Estimation and evaluation of auto-flocculated algae harvesting efficiency using the population balance in turbulence model in flotation process

Dong-Heui Kwak and Mi-Sug Kim

ABSTRACT

Algae are considered water pollutants because they form algal blooms in stagnant water. Algae harvesting technology, however, can help convert them into a useful industrial material like biomass. The core technique (flocculation) separates microalgae from other flocculants, allowing for the harvest of clean and pure algal biomass. This study aims to estimate and evaluate algal separation (removal or harvesting) efficiency (X) to concurrently obtain the objectives of algal bloom management and algal particle collection. To simulate algal separation by auto-flocculation (no flocculants) related flotation, the population balance in turbulence (PBT) model is used. Model simulations are conducted under optimal conditions provided by previous studies about the biological impact factors of algae, operating parameters of the flotation process, and so on. This modeling study determines the efficiency (X) of separating algae from the water body in the separation zone after forming auto-flocculated bubble–floc agglomerates by making them collide and attach to each other in the contact zone of the flotation tank. The X is examined as a function of size distribution of agglomerates and bubbles and of the number of initially injected bubbles. Optimal conditions for forming and harvesting the agglomerates may be found through further modeling studies.

Key words | algae harvesting, algal particle, auto-flocculation, bubble–floc agglomerate, flotation, population balance model

INTRODUCTION

Algae are known to cause water pollution and are especially problematic when algal blooms grow in stagnant water. Using algae harvesting technology, however, algal biomass can be transformed into useful industrial materials. Milledge & Heaven (2013) reviewed the microalgal harvesting techniques such as sedimentation, flocculation, flotation, centrifugation and filtration, or a combination of those. Among algal particle harvesting techniques, dissolved air flotation (DAF), a competitive process for solid–liquid separation, is a process in which flocs (or particles) are collided and attached to air bubbles (Phoochinda et al. 2004; Wiley et al. 2009). Edzwald (1995, 2010) emphasized that DAF is a superior method, having up to 99.8% removal of particles in the solid–liquid separation processes of wastewater treatment fields. This high efficiency rate is dependent, of course, on the conditions of coagulation and flocculation.

Over the last decade, increase in the popularity of DAF usage for algae separation by taking advantage of the low density of algae has revealed critical parameters including particle size, pH, zeta potential, and ionic concentration for auto-flocculation and flocculation with Na+, Mg2+, Ca2+, Al3+, Zn2+, chitosan, and so on (Zhang et al. 2015; Ge et al. 2015; Gorin et al. 2015; Pirwitz et al. 2015; Vandamme et al. 2015; Aljuboori et al. 2016; Lama et al. 2016; Ummalyma et al. 2016; Bhattacharya et al. 2017; Cassini et al. 2017; Cassini et al. 2017; Haver & Nayar 2017; Pérez et al. 2017). The efficiency of harvesting Chlorella zofingiensis via DAF at the different growth phases reached over 90% and increased with Al3+ dosage depending on dissolved organic matter (DOM); DOM cultures required
higher Al$^{3+}$ dosage compared to DOM-free cultures (Zhang et al. 2012). Based on those parameters, flotation model studies have been conducted to estimate the separation efficiency in a flotation tank (Kim et al. 2015; Kim & Kwak 2017).

Separation efficiency of auto-flocculated microalgae by DAF depends on the formation of algal floc–bubble agglomerates in the contact zone and the preservation of laminar flow in the separation zone. Hence, to obtain efficient and effective separation of microalgae in the separation zone, the bubble–floc agglomerates need to be well formed in the contact zone, and to improve the single particle collision-attachment efficiency, the number of bubbles properly attached on the floc must be improved by injecting the proper number of initial bubbles. The model in the contact zone can be classified into two types: white water bubble blanket model (WWBBM) suggested by Haarhoff & Edzwald (2004) based on Edzwald (1995), and flocculation type model (FTM) for heterogeneous flocculation without previously attached bubbles presented by Tambo et al. (1986) and Fukushi et al. (1995).

The FTM and WWBBM significantly contribute to our understanding of DAF, identifying important variables affecting design and operation (Edzwald 2010). Edzwald (2010) compared these two models as follows. Based on a single collector collision theory (SCC model) – which does not consider hydrodynamic interaction and particle effects – in laminar flow conditions, the WWBBM considers bubbles in the white water blanket existing at a dynamic steady state and at a high bubble concentration. The single collector efficiency approach in the WWBBM considers the effect of streamlines of flow diverting around the bubble and expresses Brownian diffusion using the convective–diffusion equation but differs from the flocculation trajectory approaches of Leppinen (1999).

