Numerical study on dynamic behavior of entrapped air in a partially filled pipe
Hengliang Guo, Ye Guo, Biao Huang and Jiachun Liu

ABSTRACT
Rapid filling in horizontal partially filled pipes with entrapped air may result in extreme pressure transients. This study advanced the current understanding of dynamic behavior of entrapped air above tailwater (the initial water column with a free surface in a partially filled pipe) through rigid-column modeling and sensitivity analysis of system parameters. Water and air were considered as incompressible fluid and ideal gas, respectively, and the continuity and momentum equations for water and a thermodynamic equation for air were solved by using the fourth order Runge-Kutta method. The effects of system parameters were examined in detail, including tailwater depth, entrapped air volume, driving head, pipe friction, and relative length of entrapped air and pipe. The results indicate that the presence of tailwater can mitigate the peak pressure when with identical initial volumes of entrapped air, as it can be considered to reflect a certain amount of loss of the net driving head. However, the peak pressure can increase as much as about 45% for the cases with fixed pipe length, due to the reduction in the initial entrapped air volume. The rise time for the first peak pressure was closely related to pipe friction, whereas the oscillation period (defined as the time duration between the first and second peaks) was virtually irrelevant. The applicability of the rigid-column model was discussed, and a time scale relevant indicator was proposed. When the indicator is larger than 20, the relative difference between the peak pressure estimation and experimental measurements is generally below 5%.

Key words | entrapped air, oscillation period, partially filled pipe, peak pressure, rigid-column model, tailwater

HIGHLIGHTS
- The effects of system parameters were examined in detail under four typical scenarios with tailwater.
- The difference between the rise time for the first peak and the period of pressure oscillation was clarified, and friction loss should be always considered.
- A dimensionless parameter $\lambda_2$ is proposed, which can be used to determine the applicability of rigid-column methods.

INTRODUCTION
Transient flows with entrapped air often occur in urban water systems, which can cause severe safety problems because of over-pressurization (Streeter & Wylie 1967; Martin 1976). Thus, rapid filling with entrapped air generally should be avoided. For storm sewers, however, it is difficult to control the inflow because of the complexity of boundary and initial conditions, and air entrapment may occur during the rapid transition from gravity flow to surcharge (Hamam & McCorquodale 1982; Vasconcelos & Leite 2012).
Regarding transition flow in a rapidly filling horizontal pipe, many researchers have investigated, both experimentally and theoretically, the influence of entrapped air volume on the pressurization process (Martin 1976; Cabrera et al. 1992; Zhou et al. 2002; Lee 2005; Zhou & Liu 2013). In previous studies, the rigid-column model (RCM) has been used widely in empty pipe filling (Martin 1976; Cabrera et al. 1992; Zhou et al. 2002; Lee 2005). The influence of the elasticity of water and pipe has been discussed in Abreu et al. (1992) and Guarga et al. (1996), in which indicators for the boundary between the rigid- and elastic-column theories were proposed. Theoretical models also have been developed in Malekpour (2014), Tijsseling et al. (2015), and Huang & Zhu (2020a), in which the magnitude and the period of the pressure oscillation were directly related to the parameters of the system through implicit or explicit expressions. It is difficult, however, to generalize from those ideal cases, such as a frictionless pipe.

A common case in the real world, such as in storm sewers, is that the pipe is partially filled – that is, water exists under the entrapped air (called tailwater in Zhou et al. 2002) – which has been studied on a more limited basis than empty pipe filling. Lee (2005) conducted a few experiments on partially filled pipe and found only slight differences between empty pipe and partially filled pipe filling with identical air pocket volume. The characteristics of pressure oscillation depend on system parameters, including initial tailwater depth, driving pressure, and size of entrapped air (Zhou et al. 2002; Zhou & Liu 2013). Huang & Zhu (2020b) developed an improved RCM for rapid filling in a partially filled horizontal pipe, and the computed results were in good agreement with the experimental data in Zhou & Liu (2013).

