Biogas from municipal solid waste landfills: a simplified mathematical model
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ABSTRACT

Municipal solid waste (MSW) landfills now represent one of the most important issues related to the waste management cycle. Knowledge of biogas production is a key aspect for the proper exploitation of this energy source, even in the post-closure period. In the present study, a simple mathematical model was proposed for the simulation of biogas production. The model is based on first-order biodegradation kinetics and also takes into account the temperature variation in time and depth as well as landfill settlement. The model was applied to an operating landfill located in Sicily, in Italy, and the first results obtained are promising. Indeed, the results showed a good fit between measured and simulated data. Based on these promising results, the model can also be considered a useful tool for landfill operators for a reliable estimate of the duration of the post-closure period.

Key words | biogas modelling, flux chamber, landfill gas, landfill lifetime

INTRODUCTION

Since the 1970s, municipal solid waste (MSW) landfills have been used as large biological reactors where the organic fraction of MSW undergoes anaerobic digestion producing gas and liquid emissions (Owens & Chynoweth 1993; Lissens et al. 2001; Imhoff et al. 2007; Di Bella et al. 2012). The biodegradable portions of organic compounds are hydrolyzed and eventually methanized, producing landfill gas (LFG) composed by methane (CH₄), carbon dioxide (CO₂) and trace components. MSW landfills have been identified as one of the most important anthropogenic sources of CH₄ emissions (Aronica et al. 2009; Di Bella et al. 2011; Di Trapani et al. 2013). This aspect is of particular concern, since CH₄ is a greenhouse gas with a global warming potential 28 times that of CO₂ (Kumar et al. 2016; IPCC 2007; El-Fadel et al. 2012). The utilization of the LFG would both prevent harmful atmosphere emissions and provide a source of green energy (Lee et al. 2017; Mboowa et al. 2017).

In order to optimize the sizing and the operation of the LFG extraction network in a sanitary landfill it is necessary to obtain reliable forecasts of LFG production from the beginning of a site’s operation until its closure and even in the post-closure period. This challenge has been thoroughly addressed in the technical literature by Spokas et al. (2006) and Chen et al. (2009), for example. In this context, mathematical models can represent a suitable tool for obtaining reliable estimates of LFG production over time, showing the effects of different compositions of the disposed waste as well.

In the technical literature, many examples of mathematical models have been reported for the estimation of LFG production (Panepinto et al. 2013; Fei et al. 2016). Such models can be effectively applied for predictive purposes, but they must be validated for each specific case to properly evaluate the productivity and profitability of the energy systems that use waste from traditional landfills (Arena et al. 2005; Meima et al. 2008).

Although in principle organic matter degradation/gas production is a complex process characterized by different kinetic degradation rates, the most popular models are based on a simple first-order biodegradation kinetics model. This is a suitable approximation when one of the stages is definitely rate determining; moreover, such models have the merit of being easy to use and usually provide a good fit with field data.

Among the models based on first-order reaction kinetics, it is worth mentioning the one by Andreottola & Cossu (1988). In this model, the biodegradable organic carbon content in the solid phase is obviously considered the fundamental input parameter and different degradation rates were allowed. The model takes into account the
influence of structural and operational features of the landfill, such as humidity, bulk density and size of waste, on the gas-generation process.

The model of El-Fadel et al. (1996) takes into account basic concepts from microbiology, chemistry and physics to simulate the production and transport of LFG and heat in a landfill body. This model shows the distribution of LFG and its diffusion through the landfill. Due to the complexity of this model, its application requires the knowledge of many parameters, which are often difficult to establish. Nevertheless, the ‘temperature’ parameter included in this model is one of the fundamental input parameters influencing on the intensity of processes.

The model of Hashemi et al. (2002) evaluates the gas flow split into two pathways: through the upper layer or cover and the extraction wells. This model is also very complex and difficult to apply because of the many local parameters required (more than 15).

The model of Manna et al. (1999) also considers the temperature as an input parameter for the calculation of the activation energy, necessary for starting the biodegradable processes. Another notable feature is that this model takes into account the waste layers density, which changes over time, so it represents the dynamics of the landfill processes from the beginning of the operation stage until the end of biogas production processes.

