Experimental investigations for assessing the influence of fly ash on the flow through porous media in Darcy regime
Abhishish Chandel, Vijay Shankar and M. A. Alam

ABSTRACT

Hydraulic conductivity plays a vital role in the studies encompassing explorations on flow and porous media. The study investigates the compaction characteristics of a river sand (Beas, Sutlej, and Ghaggar rivers) and fly ash mix in different proportions and evaluates four empirical equations for estimating hydraulic conductivity. Experiments show that an increase in the fly ash content results in a decrease in the maximum dry density (MDD) and an increase in the corresponding optimum moisture content (OMC) of sand–fly ash samples. MDD at optimum fly ash content was achieved at low water content, which resulted in less dry unit weight than that of typical conventional fill. In Beas, Sutlej, and Ghaggar sands the optimum fly ash content up to which the hydraulic conductivity value reduced uniformly was found to be 30, 45, and 40%, respectively. Any further increase in the fly ash content results in a negligible decrease in hydraulic conductivity value. The observed hydraulic conductivity of sand–fly ash mix lies in the range of silts, which emboldens the use of sand–fly ash mix as embankment material. Further, the evaluation of empirical equations considered in the study substantiates the efficacy of the Terzaghi equation in estimating the hydraulic conductivity of river sand–fly ash mix.

Key words | compaction, Darcy’s law, fly ash, hydraulic conductivity, porous media

HIGHLIGHTS

- The study focused on utilizing the river sand–fly ash mix for different hydraulic engineering purposes such as for making impermeable embankments and groynes.
- The study examines the influence of fly ash on the compaction and hydraulic properties of river sand.
- The compaction characteristics, strength, and stiffness of the river sand particles can be improved by mixing them with the optimum fly ash content.
- The overall result of the study is the effective use of Terzaghi’s equation to estimate the hydraulic conductivity of the sand–fly ash mix.

INTRODUCTION

Hydraulic conductivity (K) is an essential parameter considered in the construction of earth and embankment dams, in the design of the drainage systems, and is of prime significance in the determination of groundwater seepage losses, settlement computations, and stability analyses (Boadu 2000).

Flow of fluid through porous media is dependent on its hydraulic conductivity, which is governed by Darcy’s law (Ghanbarian et al. 2016). The formulation of this law indicates that the fluid discharge velocity through the saturated porous media is proportional to the hydraulic gradient. The equation includes a proportionality constant, known as hydraulic conductivity (Alabi 2011). The Reynolds number and friction factor determination of flow through porous media govern the flow regimes (Kango et al. 2018; Li et al. 2019).
In the field environment, authentic estimation of \( K \) is confined by a lack of precise knowledge of hydraulic boundaries and embankment geometry (Uma et al. 1989). Hydraulic conductivity using laboratory tests presents challenging problems in the sense of collecting representative samples and long testing times (Odong 2007; Ishaku et al. 2011). However, the estimation of \( K \) using empirical equations is recommended to overcome these problems. More importantly, information about the textural properties of the porous medium is easily obtained, resulting in a potential alternative for the estimation of \( K \) values using empirical equations (Rosas et al. 2014). Numerous investigators studied these relationships and proposed several formulae based on the experimental data, i.e., Hazen (1892), Terzaghi (1925), Kozeny (1927), Carman (1937, 1956), and Beyer (1964). The applicability of these empirical formulae depends on the type of soil.