Rapid filling in a partially filled pipe is much more complicated than in an empty pipe, as the presence of tailwater often leads to changes in other parameters, thereby resulting in unexpected consequences. Thus, correlations between the pressure characteristics and system conditions should be studied in detail. Pipe friction is often neglected in theoretical analysis (Huang & Zhu 2020a), but it has been proven to be important in most of the practical cases (Malekpour 2014). Another common misconception in rapid filling problems is that the rise time for the first peak pressure is equal to (or approximate to) one-half period of the pressure oscillation. These issues require further exploration, and the underlying physics await to be discovered.

The main objectives of this study are to investigate the dynamic behavior of the entrapped air pocket in a rapidly filling pipe with tailwater and to evaluate the effects of relevant parameters, including the depth of tailwater, the volume or length of the entrapped air pocket, the length of upstream water column, the friction coefficient, and the driving head. Because of the complexity of dealing with air-water interface evolvement in elastic-column models, the RCM developed by Huang & Zhu (2020b) is used in the present study. An indicator that can be used to assess the applicability of the RCM is also provided.

**METHOD**

A schematic diagram of entrapped air in a partially filled pipe is shown in Figure 1. The driving pressure head $H_r$ remains constant during the filling process. At the initial stage, a valve separates the upstream water column and the entrapped air pocket, which is located above the tailwater. The gauge head of entrapped air is $H_{a0}$, which is set as the atmosphere pressure head. The length of the initial water column, entrapped air, and the depth of tailwater are defined as $L_{w0}$, $L_{a0}$, and $h_{tw0}$, respectively. Rapid filling is initiated by the immediate opening of the valve. Over the process of air compression and expansion, a vertical front is assumed.

**Governing equations**

The dynamic behavior of entrapped air in a partially filled pipe is governed primarily by the continuity and momentum equations for water and the thermodynamic equation for air (Huang & Zhu 2020b). In the RCM, the elastic effects of water and pipe are neglected. The tailwater depth remains stable throughout the process, and the friction coefficient $f$ adopts the average friction (including entrance loss and frictional head loss).

The continuity equation is given as follows:

$$A(V - V_w) = A_{tw}(V_{tw} - V_w),$$

![Figure 1](http://iwaponline.com/wst/article-pdf/83/4/771/850345/wst083040771.pdf)
where \( V \) is the velocity of the water column, \( V_{aw} \) is the velocity of the vertical air-water interface, \( V_{tw} \) is the velocity of tailwater, \( A \) is the cross-sectional area of the pipe, \( A_{tw} \) is the cross-sectional area of tailwater, and \( A_{aw} \) is the cross-sectional area of entrapped air and also vertical air-water interface.

The dynamic equation for water flow in Huang & Zhu (2020b) can be reorganized as follows:

\[
\frac{dV}{dt} = \frac{g}{L_{aw} + x} \left[ h_t - h_a - \frac{A_{tw} h_c - A_{tw} \frac{v^2}{2g}}{A - A_{tw}} \right] - \frac{V}{\sqrt{g}} \left( \frac{L_{aw} + x}{D} \right) \frac{V}{\sqrt{g}}
\]

(2)

where \( L_{aw} \) is the initial length of the water column, \( x \) is the air-water interface displacement, \( L_a \) is the length of entrapped air, \( H_0 \) is the water head of the upstream reservoir, \( h_{tw} \) is the depth of tailwater, \( g \) is the gravitational acceleration, \( f \) is the average friction coefficient, \( D \) is the pipe diameter, \( H_a^* \) is the absolute air pressure, and \( t \) is time. The centroid depth of tailwater \( (h_c) \) in the dynamic equation can be calculated as

\[
h_c = \int_0^{h_{tw}} \gamma hD \sin (\arccos(1 - 2h/D)) dh/\gamma A_{tw}.
\]

When \( h_{tw} \) is equal to 0, the kinetic term \( (V - V_o)(V_{tw} - V)/L_{aw} \) vanishes and the momentum equation degenerates to that for empty pipe filling.

According to the ideal gas law, the absolute pressure head of entrapped air can be calculated according to the energy equation, as follows:

\[
\frac{dH_a}{dt} = -k \frac{H_a}{V_{aw}} \frac{dV_a}{dt},
\]

(3)

where \( k \) is the polytropic exponent, and \( V_a \) is the volume of entrapped air.