The FTM is a kinetic model based on the population balance model of bubbles and particles in a turbulent flow condition (PBT model). The PBT model was formulated by Fukushi et al. (1995) and modified by Leppinen et al. (2001) for the attachment process with bubbles and flocs as flocculation in turbulent flow and it is used to explain the distribution of bubble–floc agglomerates in the contact zone and to calculate the rise velocity of bubble–floc agglomerates. Edzwald (2010) commented briefly that the PBT model employing the FTM is still studied for describing contact zone flotation performance (Matsui et al. 1998; Leppinen et al. 2001; Leppinen & Dalziel 2004; Han 2002; Holt et al. 2004; Jung et al. 2006; Kim & Kwak 2014a; Kim et al. 2015; Kim & Kwak 2017).

However, research effort to improve the algae removal technique is still ongoing. This study focuses on separation efficiency related to auto-flocculated microalgae by DAF to achieve two synchronous objectives: algal bloom management and algal biomass production. The purpose of this study is to provide optimal parameters and operation conditions to improve algal separation (removal or harvesting) efficiency by the flotation process (DAF) related to auto-flocculation (no flocculants), using the PBT model that describes by how much agglomerates of auto-flocculated algal floc and bubbles are formed in the contact zone. Model simulations are conducted under simulation conditions regarding biological impact factors of algae (zeta potential and particle size), operating parameters of the flotation process (bubble size, initial number of bubble, contact time, etc.), and so on.

**METHODOLOGY**

This study uses the PBT model to establish optimal parameters and operation conditions for improving separation efficiency of auto-flocculated algal flocs. This section consists of three subsections for more detailed methodology: model description of the PBT model, model design and application factors, and model simulation conditions.

**Conceptual model description: PBT model**

In Figure 1, the flotation tank consists of the contact zone and the separation zone, with separation efficiency mostly depending on agglomerates formed in the contact zone as a function of the auto-flocculation capacity of algae. Many agglomerates formed in the contact zone are transported into the separation zone and rise to the top of the water surface layer based on a balance between the bubble rising velocity (buoyancy) and the bubble falling velocity according to the downward water flow (gravity) in the separation zone.

The water, supersaturated with air, is injected into the contact zone of the flotation tank. The resulting pressure reduction releases air from the solution in the form of micro-bubbles. The mass of air released can be expressed in terms of bubble volume concentration. Bubble volume concentration is defined as the bubble number concentration multiplied by the average volume of a single class of bubbles with a volumetric mean diameter ($d_b$). These bubbles collide with the flocculated particles in the contact zone forming bubble–particle agglomerates. As a result of bubble attachments, the agglomerates rise in the separation zone due to the apparent density of the formed algal
floc–bubble agglomerates being reduced below that of water. The rise velocity of the algal floc–bubble agglomerate could be calculated, assuming Stokes drag conditions, i.e., Reynolds number of the algal floc–bubble agglomerate, \( \text{Re}_{fb} \leq 1.0 \).

According to Edzwald (2010), all model types in the contact zone begin with second order rate kinetics in the contact zone to describe the rate of particle change caused by collision and attachment to bubbles as a function of the rate coefficient (\( k_c \)), and the floc (\( N_f \)) and bubble (\( N_b \)) concentrations (Equation (1)).

\[
\frac{dN_f}{dt} = -k_c N_b N_f \tag{1}
\]

For describing the process of bubble–floc collision and attachment, the PBT model is formulated by counting the number of flocs attached to \( i \)-bubbles, \( N_{fb} \) at mixing time \( t \), considering that bubbles and particles form agglomerates with first order reaction in the contact zone considering constant initial \( N_b \). Thus, the kinetic equation in Equation (2) expresses a linear relationship when combining the flocs in the water with bubbles formed by the saturator.

\[
\frac{dN_f}{dt} = -(\alpha_{fb} \eta_T) A_b U_{fb} N_b N_f \tag{2}
\]

\( \alpha_{fb} \) = initial attachment efficiency (non-dimension)

\( \eta_T \) = collision efficiency (non-dimension)

\( t \) = time (sec)

\( A_b \) = the area of the released bubbles (m\(^2\))

\( U_{fb} \) = approach velocity between the floc and bubble (m s\(^{-1}\))

\( N_b \) = bubble concentration (number m\(^{-3}\))