Fly ash is a waste product resulting from the ignition of naturally pulverized coal. The particles of fly ash are extracted with the help of precipitators in the smoke ducts to reduce its harmful impact on the environment (Gupta & Alam 2004; Sivapullaiah & Lakshmikantha 2004). In India, growing demand for electrical energy resulted in the installation of several coal-based thermal power plants. The primary disposal of this waste is landfilling, but due to space constraints and large treatment cost, alternative utilization of fly ash is encouraged (Muhunthan et al. 2004). Hydraulic conductivity is the governing parameter that determines the suitability of soil–fly ash mix at the optimum fly ash content. The optimum fly ash content corresponds to the maximum dry density (MDD) of sand–fly ash mix at a low moisture content characterized with significantly reduced hydraulic conductivity. The compaction characteristics, strength, and stiffness of the river sand particles can be improved by mixing the optimum fly ash content that results in reduced pore space and hydraulic conductivity, thereby substantiating the use of sand–fly ash mix for making impermeable embankments and dams (Mir & Sridharan 2014). Numerous investigations have been carried out to utilize fly ash in flexible dam construction and water retaining embankments. Kumar & Stewart (2003) performed laboratory investigations to govern the feasibility of dry bottom ash as a landfill liner. The hydraulic conductivity of a bentonite and bottom ash mix was minimum with 35% bottom ash as optimum, for its use as a hydraulic barrier. Gupta & Alam (2004) experimentally examined the seepage characteristics of fly ash with different proportions of lime and cement. The hydraulic conductivity of fly ash at 40% lime and 15% cement content was found to be optimum. Muhunthan et al. (2004) examined the potential use of fly ash mixes. The hydraulic conductivity of bottom ash with 40% fly ash content is relatively comparable to that of silt and clay. Marto et al. (2011) studied the hydraulic conductivity and compaction characteristics of coal ash mixes. The hydraulic conductivity of compacted ash mixtures was found to decrease with increasing fly ash content. Amiralian et al. (2002) investigated the influence of fly ash (5–15%) and lime (1–3%) on the hydraulic conductivity of Baldivis Yellow Concrete Sand. Ige & Ajamu (2015) examined the hydraulic conductivity and compressive strength of a sand–fly ash mix. Fly ash content of 40% resulted in the lowest \( K \) value and also had the highest impact on the increase of the strength of sand. Galupin & Dungca (2015) examined the \( K \) characteristics of soil–fly ash mixes and developed a relationship between \( K \) and the percentage of fly ash content. Samanta (2018) studied the influence of varying proportions of cement (0–10%) on the \( K \) value of fly ash. Coarser porous media, especially river sand with fly ash content (varying from 0 to 50%), were suitable for making structurally stable impermeable embankments having \( K \) value varying between \( 10^{-5} \) and \( 10^{-3} \) cm/s whereas more than 50% fly ash content in porous media resulted in the reduction of the strength of impermeable groynes (Sreekantan et al. 2019). Wasil (2020) studied the effect of different proportions of bentonite (0–15%) on the hydraulic conductivity of fly ash.

Literature indicates that many researchers worked on the use of fly ash, i.e., fly ash mixed with lime, cement, bentonite, and gypsum, to reduce the hydraulic conductivity value. However, there is a need to study the effect of the addition of different proportions of fly ash by weight on the hydraulic conductivity of porous media, particularly river sand, followed by standard compaction conditions. The present study investigates the influence of fly ash on the compaction and hydraulic properties of river sand. The specific objectives of the study are (1) to determine the optimum fly ash content, i.e., the lowest fly ash content in sand–fly ash mix, at which the hydraulic conductivity reduces to a significant value, and (2) to evaluate the efficacy of four existing empirical equations in estimating hydraulic conductivity of sand–fly ash mix.

**MATERIALS AND METHODOLOGY**

**Materials**

In the present study, fly ash (class-F) was collected from Guru Gobind Singh Super thermal power plant situated in
Ropar district of Punjab and sand samples were collected from three different riverbanks in India. The first and second samples of sand were collected from Beas River in Amritsar district and Sutlej River in Ropar district of Punjab state, respectively, and the third from Ghaggar River in Panchkula district of Haryana state.

**Engineering and physical properties of materials**

To accomplish the objectives of the study, initially sieve analysis of collected sand samples and fly ash was done to determine the grain size ($d_{10}$, $d_{50}$, and $d_{90}$), uniformity, and curvature coefficient (ASTM 2007). Basic properties of the different sands and fly ash were determined using standard procedures in the laboratory, and are shown in Table 1.

**Methodology**

The standard Proctor compaction test (SPCT) was conducted on the samples prepared by mixing sand with different proportions of fly ash added by weight. For SPCT water was added to 3 kg of oven dried sample to bring the moisture content to about 6%. The sample thus prepared was placed in an airtight container for about 18 hours for maturing. After this, the sample was compacted in a mould in three equal layers by providing 25 blows with the help of a rammer of mass 2.6 kg in every one-third layer from a 30.48 cm drop height. All blows were equally distributed over the surface of each layer. The collar of the compaction mould was removed, and the excess sample was trimmed off. The weight of the mixture in the mould was recorded. Two samples from the top and the bottom of the mixture were collected and placed inside the oven at 105 °C for about 24 hours. The average of these samples was taken to determine the water content in each sand-fly ash mix. For each mix, about four to six data points were obtained. Further, a plot between dry density and water content was drawn to determine the MDD (kN/m³) and optimum moisture content (OMC, %) (ASTM 2012).