### Scenarios under study

Four typical scenarios are numerically studied: (a) Scenario 1, keeping the length of the entrapped air pocket \( L_0 \) fixed and increasing the tailwater depth, which is common in real horizontal pipes; (b) Scenario 2, maintaining a constant air volume by varying both the initial air length and the tailwater depth, and keeping the initial length of the water column unchanged; (c) Scenario 3, maintaining a constant air volume by varying both the initial air length and the tailwater depth, and keeping the total length of pipe unchanged; and (d) Scenario 4, only changing \( L_0 \) and keeping a constant \( h_{tw} \) to explore the effect of the initial air length on the maximum pressure. In addition, the experiments conducted in Zhou & Liu (2013) are also simulated, and the four scenarios are denoted by V1, V2, V3, and V4. The description of these scenarios is given in Table 1.

### COMPARISON WITH EARLIER STUDIES

Numerical results of the RCM are first compared with experimental observations in Zhou et al. (2002), Lee (2005), and Zhou & Liu (2013), of which four conditions are shown in Figure 2. The boundary conditions of the test cases are \( P_r = 137 \text{ kPa}, \ L_0 = 5 \text{ m}, \ L = 10 \text{ m}, \ D = 35 \text{ mm}, \) and \( f = 0.032 \) in Zhou et al. (2002); \( P_r = 2 \text{ H}_\text{atm}, \ L_0 = 4.947 \text{ m}, \) \( L = 11.0429 \text{ m}, \ D = 26 \text{ mm}, \) and \( f = 0.032 \) in Lee (2005); and \( P_r = 120 \text{ kPa}, \ L_0 = 3.25 \text{ m}, \ L = 8.82 \text{ m}, \ D = 40 \text{ mm}, \) \( h_{tw} = 20 \text{ mm}, \) and \( h_{tw} = 25 \text{ mm} \) in Zhou & Liu (2013). The polytropic exponent \( k \) was set as 1.4 in all of the cases, and the friction was 0.085, according to Zhou & Liu (2013).

The test cases computed by the RCM compared well with experiments for both peak pressure and period, as shown in Figure 2. When \( h_{tw} = 0 \), the momentum equation

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**Table 1 | Scenarios for numerical simulations with rigid-column model (RCM)**

<table>
<thead>
<tr>
<th>Scenario no.</th>
<th>( H_{max} ) (m)</th>
<th>( h_{tw} ) (m)</th>
<th>( L_0 )</th>
<th>( L ) (m)</th>
<th>( V_{aw} ) (m³)</th>
<th>( D ) (m)</th>
<th>( f ) (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>30.360</td>
<td>0–0.396</td>
<td>30</td>
<td>100</td>
<td>0.009–3.770</td>
<td>0.40</td>
<td>0–0.200</td>
</tr>
<tr>
<td>S2</td>
<td>30.360</td>
<td>0–0.263</td>
<td>30–99</td>
<td>100–169</td>
<td>3.770</td>
<td>0.40</td>
<td>0–0.200</td>
</tr>
<tr>
<td>S3</td>
<td>30.360</td>
<td>0–0.263</td>
<td>30–99</td>
<td>100</td>
<td>3.770</td>
<td>0.40</td>
<td>0–0.200</td>
</tr>
<tr>
<td>S4</td>
<td>30.360</td>
<td>0.200</td>
<td>10–80</td>
<td>100</td>
<td>0.628–5.027</td>
<td>0.40</td>
<td>0–0.200</td>
</tr>
<tr>
<td>V1</td>
<td>18.539</td>
<td>0–0.035</td>
<td>1.058–6.450</td>
<td>8.824</td>
<td>9.498e-05–8.105e-03</td>
<td>0.04</td>
<td>0.085</td>
</tr>
<tr>
<td>V2</td>
<td>22.629</td>
<td>0–0.035</td>
<td>1.058–6.450</td>
<td>8.824</td>
<td>9.498e-05–8.105e-03</td>
<td>0.04</td>
<td>0.085</td>
</tr>
<tr>
<td>V3</td>
<td>26.719</td>
<td>0–0.035</td>
<td>1.058–6.450</td>
<td>8.824</td>
<td>9.498e-05–8.105e-03</td>
<td>0.04</td>
<td>0.085</td>
</tr>
<tr>
<td>V4</td>
<td>18.539; 22.629; 26.719</td>
<td>0.035</td>
<td>1.067–6.450</td>
<td>8.824</td>
<td>9.675e-05–5.847e-04</td>
<td>0.04</td>
<td>0.085</td>
</tr>
</tbody>
</table>
was identical to a dry front model (Zhou et al. 2002), and the prediction was in good agreement with the experimental observations. The maximum error of pressure peak between the numerical and experimental results was less than 7%. This discrepancy could be attributed to a number of factors that may have nontrivial effects on the pressurization process, including local loss and heat transfer.