\( N_f \) = floc concentration (number m\(^{-3}\))

To determine the initial efficiency of collision (\( \eta_T \)) and attachment (\( \alpha_{fb} \)) as input parameters of the PBT model, the PBT model links to a mathematical model, an SCC model described in Kim & Kwak (2014b) using theory of collision efficiency and trajectory analysis. Given that the settling velocity of a floc is very low compared to the rising velocity of a bubble, the approach velocity (\( U_{fb} \)) between each bubble and floc is briefly expressed as Equation (3) using Stokes’ law for a bubble size smaller than 150 \( \mu \)m; that is, the Reynolds number is less than 1 in the laminar zone:

\[
U_{fb} = \frac{g(\rho_f) d_b^2}{18\mu} \tag{3}
\]

where bubble diameter (\( d_b \)), algal floc density (\( \rho_f \)), viscosity of water (\( \mu \)) and acceleration of gravity (\( g \)).
Zeta potential of different types of gas bubbles (air, N₂ and CO₂) at varying pH (2.0–12.0) becomes more negative as pH increases up to 10 at which point the zeta potential starts to become less negative (Yang et al. 2001; Oliveira & Rubio 2011; Kwak & Kim 2015; Kim & Kwak 2017). More specifically, the positive value appears when pH is about 3 and the negative bubble zeta potential appears when pH is above 4.0. Over pH 7.0, the bubble zeta potential is strongly negative within the range of -31.25–36.88 mV. When examining the change in zeta potential corresponding to the surfactant type (cationic cetyltrimethyl-ammonium bromide (CTAB) and non-ionic polyoxyethylene glycol octylphenol ethers (Triton X-100)), the change of bubble zeta potential is less than 5 mV above pH 10.0 (Kwak & Kim 2015).

### Algal particle zeta potential

The flotation process considers solid (algal particle), liquid (water) and gaseous (bubble) phases. It conducts oxidation, hydration and hydrolysis, which change the properties of the algal surface due to two factors: interaction of water molecules with algal surface, and electrical double layers in the algal particle–bubble interaction or algal particle–water interaction. One of the important parameters of the electrical double layer is algal particle zeta potential depending on pH values and controlling the interaction between algal particle and water or between algal particle and bubble. Thus, the zeta potential needs to be controlled to reduce the particle distance to achieve flocculation.

Henderson et al. (2008) suggest that the particle zeta potential attached on the negative bubble surface in the flotation process is one of the major parameters, and the optimal zeta potential of the algal particle separation ranges between -8 mV and +2 mV. The typical range of the zeta potential is +10 to +200 mV, generally the flocculation effortlessly occurs under +20 to +30 mV, and the most effective flocculation occurs in the range of 0 ± 5 mV (Kim & Kwak 2014b). Kwak & Kim (2015) report that the zeta potential of algal particles is affected by pH and the number of cultivation days by auto-flocculation without coagulant additive. The more negative zeta potential of algal particles gradually becomes positive when forming auto-flocculation according to the algal (Anabaena sp.) growth as pH increases between 8 and 9. Also, the flotation efficiency is improved as the surfactants generate a small bubble size from the saturator.

### Model simulation condition

Model simulations, conducted under optimal conditions described in Tables 1 and 2, are provided from previous studies (Millero 1985; Yoon & Yardan 1986; Reid et al. 1987; Levine et al. 1991; Ackler et al. 1996; Liers et al. 1996; Shawwa & Smith 1998; Han & Dockko 1999; Yoon 2000; Rodrigues & Rubio 2003; Taki et al. 2004; Kim & Kwak 2014a). The PBT model linked to the SCC model is written with the language software MATLAB (version 7.1) and performed numerical trajectory analysis for collision-attachment efficiency between algal particle and water or between algal particle and bubble.
Several simulations were run to estimate efficiency (X) of algal separation under the conditions as described in Tables 1 and 2. Table 1 provides input data of DAF operating parameters that may be used for running the PBT model. The bubble size was in the range of 10–300 μm according to Rodrigues & Rubio (2003) and most bubbles in the separation zone were distributed from 50 to 150 μm as a function of tank length and depth according to Leppinen & Dalziel (2004). Exceeding the maximum bubble size of 150 μm reduces the separation efficiency in the separation zone by disturbing the laminar flow. Based on those researches, bubble sizes are selected from 40 to 150 μm for simulations. In Table 2, the auto-flocculated algal floc size (df) ranges from 8 μm to 1,650 μm given that the ratios of the auto-flocculated algal floc sizes and bubble sizes range from 0.2 to 11 based on bubble sizes (db) ranging from 40 to 150 μm. Using input data described in Tables 1 and 2, the model simulation is performed to estimate and evaluate the X in the contact zone of the flotation tank as a result of the auto-flocculated floc size (df) and bubble size (db), and the initial number of bubbles (N0) inserted.