For the determination of hydraulic conductivity of sand-fly ash mix, a constant head permeameter having diameter 15.3 cm and height 46.5 cm was used as shown in Figure 1. The samples were prepared by mixing sand with varying fly ash content increasing from 10 to 50% in increments of 10%. The percentage content up to which the K value reduced uniformly was termed as optimum value. In the second phase of investigations, the fly ash content was varied in increments of 5% from 25 to 45% for the refinement of the investigation on K value. The various combinations of the sand-fly ash mix thus prepared were compacted in the permeameter to achieve the MDD which was determined during the compaction test. The top inlet of the reservoir was connected to the specimen. Before measuring the K value, the specimen was saturated by allowing the water to flow inside the permeameter for about 3 hours to obtain a steady flow condition. A constant head was maintained in the standpipe and, for an appropriate time interval, the quantity of flow was measured. The same procedure was repeated at three different constant heads (ASTM 2006). The average discharge values were taken to determine the K values in all three sand-fly ash mixes. Specific gravity studies were carried out by the pycnometer method to determine the porosity of all samples. The temperature of the water was also measured at the beginning and the end of each analysis.

**Empirical equations for hydraulic conductivity estimation**

Several empirical equations are available for predicting the hydraulic conductivity of fly ash mixed porous media based on grain size analysis. Kasenow (2002) and Song et al. (2009) summarized different empirical relationships from previous studies and gave a general equation:

$$K = \frac{k}{\phi} f(n) \cdot C \cdot d_e^2$$

(1)
where $K$: hydraulic conductivity (m/s), $g$: gravitational acceleration (m/s²), $\theta$: kinematic viscosity (m²/s), $C$: sorting coefficient, $f(n)$: porosity function, and $d_e$: effective grain size (m).

Various researchers have presented different empirical equations which take the standard form as shown in Equation (1) but with different values of $C$, $f(n)$, and $d_e$. Table 2 shows the empirical equations considered in the present study, with their boundary conditions for the estimation of hydraulic conductivity.

### Statistical parameters

The performance of different empirical equations was evaluated quantitatively using different statistical parameters, i.e., bias, SI (scatter index), $R^2$ (determination coefficient), $I_a$ (agreement index), RMSE (root mean square error), and MAE (mean absolute error) defined as:

$$\text{Bias} = \sum_{i=1}^{N} \frac{1}{N} (P_i - M_i)$$

$$\text{SI} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_i - M_i)^2}$$

$$R^2 = \left[ \frac{\sum_{i=1}^{N} (M_i - M)(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{N} (M_i - M)^2 \sum_{i=1}^{N} (P_i - \bar{P})^2}} \right]^2$$

*Figure 1 | Experimental setup of the hydraulic conductivity measuring apparatus.*
\[ I_a = 1 - \frac{\sum_{i=1}^{N} (P_i - M_i)^2}{\sum_{i=1}^{N} |P_i - M| + |M_i - M|} \]  
(5)

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (M_i - P_i)^2}{N}} \]  
(6)

\[ \text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |M_i - P_i| \]  
(7)

where \( M_i \) and \( P_i \) represent the experimentally measured and empirically predicated \( K \) values, respectively, and \( N \) is the number of observations. \( M \) and \( P \) are the mean values of measured and predicted parameters, respectively.

### RESULTS AND DISCUSSION

Sieve analysis and SPCT were performed on the sand–fly ash samples. A total number of 51 experimental sets (24 SPCT and 27 hydraulic conductivity tests) were performed on different sand–fly ash mixes. Variation of friction factor (\( \text{Fr} \)) with Reynolds number (\( \text{Re} \)) was studied and experimentally obtained values of \( K \) were compared with empirically obtained values for all three sand types with different fly ash content.

