using the two approaches. The results of the hierarchical prediction model under medium water demand scenario and GPM (1, 1) reveal that Tianjin’s industrial water demand will continue to increase into the future, reaching 600 million m³, 800 million m³, and 1 billion m³ by 2015, 2020, and 2025, respectively. If no measures are taken, Tianjin will face another water crisis; thus, urgent action is required to resolve the industrial water shortage. It is concluded that Tianjin needs to consider adjusting its industrial structure, targeting industrial water conservation, as well as seeking new water sources.

**Further discussion**

Industrial water demand is influenced by output, technological, and structural drivers. The output drivers primarily accelerate the growth of industrial water demand, while the technological parameters primarily restrict its growth. Structural issues exert both accelerative and inhibitive effects according to changes in the industrial structure. At different times, all of these driving forces act in different ways, and their fluctuations subject industrial water demand to complex trends. The findings described here demonstrate that hierarchical prediction theory can achieve accurate and highly precise predictions of Tianjin’s industrial water demand. However, the theory’s scope of application, simulation limitations, and means of detection are not clear. Moreover, this theory still requires empirical testing in future studies, and a variety of traditional models should be applied to verify the theory and delimit the scope of its application. Nevertheless, the present study provides a new method and perspective for predicting long-term industrial water demand.

The hierarchical prediction approach has particular advantages for predicting long-term water demand, as a variety of development scenarios can be used based on practicability. Furthermore, this process can accurately determine the range of fluctuation of future water demand. Therefore, hierarchical prediction has greater flexibility
compared to GPM (1, 1). The hierarchical prediction approach is based on the refined Laspeyres decomposition model, which may have applications other than the long-term prediction of industrial water demand that require exploration. For example, water-saving potential is an important index for the evaluation of local water-saving conditions. An accurate understanding of the water-saving potential of a region can provide a scientific basis for appropriate departments to implement water-saving management plans and optimize water allocation. The method documented here for calculating industrial water-saving potential is relatively simple and has often ignored the inertia effects of the internal drivers of industrial water demand. A further study is planned, which will examine whether the decomposition results of these drivers can be applied to improve the scientific validity and reasonability of calculated water-saving potentials.

**CONCLUSION**

Based on a quantitative analysis of the historical evolution of Tianjin’s industrial water demand, Shang *et al.* (2017) established a refined Laspeyres decomposition model in accordance with the historical output and water data of all industrial sectors, to determine the factors driving Tianjin’s industrial water demand from 2003 to 2012. From that foundation, the present study: layered Tianjin’s historical industrial water data; proposed a hierarchical prediction theory; predicted Tianjin’s industrial water demand by 2015, 2020, and 2025 using this theory; and employed the GPM to verify the theory. The conclusions of this study are as follows.

(1) In essence, the hierarchical prediction model generates a combined prediction. The industrial water curve exhibits a complex trend due to the influences of many factors. In general, the traditional trend analysis method is unsuitable for predicting the long-term industrial water demand of a country or region. Using previous studies, Shang *et al.* (2017) found that, if the refined Laspeyres decomposition model is used for the hierarchical decomposition of Tianjin’s industrial water demand, the decomposed hierarchical water data will become regular and can completely satisfy the conditions of the traditional analysis method. This study created and tested a hierarchical prediction model based on the refined Laspeyres decomposition model and existing mature prediction methods. The hierarchical prediction model has the following advantages: It adopts a traditional, mature prediction method, ensures calculation precision, and simultaneously considers the mechanism driving industrial water demand changes. Its advantages are especially pronounced for medium- and long-term
water demand prediction because it can establish scenarios based on actual conditions and apply the historical driving mechanism to the prediction. It is concluded that the model accurately determines the range of fluctuation of future water demands.

(2) This study first used the trend analysis and scenario methods to predict water demand for the output, technological, and structural hierarchies, respectively, and then formulated three water demand scenarios (high, medium, and low) corresponding to possible future industrial structures. The water demand for the structural hierarchy in these different scenarios was then predicted. The predictions of the three different hierarchies were superimposed, which defined the numerical range of future water demand, and obtained predictions for Tianjin’s overall industrial water demand. In this study, we used the mature GPM (1, 1) to verify the hierarchical predictions, and found that the error between the two models was less than 10%, thereby verifying the reliability of the hierarchical approach. The hierarchical prediction model can integrate the driving factors of water demand into the prediction mechanism and predict future water demand under different management scenarios. It is concluded that the hierarchical prediction method based on a refined Laspeyres decomposition model is more scientifically robust and effective than the models used at present.

(3) Tianjin’s industrial structure is moving toward greater water demand, which is expected to reach 580 million m$^3$, 776.38 million m$^3$, and approximately 1,093.8 million m$^3$ by 2015, 2020, and 2025, respectively. With appropriate restructuring of the industrial sector, Tianjin’s industrial water demand can be reduced to 660.91 million m$^3$ and 927.7 million m$^3$ by 2020 and 2025, respectively. Conversely, if industries with high water demand continue to expand, the industrial water demand of Tianjin may reach 1.0 billion m$^3$ and 1.493 billion m$^3$ by 2020 and 2025, respectively. It is predicted that Tianjin will soon face another water crisis. It is concluded that Tianjin needs to adjust its industrial structure, with the primary goal of water savings; and actively seek new sources of water in order to ensure the sustainable development of its economy.

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