RESULTS OF PARAMETRIC ANALYSIS

The shape of the entrapped air pocket was associated with air length, La0, and tailwater depth, htw. To investigate the specific effects of different parameters, including htw, La0, Lw0, and f, on the peak pressure Pmax, the oscillation period T, and the time for first peak Tp, four typical scenarios (Scenarios 1–4) were numerically simulated, as listed in Table 1.

Effect of initial tailwater depth

For S1, the computed cases were of various tailwater depths and fixed air pocket length (see Table 1). Under such a typical condition, an increase in initial tailwater depth led to a reduction in the initial volume of entrapped air. The pressure patterns in Figure 3(a) indicate that as the initial air volume decreases, both the peak value and the frequency become larger. This agrees with earlier findings in empty pipe filling problems (e.g. Malekpour 2014), in which the elastic effects were ignored.

The comparisons between cases of V1, V2, and V3, and experimental data in Zhou & Liu (2013) are presented in Figure 3(b)–3(d), respectively. Most of the cases are in good agreement at different driving pressures, 80 kPa, 120 kPa, and 160 kPa. In general, as the driving pressure increased, the peak pressure became larger with the same initial tailwater depth. Similar trends occurred under the conditions with La0 equal to 3.25 m and 6.45 m, which confirmed that the lower the initial air volume, the higher the peak pressure. When La0 = 1.058 m, however, the observed peak pressures first increased and then decreased. In the numerical study, all of the peak pressures increased monotonically with an increase in tailwater depth at the same boundary conditions. The calculated values were higher than the experimental measurements when the initial air volume was small.
As in Zhou & Liu (2013), the trend change for small air volume was attributed to the fact that the initial entrapped air pocket would break up into several small air bubbles. In fact, a critical value exists for entrapped air volume at which the maximum pressure occurs and below which the peak pressure decreases, as reported in Zhou et al. (2014) and Malekpour (2014). The elasticity of pipe walls and the water column introduced considerable errors, as shown in Figure 3(b)–3(d) with $L_0 = 1.058$ m. As air volume decreased, the elastic effects became dominant. Thus, such conditions were out of the applicability domain of the RCM.

The change of pressure pattern was caused by the combined effects of tailwater and entrapped air. According to the governing equations, the influence of tailwater could be considered as a certain amount loss of the net driving pressure (Huang & Zhu 2020b). On the one hand, an increase in tailwater depth resulted in the reduction of peak pressure. On the other hand, a decrease in air volume led to a rise in peak pressure under the rigid-column assumptions. From Figure 3, it is implied that the initial volume of entrapped air had a much larger impact than the tailwater depth on the peak pressure.

It seems that the friction played an unexpected role in determining the peak pressure in Figure 4(a). When the friction was small, the peak pressure first decreased and then increased as the initial tailwater depth increased. When the friction was large enough, a monotonical trend of pressure peak with increasing tailwater depth appeared. For the calculated cases of S1, a critical value $f_c$ can be seen in Figure 4(a), which was approximate to 0.03. Friction should always be associated with energy loss for the problem under study, and thus the peak pressure of the system was determined by the balance of friction, tailwater, and entrapped air volume. As a result, the critical value of friction was dependent on other relevant parameters.