Recently, other models have been proposed, taking into account operation actions such as the installation of the covering cap and the recirculation of leachate (Berge et al. 2003; Sinan Bilgili et al. 2009). Indeed, it is worth noting that one must differentiate between the algorithms that enable the evaluation of LFG production and the commercial software that allow users to also take into account the different landfill management strategies (Sharff & Jacobs 2006; Wangyao et al. 2010). In this sense, the application results of several commercial software packages have been reported in the technical literature, such as, for instance, GasSim (Gregory et al. 1999, 2005), LandGEM (USEPA 2005) and IPCC (2006) models. Nevertheless, among the models summarized above and several others reported in the technical literature, only a few have been applied to real landfill sites. Indeed, the proper application of models is strongly site specific, depending in particular on the waste moisture content, the thermal profile inside the landfill body, the density profile with the landfill depth and the covering system features. This is a crucial aspect, since the need to consider site-specific landfill operational features requires the collection of a large amount of experimental data, which is not easy to acquire.

Bearing these considerations in mind, the present paper presents an application study of a mathematical model able to simulate the amount and rate of LFG production from an MSW landfill, inspired by one of the conceptual models cited above – Manna et al. (1999). The model consists of two parts: material balance and energy balance. The equations of material balance take into account the amount of biodegradable carbon fraction in the different waste layers involved in the anaerobic reactions, as well as the changes of density of the layers over time. Equations of energy balance calculate temperature changes over time, resulting from the anaerobic processes and thermal exchanges between layers and towards the landfill boundaries.

The model was first run and tested with fictitious data, then applied to an actual operating landfill located in Sicily, in Italy, as a case study. The main goal was to compare the model results with experimental data provided by the landfill operator in order to validate the model and to optimize the model parameters.

MATERIALS AND METHODS

Model description

As mentioned previously, the paper presents a mathematical model application aimed at simulating the LFG production in an MSW landfill. The model was inspired by the conceptual model proposed by Manna et al. (1999). Different aspects have been taken into account: the temperature profile, by applying a thermal balance within the landfill body, which influences the organic carbon that is available for the conversion process; the model also considers the material conversion for the gas generation process. The mathematical model is able to simulate landfill behavior under three different periods:

- Period I ($t < t_{\text{cult}}$): MSW is discharged into a cell and consequently the landfill depth increases as a function of time ($L(t)$). It is possible to distinguish two zones within the waste layer: an inhibition zone and a reaction zone. Briefly, in the inhibition zone, characterized by a residence time $t < t_{\text{in}}$ and $0 < z < L_{in}(t)$, no biochemical reactions will occur (as outlined in more detail below), whereas in the reaction zone $L_{in}(t) < z < L(t)$ LFG production will occur (Figure 1(a)).
- Period II ($t_{\text{cult}}< t < t_{\text{cult}} + t_{\text{in}}$): active landfilling has ended and the cell is covered with a clay layer. The whole depth is constant and the landfill can be divided...
into: clay layer, inhibition zone and reaction zone (Figure 1(b)).

- Period III ($t > t_{cut} + t_{in}$): there is no inhibition zone ($L_{in} = 0$) since each layer is characterized by a residence time higher than inhibition time (Figure 1(c)).

In particular, the inhibition zone has been evaluated according to the lag time $t_{in}$: the time from the placement of waste to the beginning of significant gas production. The absence of significant biochemical reactions could be due to different aspects (including initial microbial acclimation period, biodegradable substrate unavailability, etc.). In the present study, the lag time was set at 1 year, based on Cossu et al. (1996).

### Mass balance equations

The model is able to analyze the landfill body as a batch reactor and is based on first-order biodegradation kinetics for the organic fraction of MSW, according to the following equation:

$$\left(\frac{\partial \omega_i}{\partial t}\right)_z = -A_k \omega_i - \left(\frac{\partial \omega_i}{\partial z}\right)_t \left(\frac{\partial z}{\partial t}\right)_m$$

(1)

where $\omega_i$ is the biodegradable carbon fraction (kg kg$^{-1}$ MSW), $A_k$ is a dimensionless coefficient, $t$ is the simulation time (s), $z$ is the depth from the landfill top (m) and $k$ is the kinetic constant ($s^{-1}$), expressed as:

$$k = k_0 e^{-\frac{E}{R} \left(1 - \frac{T_0}{T_w}\right)}$$

(2)

where $E$ is the activation energy ($J/(molK)$), $R$ is the ideal gas constant ($J/(molK)$), $T_0$ is the initial temperature value (K) and $T_w$ is the waste temperature (K).