### RESULTS AND DISCUSSION

#### Effect of auto-flocculated algal floc size

The flotation process presents the opportunity to form auto-flocculated bubble–floc agglomerates, which are formed by collision and attachment in the contact zone, to separate algae particles from the water body once they reach the separation zone. Separation efficiency is examined as a function of size distribution of agglomerates between auto-flocculated flocs and bubbles (dfb) and the initial number of injected bubbles (N0).

The PBT model simulates conditions as follows: the auto-flocculated algal floc with diameters (df) of 50, 100, 125, 200, and 300 μm; the initial number of bubbles (N0) being 105 m−3; and bubble diameters (db) of 150 and 50 μm. The simulation results are presented in Figure 2. When using the larger bubble (db = 150 μm) in turbulent flow as shown in Figure 2(a), the lowest X occurred in the trial with the smallest df, while the highest X resulted when using the largest df. In the trials of df with 100, 125, and 200 μm, which are similar to the bubble radius, X values lie between the highest and the lowest but they do not follow the expected trend; instead, X decreases with

### Table 1 | Information of parameters used when simulating PBT model linked with SCC model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
<th>Unit</th>
<th>Reference</th>
<th>Selected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algae diameter, df</td>
<td>2–200</td>
<td>μm</td>
<td>Levine et al. (1999)</td>
<td>Table 2</td>
</tr>
<tr>
<td>Ion strength, I</td>
<td>0.01–0.00001</td>
<td>mole m−3</td>
<td>Millero (1985)</td>
<td>0.005</td>
</tr>
<tr>
<td>Viscosity, μ</td>
<td>0.001</td>
<td>kg m−1 s−1</td>
<td>Reid et al. (1987)</td>
<td>0.001</td>
</tr>
<tr>
<td>Hydrophobic constant, K132</td>
<td>6.30 × 10−19</td>
<td>J</td>
<td></td>
<td>6.30 × 10−19</td>
</tr>
<tr>
<td>Water density, ρw</td>
<td>1,000</td>
<td>kg m−3</td>
<td></td>
<td>1,000</td>
</tr>
<tr>
<td>Bubble density, ρb</td>
<td>1.174</td>
<td>kg m−3</td>
<td>Liers et al. (1996)</td>
<td>1.174</td>
</tr>
<tr>
<td>Algae density, ρf</td>
<td>473.4</td>
<td>kg m−3</td>
<td></td>
<td>473.4</td>
</tr>
<tr>
<td>Initial bubble number, N0</td>
<td>1 × 107</td>
<td>number m−3</td>
<td>Kim &amp; Kwak (2017)</td>
<td>1 × 105</td>
</tr>
<tr>
<td>Initial algal particle number, Nf</td>
<td>1 × 102</td>
<td>number m−3</td>
<td>Kim &amp; Kwak (2017)</td>
<td>1 × 102</td>
</tr>
<tr>
<td>Contact time, t</td>
<td>60–180</td>
<td>sec</td>
<td>Shawwa &amp; Smith (1998)</td>
<td>30</td>
</tr>
</tbody>
</table>
increasing \( d_f \). Otherwise, when using the smaller bubble size \( (d_b = 50 \, \mu m) \), \( X \) for this simulation, as shown in Figure 2(b), improves due to the increasing of algal floc size given that the bubble size is always smaller than the algal floc size, \( d_b < d_f \). Hence, the larger auto-flocculated algal flocs may be removed or harvested more effectively than the smaller ones due to the small bubbles. \( X \) from large bubbles is far better than that of small bubbles but it fluctuates irregularly depending on the algal floc size. The results show that the obtained algae separation efficiency agrees with investigations reported in the literature that the sizes of algae affect the removal process of bubble flotation (Bui et al. 2015).