A common misunderstanding regarding the pressurization process is that the time for the first peak pressure is the first half cycle of the oscillation. The two should not be used interchangeably. In fact, the pressure oscillation period should be better defined as the time duration between the first and second peaks, as reported in Huang & Zhu (2020a). Correlations between the pressure characteristics of the filling process, including the time for the first peak $T_p$ and the oscillation period $T$, and the tailwater depth, are shown in Figure 4(b) and 4(c). Both $T_p$ and $T$ decreased with the increasing depth of the tailwater under various friction coefficients. For a case with a specific tailwater depth, however, $T_p$ increased with the friction.
coefficient $f$, and it was more profound with a lower tailwater depth. The oscillation period was almost independent of $f$, which agreed with Huang & Zhu (2020a).

**Effect of initial air length with fixed air volume**

The effect of initial tailwater depth was examined by keeping constant entrapped air volume in S2 (Table 1). In this way, only the length of the entrapped air pocket varied corresponding to tailwater depth change. The pressure variations with regard to time for all four cases are shown in Figure 5(a). The differences among the four pressure oscillation processes were not as large as those in the previous scenario S1. The results also suggested that with identical entrapped air, pressure transients could be mitigated by the presence of tailwater. As the length of the air pocket increased, the peak pressure was reduced by friction as well as tailwater.

The friction coefficient was 0.032 and 0.085, respectively, in Lee (2005) and Zhou & Liu (2015), which generally was between 0 and 0.2 in engineering. The results in S2 showed that with a fixed volume of entrapped air, the peak pressure induced by rapid filling decreased with the increasing depth of tailwater, regardless of friction (Figure 5(b)). Note that in the absence of tailwater (i.e. for empty pipe filling), the friction had a remarkable effect on the peak value. This must be kept in mind when choosing simplified models, as in Malekpour (2014) and Huang & Zhu (2020a). In contrast, for $f = 0.032$, the difference in peak pressure was much less, which agreed with the finding of Lee (2005). If the friction was as large as 0.2, the change of peak pressure with tailwater was negligible. Under this condition, the kinetic energy increased because the initial air pocket length growth corresponding to tailwater rise was well balanced by friction loss (Malekpour 2014).

Results on the time for the first peak pressure $T_p$ and the oscillation period $T$ are given in Figure 5(c) and 5(d). It is expected that along with the increase in tailwater depth, the initial air pocket length increased and thus both $T_p$ and $T$ increased. The larger the friction, the longer the $T_p$, as shown in Figure 5(c). The influence of friction on $T$, however, was limited (Figure 5(d)).

**Effect of initial air length with fixed air volume and total pipe length**

For test cases in S3, the entrapped air volume was maintained as constant, as in the previous scenario S2, and the total length of pipe was also fixed, so that the increase in
tailwater depth caused the extension of the entrapped air pocket and corresponding shortening of the upstream water column. The computed pressure patterns are shown in Figure 6(a), in which the combined influence of the upstream water column length $L_{w0}$ and the air pocket length $L_{a0}$ is presented. By comparing Figure 6(a) with Figure 5(a), it is clear that $L_{w0}$ had a notable effect on the pressure characteristics, as reported in Huang & Zhu (2020b). The peak pressure decreased with an increase in the tailwater depth and was much more sensitive when friction tended to 0 (Figure 6(b)), which was similar to S2. The correlation between the time for first peak $T_p$ and tailwater depth varied with the friction coefficient (Figure 6(c)). A critical value $f_c$ also appeared among the test cases, and it was around 0.049. The time for the first peak decreased as the tailwater depth rose when $f > f_c$, but it increased when $f < f_c$, as a result of the balance between $L_{w0}$, $L_{a0}$, and $h_{tw}$. Again, the critical friction value depended on the system conditions. Unlike S2, the oscillation period became shorter along with an increase in tailwater depth, which was mainly due to the decrease in $L_{w0}$, according to the formulation for the oscillation period for rapid filling problems in Huang & Zhu (2020a).