Equation (1) is valid for landfill depths in the anaerobic reaction zone ($L_{cut}(t) < z \leq L(t)$) and depends on the variation of depth $z$ as a function of time according to:

$$\left(\frac{\partial z}{\partial t}\right)_m = \phi_m(t) / A \left(\frac{\rho_w(z, \mathcal{Z})}{\rho_w(z_0, \mathcal{Z})}\right)$$

(3)

The landfill depths are evaluated on the basis of the amounts of disposed waste:

$$m_0(t) = \int_0^t \phi_m(t) dt = A \int_{z_0}^{L(t)} \rho_w(z) dz$$

(4)

$$m_{in}(t) = \int_{t_{in}}^t \phi_m(t) dt = A \int_{z_0}^{L_{in}(t)} \rho_w(z) dz$$

(5)

The waste components that have been taken into account for gas production are: carbohydrates, fats and proteins. The first degradation process is the chemical/ enzymatic hydrolysis of organic matter. Subsequently, the hydrolyzed compounds are subjected to biochemical transformations with volatile fatty acid (VFA) production and then metabolized with LFG production.

LFG production strictly depends on waste composition, moisture content, waste temperature and density. Referring to waste composition, the organic carbon fraction and the total available organic carbon fraction for the component $i$...
has been assessed by the following equation, which takes into account its strict dependence on temperature values, according to Tabasaran (1982):

\[ \omega_i = [0.014 \cdot (T_w - 273.15) + 0.28] \cdot \omega_{0i} \]  

(6)

where \( \omega_{0i} \) is the total organic carbon fraction of biodegradable component \( i \) (kg/kg MSW), while the other symbols have been already defined.

The variation of waste density has been expressed according to the following expression, only depending on depth \( z \):

\[ \rho_w = \rho_0 + (\rho_{\infty} - \rho_0) \cdot \frac{z}{z + \beta} \]  

(7)

where \( \rho_0 \) is the starting density value (kg/m\(^3\)), \( \rho_{\infty} \) the maximum density value corresponding to an infinite specific load (kg/m\(^3\)) and \( \beta \) a numeric coefficient obtained after a calibration procedure.

**Thermal balance equations**

As mentioned previously, the model is able to simulate temperature variations due to biochemical reactions within the landfill body. In detail, the energy balance has been computed by the following equation, which is valid into the anaerobic reaction zone (\( L_{in}(t) < z \leq L(t) \)):

\[ \rho_w C_{pw} \left( \frac{\partial T_w}{\partial t} \right)_z = \lambda_w \left( \frac{\partial^2 T_w}{\partial z^2} \right)_t - \frac{U(t)P}{A} (T_w - T_{ground}) + \rho_w n_{\text{gas}} (-\Delta H) \]  

(8)

where \( P \) is the perimeter of the cell, \( n_{\text{gas}} \) is the gas volume producible per kilogram and per second (Nm\(^3\)/kg MSW/s), \(-\Delta H\) is the enthalpy of anaerobic reaction (J/Nm\(^3\)), \( U(t) \) is the overall heat transfer coefficient (W/m\(^2\)/K) and \( C_{pw} \) is the specific heat (J/kg/K). The other symbols have been already defined.

Conversely, for the inhibition zone (\( z_0 < z < L_{in}(t) \)), the energy balance can be expressed as:

\[ \rho_w C_{pw} \left( \frac{\partial T_w}{\partial t} \right)_z = \lambda_w \left( \frac{\partial^2 T_w}{\partial z^2} \right)_t - \frac{U(t)P}{A} (T_w - T_{ground}) \]  

(9)

For the superficial zone (\( 0 < z \leq z_0 \)), characterized by the presence of the clay layer, the following equation can be applied:

\[ \rho_c C_{pc} \left( \frac{\partial T_c}{\partial t} \right)_z = \lambda_c \left( \frac{\partial^2 T_c}{\partial z^2} \right)_t - \frac{U(t)P}{A} (T_c - T_{ground}) \]  

(10)

**Model application**

Referring to the numerical formulation, the solution of the equations for the material balance was assessed by an explicit finite difference. Specifically, the combination of Equations (1) and (3) gives:

\[ \omega_i^{k+1} = \omega_i^k + \left[ \frac{-A_k \rho_0}{\Delta \omega_i} \right]^{k-1/2} \left[ \frac{\theta_m(t)}{A \cdot (\rho_{\infty} + z_l \rho_i \cdot \rho_{\infty} - \rho_{0i} - \rho_i \cdot \rho_{\infty} - \rho_{0i})} \right]^{k} \cdot \Delta t \]  

(11)

The following boundary and continuity conditions have been imposed, with the aim of taking into account the different mechanisms of energy transfer. In detail, the model imposes two continuity conditions:

- the first one is at the waste–waste interface (between the active waste, subjected to biochemical processes, and non-active waste, in the inhibition zone) that is valid for a depth equal to \( L_{in}(t) \):

\[ \frac{\partial T_w}{\partial z} \bigg|_{z=L_{in}(t)} = \frac{\partial T_w}{\partial z} \bigg|_{z=L_{in}(t)} \]  