**Effect of bubble size**

Based on the simulation results examining the effect of the algal floc size, the next simulation is required to estimate the effect of the bubble size on \( X \). Thus, the simulation is conducted with several different bubble diameters (75, 150, 175, and 200 \( \mu m \)), the initial number of bubbles \( (N_{0b}) \) being \( 10^5 \, m^{-3} \), and two different auto-flocculated algal floc diameters (250 and 50 \( \mu m \)) as shown in Figure 3. When \( d_f \) equals 250 \( \mu m \), which is greater than \( d_b \), similar \( X \) values in Figure 3(a) are determined for the different bubble sizes, with the greatest effectiveness estimated by the bubbles size of 175 \( \mu m \) compared to all bubble sizes applied in this simulation. If \( d_f \) equals 50 \( \mu m \) and is less than \( d_b \), \( X \) increases as bubble size increases as shown in Figure 3(b). The highest \( X \) among the entire simulation resulted when applying the largest bubble size \( (d_b = 200 \, \mu m) \) and the smaller floc size \( (d_f = 50 \, \mu m) \). All bubble diameters in this simulation, however, are greater than 120 \( \mu m \), which will cause turbulence flow in the separation zone and reduce separation efficiency. These results concur with a previous study in that the highest efficiency is obtained when the floc size is larger than the bubble size in the DAF process followed by chemical coagulation (Han et al. 2007).

**Effect of ratio between \( d_b \) and \( d_f \)**

In addition to knowing that the size of the bubbles and algal flocs influences \( X \), it is necessary to understand the interaction between the bubbles and algal flocs regarding \( X \). The next simulation was conducted to comprehend how the two components relate to each other. When the auto-flocculated algal flocs make contact with the initial bubble number of \( 10^5 \, m^{-3} \) in various bubble diameters...
(\(d_b\)) of 40, 60, 80, 100, 120, and 150 \(\mu m\) for a duration of 30 s, and ratio (\(R_{fb}\)) of auto-flocculated algal flocs (\(d_f\)) to bubbles (\(d_b\)) ranging from 0.2 to 11, we can estimate various values of \(X\) using the PBT model based on the SCC model as shown in Figure 4. Figure 4(a) shows \(X\) is higher when the \(d_f\) is smaller or larger than the \(d_b\). In Figure 4(b), \(X\) is higher when \(R_{fb}\) decreases when \(d_f < d_b\), while it increases when \(d_f > d_b\). When \(d_f\) and \(d_b\) are in close proximity, \(X\) reaches the minimum, near zero. Hence, \(X\) is more effective if \(d_b\) is much bigger than \(d_f\) or if \(d_f\) is much larger than \(d_b\), that is, \(R_{fb} < 1\) or \(R_{fb} > 1\). When \(R_{fb}\) is greater than 6, the most effective separation efficiency is determined for the agglomerates of the auto-flocculated algal flocs and bubbles with values highlighted in Table 2. The effective bubble sizes are distributed from 80 to 150 \(\mu m\) but larger bubble sizes (\(d_b > 125 \mu m\)) generate turbulent flow, which disturbs the flotation process. If smaller bubbles are used, a greater number of bubbles and contact time are required to approach the effective separation efficiency.

Figure 2 | Effect of auto-flocculated algal floc sizes, \(d_f\), on separation efficiency with given bubble sizes: (a) larger bubble (\(d_b = 150 \mu m\)) and (b) smaller bubble (\(d_b = 50 \mu m\)) in the turbulent flow.
Comparison of contact time

The PBT model simulation results indicate that X improves with contact time change. Figure 5 shows the comparison of X values as a function of $R_{fb}$: (a) for a contact time of 1 s and (b) for a contact time of 30 s. When comparing the X values depending on the contact time, 1 s and 30 s, the X values show a similar tendency of change with $R_{fb}$, although the X values are vigorously improved after contact for the duration of 30 s compared to those for 1 s. For the contact time of 30 s, the X values of the higher $R_{fb}$ reach over 90% for all bubble sizes except 40 and 60 μm. Thus, the effectiveness of X is dependent on longer contact time to form the agglomerates of the auto-flocculated algal flocs and bubbles.

Figure 5 Effect of bubble sizes, $d_b$, on separation efficiency with given auto-flocculated algal floc sizes: (a) larger than bubble sizes ($d_f = 250$ μm) and (b) smaller than bubble sizes ($d_f = 50$ μm).
Effective size distribution for maximum X

When the auto-flocculated algal flocs contact with the initial bubble number of $10^5 \text{ m}^{-3}$ in various bubble diameters for 30 s, each bubble size has a specific floc size to reach the maximum X as shown in Figure 6; the most effective floc diameters are 160, 300, 480, 700, 960, and 1,500 μm for the bubble diameters of 40, 60, 80, 100, 120, and 150 μm, respectively. The maximum of X increases with bubble size increase for effective floc size. However, X flocculates at a certain floc size, $d_f = 120 \mu m$, as a function of bubble size even though the values of X are very low. For $d_b$ less than 100 μm, X increases as bubble size increases but it decreases when the bubble size is greater than or equal to the certain floc size.