#### Gradation analysis

Sieve analysis was conducted for fly ash and Beas, Sutlej, and Ghaggar sands. Figure 2 shows the gradation curve for different sands and fly ash. From the gradation curve, it is evident that nearly 50% of fly ash is retained on 75-micron size. The values of \( d_{10} \), \( d_{30} \), and \( d_{60} \) particle sizes for all three sands and fly ash are given in Table 1. Ghaggar

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**Table 2: Empirical equations for hydraulic conductivity estimation**

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Equation</th>
<th>( C )</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hazen (1892)</td>
<td>( K_{\text{Hazen}} = \frac{g}{v} C \frac{1 + 10(n - 0.26)}{n} d_{10}^2 )</td>
<td>( 6 \times 10^{-4} )</td>
<td>( 0.1 \text{ mm} &lt; d_{10} &lt; 3 \text{ mm}, C_u &lt; 5 )</td>
</tr>
<tr>
<td>Terzaghi (1925)</td>
<td>( K_{\text{Terzaghi}} = \frac{g}{v} C \frac{n - 0.13}{\sqrt{1 - n}} d_{10}^2 )</td>
<td>( 10.7 \times 10^{-3} ) for smooth grains</td>
<td>Large grain sand</td>
</tr>
<tr>
<td>Kozeny (1927), Carman (1937, 1956) (Kozeny–Carman)</td>
<td>( K_{\text{Kozeny}} = \frac{g}{v} C \frac{n^3}{(1 - n)^2} d_{10}^2 )</td>
<td>( 8.3 \times 10^{-3} )</td>
<td>( d_{10} &lt; 3.0 \text{ mm} ) suitable for gravel, sand and silty soil</td>
</tr>
<tr>
<td>Beyer (1964)</td>
<td>( K_{\text{Beyer}} = \frac{g}{v} C \log \frac{500}{C_u} d_{10}^2 )</td>
<td>( 6 \times 10^{-4} )</td>
<td>( 0.06 \text{ mm} &lt; d_{10} &lt; 0.6 \text{ mm}, 1 &lt; C_u &lt; 20 )</td>
</tr>
</tbody>
</table>

\( C_u \): uniformity coefficient \( (d_{60}/d_{10}) \); \( n \): porosity, and \( d_{10} \) and \( d_{60} \): grain sizes.

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**Figure 2**: Gradation of different sands and fly ash.
sand and fly ash both had more than 10% silt content, so their effective size, i.e., $d_{10}$, was computed using hydrometer analysis. However, in the case of Beas and Sutlej sand $d_{10}$ particle size was determined using the gradation curve. The average value of uniformity coefficient ($C_u$) and curvature coefficient ($C_c$) for fly ash is found to be 3.67 and 1.09, whereas for Beas, Sutlej, and Ghaggar sands the values are 2.78 and 1.60, 3.43 and 1.17, and 3.0 and 0.59, respectively. Generally, for well-graded sand $C_u$ is greater than 6, and $C_c$ values lie between 1 and 3 (ASTM 2007). The $C_u$ values are less than 6 in all three sands, which indicates that all three sands, i.e., Beas, Sutlej, and Ghaggar, are poorly graded.

**Compaction performance**

SPCT was conducted initially for samples with the fly ash content incrementally increasing by 10% from 0 to 50%, dry weight basis, in the Beas, Sutlej, and Ghaggar sands. In the second phase, the fly ash was varied in increments of 5% from 25 to 45% for all three river sands. Figure 3 shows the variation of the dry density in respect to the moisture content through the compaction curves. The plots indicate that with an increase in moisture content the density increases initially. The curve attains a peak value at the OMC and then begins to drop with further increase in moisture content. For Beas, Sutlej, and Ghaggar sands with the 10% incremental increase in the fly ash content by weight from 10 to 50%, the MDD decreases from 13.6 to 11.5 kN/m$^3$, 14.52 to 11.64 kN/m$^3$, and 14.78 to 10.46 kN/m$^3$ and the corresponding OMC increases from 9.1 to 15.2%, 12.4 to 16.2%, and 13.2 to 17.4%, respectively. In the second phase, for Beas, Sutlej, and Ghaggar sands the MDD decreases from 12.68 to 11.71 kN/m$^3$, 13.54 to 12.1 kN/m$^3$, and 13.04 to 10.93 kN/m$^3$ and the corresponding OMC increases from 11.43 to 14.18%, 13.66 to 15.65% and 14.66 to 16.76%, respectively. The decrease in the MDD as the fly ash content increases is linked with the conception that fly ash is lightweight as compared to sand. Also, the OMC increases because fly ash particles with dust-like appearance have much more surface area to be covered with water. Subsequently, they absorb more water, leading to an increase in moisture content in all sand–fly ash samples. Muhunthan et al. (2004) conducted a study on investigating the moisture density of fly ash mixes and concluded, that the optimum fly ash mix resulted in MDD of 12.5 kN/m$^3$ and OMC of 25.8%. From the comparative analysis of these values with the MDD and OMC values at optimum fly ash content in the present study (mentioned in ‘Estimated hydraulic conductivity’ section), it can be seen that MDD at optimum fly ash content attained nearly the same value whereas the corresponding OMC is 10% lower. The significant variation in the OMC of sand–fly ash mix highlights that the fly ash used in the present study is of silty texture whereas that in the Muhunthan et al. (2004) study is of silty loam texture.