(12)

- the second one at the waste–clay layer interface, that is valid for time periods longer than the cultivation time \( t_{cul} \):

\[ \frac{\partial T_c}{\partial z} \bigg|_{z=z_0} = \frac{\partial T_w}{\partial z} \bigg|_{z=z_0} \]  

(13)

**Figure 2** shows a flow chart of the model to enable better understanding of it.
The boundary conditions that have been imposed are:

- at the waste–ground interface:
  \[ U(t) \cdot \left( T_w(z=L) - T_w(z=0) \right) = -\frac{\lambda_w}{\partial z} \frac{\partial T_w}{\partial z} \bigg|_{z=L(t)} \]  
  (14)

- at the waste–air interface (valid for time periods shorter than the cultivation time \( t_{cult} \)):
  \[ -\lambda_w \frac{\partial T_w}{\partial z} \bigg|_{z=0} = h_{air} (T_{air} - T_w(z=0)) \]  
  (15)

- at the air–clay interface (valid for time periods longer than the cultivation time \( t_{cult} \)):
  \[ -\lambda_c \frac{\partial T_c}{\partial z} \bigg|_{z=0} = h_{air} (T_{air} - T_c(z=0)) \]  
  (16)

Tables 1 and 2 summarize the solutions of the equations for the energy balance by means of explicit finite difference equations, for the waste layer and the clay layer, respectively.

The model parameters used for model application are summarized in Tables 3–5.

The landfill site

The case study landfill site is located in south-central Sicily at about 260 m above sea level (Figure 3). It covers an overall area of 18 ha. At present, it is composed of five disposal cells (named VE, V1, V2, V3, V4), only one of which (V4) is in the operational phase, whereas the remaining cells are in the post-operational phase and permanently covered with a multi-layered covering system. The mathematical model was applied to cell V3, characterized by a volume of 1,240,000 m³ and in operation from 2004 to 2011. The amounts of disposed waste (model input), the LFG emissions relating to 2012 and 2013, and the LFG recovery data have been provided by the owner of the landfill. Table 6 shows the waste quantities disposed of in cell V3. Moreover, direct measurements of methane flux have been carried out by the authors in cell V3 in two different field gathering campaigns, as outlined in the following section.

Direct measurement of methane flux

The methane flux emission measurements were carried out by means of the flux chamber (static non-stationary)
method. Briefly, a total of 191 and 126 sampling points were measured in the cell V3 during in the experimental campaigns carried out in September 2014 and November 2015, respectively. In order to increase the experimental dataset available for the present study, historical data have been retrieved by the landfill operator, referring to 2012 and 2013 (previous data would be not of interest, since the landfill was still in the operational period and only a portion of the waste would be active in the LFG production process).

Figure 3 depicts the aerial view of the landfill and, as an example, the detail of the sampling points during the campaign of September 2014. The surface methane flow was determined by measuring the temporal change in methane concentration inside the chamber (LANDBOX HV30, LabService Analytica s.r.l.), using a portable flame ionization detector (Telegan Gas-Tec®), connected to a notebook for instantaneous data recording. The flux chamber had a volume of 0.026 m³ and covered a 0.08 m² surface area on the ground; it was equipped with a small fan for gas mixing in the internal volume and it was properly sealed to the ground.

Methane fluxes were evaluated using the following expression:

$$Q = \frac{V}{A} \cdot \frac{dc}{dt}$$

(17)

where $Q$ is the CH$_4$ flux (mg CH$_4$ m$^{-2}$ s$^{-1}$); $V$ (m$^3$) and $A$ (m$^2$) are the volume and footprint of the flux chamber respectively; $c$ is the CH$_4$ concentration (mg CH$_4$ m$^{-3}$) and $t$ represents the time step (s). More specifically, the run time for each measurement was based on a flattening of the concentration/time curve. The temporal acquisition frequency was set to 1 s, since the expected fluxes were not so high. For further details on the procedure, the reader is referred to Di Trapani et al. (2013).
It is worth noting that the number of required sampling points, depending on the size of the investigated area, was evaluated by means of the following expression (USEPA 1986):

\[ n = 6 + 0.15 \cdot \sqrt{Z} \]  

(18)

where \( n \) is the number of field measurements and \( Z \) is the size of the investigated area, expressed in \( m^2 \).