**Effect of initial air length/volume with fixed tailwater depth and total pipe length**

For S4, the tailwater depth was set as half of the pipe diameter, and only the entrapped air volume changed through the initial air length. The pressure variations are shown in Figure 7(a). As expected, the initial air length or volume was dominant over other parameters on the pressure characteristics. Similar to the previous comparisons, it was found that the results of the numerical and experimental conditions (V4) had notable differences when applying a small entrapped air volume; for instance, when $L_{a0}/L$ was about 0.12 (Figure 7(b)). This further confirmed that the elasticity of the system cannot be ignored under such conditions. The validity range of RCM is discussed next.

The correlations between pressure characteristics and the length ratio of air pocket and pipe are presented in Figure 8. The results indicated that the peak pressure was reduced dramatically by increasing $L_{a0}/L$ and the friction of pipe $f$. Both the rise time for first peak $T_p$ and oscillation period $T$ increased with $L_{a0}/L$, whereas the former was closely related to $f$ and the latter was not.
DISCUSSION

As for empty pipe filling problems, studies on the comparison between rigid-column models and elastic-column models have been conducted in Abreu et al. (1992), Malekpour (2014), and Huang & Zhu (2020b), among others. It is well recognized that when the initial air volume is small, the error from RCMs can be considerably large because the elastic energy stored in the water column and pipe walls cannot be ignored, compared with that of entrapped air. Thus, before using a RCM, the validity range should be first examined.

An indicator has been proposed in Huang & Zhu (2020b) to determine whether or not a RCM can be used for rapid filling in an empty horizontal pipe, given as follows:

$$\lambda_t = \frac{1}{4} \frac{\sqrt{L_{a0} / L}}{a \pi L_{a0} / D + x_e} \sqrt{\frac{gkH_R}{H_{a0}}} \left( \frac{H_R}{H_{a0}} \right)^{\frac{1}{18}}$$  \hspace{1cm} (4)

Figure 6 | Correlations of pressure characteristics and tailwater depth for SC3: (a) pressure variations with regard to time; (b) peak pressure; (c) time for first peak; and (d) oscillation period.

Figure 7 | Effect of initial length of entrapped air: (a) pressure patterns for S4; and (b) comparisons between this study and Zhou & Liu (2013) for V4.
where $a$ is wave speed in water, and $x_e$ is the equilibrium point location at which the air pressure is exactly equal to the driving pressure. The relative errors, defined as $\varepsilon = |P_{ECM} - P_{RCM}|/P_{ECM}$ or $|P_{Exp} - P_{RCM}|/P_{Exp}$, with respect to the corresponding values of $\lambda_t$ for the test cases in Malekpour (2014), are plotted in Figure S1. Generally, the relative error fell below 5% when $\lambda_t > 6$, as suggested by Huang & Zhu (2020b).

The indicator in Equation (4) should be revised, however, to take tailwater into account for partially filled piping problems. By replacing $L_{a0}$, $x_e$, and $L_{a0}$ as $L_{a0}/A_w/A$, $x_e/A_w/A$, and $L - L_{a0}/A_w/A$, respectively, the revised equation becomes:

$$\lambda_t = \frac{1}{4} \frac{a \pi}{\sqrt{L_{a0}/A_w - L_{a0} + x_e}} \sqrt{g k H_R \left( \frac{H_R}{H_{a0}} \right)^2}.$$ (5)

The computational results are given in Figure S2, which indicates that when $\lambda_t > 20$, the RCM can be used to predict the pressure transient during the rapid filling for a partially filled pipe.

**CONCLUSIONS**

Pressure transients can be induced over rapid filling in partially filled pipes. The correlations between pressure characteristics and system parameters were investigated in detail using a RCM for four typical scenarios. The following conclusions are supported by the findings of this study:

1. The presence of tailwater can mitigate the peak pressure as it can be considered as a certain amount of loss of the net driving pressure; however, the increase of tailwater is always accompanied by other changes, such as entrapped air volume or length variation, which also has an appreciable influence on the pressure pattern.
2. The friction loss should be always considered, as it may cause larger errors than expected.
3. The rise time for the first peak pressure is different from the half cycle of the pressure oscillation, and the former depends on the pipe friction, whereas the latter is almost insensitive.
4. A dimensionless parameter $\lambda_t$ is given for partially filled pipe filling and when it is larger than 20, RCMs can be used without much loss in accuracy.
ACKNOWLEDGEMENTS

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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