Figure 4 | Comparison of separation efficiency depending on various sizes of bubbles and algal flocs: (a) expressed by algal floc size ($d_f$) and (b) expressed by ratio $R_{fb}$. 

(a) 

(b)
Effect of the initial number of bubbles for improving efficiency

Based on the above simulation results, the minimum $X$ ($X_{\text{min}}$) of the agglomerates formed by $d_b$ (40 μm) and $d_f$ (8 μm) with $N_{b0} = 10^5$ m$^{-3}$ was chosen to examine the effect of the initial number of bubbles, $N_{b0}$. In this study, $N_{b0}$ is inserted from $10^6$ to $10^9$ m$^{-3}$ for each simulation to compare with the $X_{\text{min}}$ when using $N_{b0} = 10^5$ m$^{-3}$. In Figure 7, the $X_{\text{min}}$ is obviously increased up to 100% by increasing $N_{b0}$ from $10^6$ to $10^9$ m$^{-3}$. Therefore, the separation efficiency of the auto-flocculated algae is influenced not only by the size of the bubbles and/or algal flocs but also the initial number of bubbles inserted.
CONCLUSIONS

This study conducted model simulations using the PBT model under optimal conditions to estimate and evaluate algal separation (removal/harvesting) efficiency of flotation related to auto-flocculation (no flocculants) for algal bloom management and algal biomass production. The model simulation findings are as follows.

1. Because more bubbles collide and attach on the surface of larger flocs, larger auto-flocculated algal flocs with the small bubbles may be separated more effectively than smaller ones with the small bubbles. The algal separation efficiency has been estimated using auto-flocculated algal floc with five different diameters, 50, 100, 125, 200, and 300 μm, and bubbles with two different diameters of 150 and 50 μm. For the bubble size of 50 μm, larger flocs may be more effective than smaller flocs. For flocs over 200 μm,
the efficiency of the larger bubble (150 μm) is slightly higher than that of the smaller one (50 μm). Exceeding the maximum bubble size of 150 μm reduces the separation efficiency in the separation zone by disturbing the laminar flow in the separation zone. Bubble sizes greater than 150 μm are the limiting value of the laminar flow on the basis of Stokes’ law. Beyond 150 μm, the rising velocity becomes rapid and causes a transition flow or turbulent flow; that is, Reynolds number becomes greater than 1. For Reynolds number > 1, the separation efficiency is reduced as it disturbs the laminar flow in the separation zone.

2. This study examined two cases, the traditional DAF case (Rfb > 1) and the opposite case to the traditional one (Rfb < 1) to find the flotation efficiency under auto-floculation condition. The traditional case used small bubbles, with respect to the size of the particles to be removed. The opposite case considered that the auto-flocculated algal floc sizes are much smaller than the flocs formed by chemical flocculants and that they are smaller than the bubble sizes. When examining the influence of the relationship between the bubbles and the algal flocs on efficiency, the traditional case proves to be much more effective than the opposite case if a significant size disparity exists between the flocs and the bubbles, i.e., \( R_{fb} < < 1 \) or \( R_{fb} > > 1 \).

3. The maximum efficiency increases as the bubble size increases, given that the bubble is coupled with the effective floc size; that is, \( R_{fb} \) increases for \( d_f > d_b \). The maximum efficiency may be improved by setting the initial conditions as a higher initial number of injected bubbles and longer contact time (\( t_c \)).

4. Also, the effectiveness of X can be controlled by lengthening the contact time for the auto-flocculated algal flocs and bubbles to form agglomerates.

5. Finally, the efficiency of removing or harvesting auto-flocculated algae is influenced not only by bubble size and/or floc size but also the number of initially inserted bubbles. A higher initial number of bubbles may result in improved efficiency.

In conclusion, the PBT model simulation gives the effective separation efficiency of auto-flocculated microalgae given the optimal conditions of bubble size, algal floc size, contact time, and initial number of bubbles in the contact zone of the flotation tank. Continuous PBT modeling studies may find more optimal conditions for forming and harvesting agglomerates of the auto-flocculated algal flocs and bubbles based on algal characteristics.

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