With the aim of comparing the model results with experimental data, the following balance equation was applied, according to Jacobs & Scharff (2001):

\[ P = E + R + O \]  

(19)

where the term \( P \) (production) is the methane produced by the anaerobic digestion of organic matter; \( E \) (emission) is the diffuse methane emission from the landfill surface; \( R \) (recovery) is the methane emission conveyed by the gas collection system (measured by the landfill operator) and \( O \) (oxidation) represents the methane oxidized when crossing the covering system of the landfill.

As mentioned previously, \( E \) was measured directly by means of the flux-chamber method, \( R \) data was provided by landfill operator, whilst \( O \) was set to the default value of 10%, according to Di Bella et al. (2011).

RESULTS AND DISCUSSION

Comparison between model result and experimental data

The model was applied to a real solid waste landfill to calibrate it by employing real data of methane gathered from the field campaigns. Specifically, a semi-automatic model calibration was carried out adopting a minimization of differences between measured and simulated values by means of the conjugate gradient.

Table 7 summarizes the field investigations, showing the resulting values of the most commonly used statistical indices. The CV (coefficient of variation) values confirmed the high spatial variability of the instantaneous emission rates for both campaigns.

In the different experimental campaigns, the highest emission zones were located in some sampling points close to LFG collection wells, which were characterized by faults between the liner and the well head, as previously mentioned.

Referring in particular to the field campaign carried out in 2014, starting from the instantaneous flux values, the overall methane emission from the investigated area was derived, with an average value as almost equal to \( 0.74 \times 10^4 \) Nm\(^3\)/month. Concerning the recovery data (measured values) provided by the landfill owner, it was \( 5.04 \times 10^6 \) Nm\(^3\)/month. Methane subjected to oxidation through the superficial cover soil (O) was \( 0.07 \times 10^4 \) Nm\(^3\)/month.

Table 6 | MSW amounts disposed of year by year

<table>
<thead>
<tr>
<th>Year of disposal</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>( 10^3 ) kg</td>
<td>10,201</td>
</tr>
<tr>
<td>2005</td>
<td>( 10^3 ) kg</td>
<td>64,246</td>
</tr>
<tr>
<td>2006</td>
<td>( 10^3 ) kg</td>
<td>72,061</td>
</tr>
<tr>
<td>2007</td>
<td>( 10^3 ) kg</td>
<td>155,148</td>
</tr>
<tr>
<td>2008</td>
<td>( 10^3 ) kg</td>
<td>228,283</td>
</tr>
<tr>
<td>2009</td>
<td>( 10^3 ) kg</td>
<td>311,985</td>
</tr>
<tr>
<td>2010</td>
<td>( 10^3 ) kg</td>
<td>325,815</td>
</tr>
<tr>
<td>2011</td>
<td>( 10^3 ) kg</td>
<td>125,833</td>
</tr>
</tbody>
</table>
Therefore, by applying a mass balance, it was possible to evaluate the net methane produced, which was $5.12 \times 10^6 \text{Nm}^3/\text{month}$.

The latter was compared with the simulated value deriving from the mathematical model, referring to the specific month of investigation. The results obtained are summarized in Table 8.

Figure 4 shows the CH$_4$ production curve, obtained using the model, as well as the CH$_4$ production in the months of investigation, achieved through mass balance equation (Equation (19)), from measured data.

Figure 4 shows a good fit between simulation results and methane production evaluated with experimental data, even if the model result is slightly higher than the experimental data. This result is in agreement with previous data, which have highlighted that mathematical models usually give an overestimate of LFG production (Cossu et al. 1996). Nevertheless, the proposed model can be applied for reliable estimates of LFG production, thus representing a useful tool for landfill operators to evaluate a reliable duration of the post-closure period. The estimated LFG production increases until cell V3 is characterized by active landfilling, while afterwards there is a sudden production depletion; this behavior is consistent with previous findings. However, it has to be stressed that despite the good reliability of the model, further validation for a longer period (multi-year analysis) should be provided for future applications.

**CONCLUSIONS**

This paper presents the results of a mathematical model application aimed at predicting LFG production from an MSW landfill. The model was applied to an actual operating landfill located in Sicily and its results were compared with...
experimental data. In particular, LFG recovery data were provided by landfill operator, whereas data of diffuse emission from the landfill surface were acquired directly by the authors. The results obtained showed a good fit between model simulation and experimental data, with only a slight deviation of model predictions compared with experimental data. This has highlighted the importance of direct measurements for the calibration/validation of the proposed model. Nevertheless, the proposed model can be applied for reliable estimates of LFG production, thus representing a useful tool for landfill operators to evaluate a reliable duration of the post-closure period.

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