**Variation of friction factor with Reynolds number**

The variation between the dimensionless quantities $F_r$ and $Re$ for Beas, Sutlej, and Ghaggar sand–fly ash mixes was studied and plotted on a logarithmic scale as shown in Figure 4. $F_r$ and $Re$ are given as:

$$F_r = \frac{2g d_{50} \sin i}{V^2}$$  \hspace{1cm} (8)

$$Re = \frac{V d_{50}}{v}$$  \hspace{1cm} (9)

where $g$: gravitational acceleration, $i$: hydraulic gradient, $V$: average velocity of flow through pores, $d_{50}$: mean grain diameter, and $v$: kinematic viscosity of the fluid.
Figure 4 indicates a straight-line variation between $Fr$ and $Re$ for Beas, Sutlej, and Ghaggar sand–fly ash mixes, signifying a linear regime of flow having $Re$ value less than 1, thus validating Darcy’s regime (Alabi 2011).

Estimated hydraulic conductivity

Hydraulic conductivity of different sand–fly ash samples was determined at OMC. For Beas sand, $K$ was found to be $7.33 \times 10^{-3}$ cm/s; however, with the addition of 30% fly ash, the $K$ value reduced to $2.11 \times 10^{-3}$ cm/s and an insignificant decrease in the $K$ value was noticed with further increase in fly ash content in steps of 10% up to 50%. For Ropar and Ghaggar sand, the $K$ value uniformly reduced from $5.64 \times 10^{-3}$ to $1.83 \times 10^{-3}$ cm/s and from $4.52 \times 10^{-3}$ to $1.29 \times 10^{-3}$ cm/s with the addition of fly ash content up to 40% initially. In the second phase, when the fly ash was varied in increments of 5% from 25 to 45%, it was observed that $K$ value reduced uniformly from $3.40 \times 10^{-3}$ to $1.65 \times 10^{-3}$ cm/s up to 45% fly ash content in the case of Sutlej sand, whereas for Beas and Ghaggar sands there was no significant change beyond 30 and 40% fly ash content, respectively, as shown in Figure 5. In Beas, Sutlej, and Ghaggar sands the optimum fly ash content was found to be 30, 45, and 40%, respectively. MDD and OMC in Beas, Sutlej, and Ghaggar sands at the optimum fly ash content (30, 45, and 40%) were found to be 12.39 kN/m$^3$ and 12%, 12.1 kN/m$^3$ and 15.65%, and 11.38 kN/m$^3$ and 16.1%, respectively. The results of the compaction study indicate that sand–fly ash mixes at optimum fly ash content achieved much lower unit weight than those of clay and silt.

The decrease in the value of $K$ up to the optimum fly ash content is due to an increase in the specific surface of porous media with the increase in the content of fines particles, providing more resistance to the water flow through voids (Galupino & Dungca 2015). Mir & Sridharan (2014) reported comparatively higher values ($6.8 \times 10^{-2}$ to $9.3 \times 10^{-2}$ cm/s) of $K$ for sand–fly ash mixes. Also, it was not described how the sand–fly ash samples were prepared and saturated to determine the $K$ value. These issues can expressly affect the $K$ of the sand–fly ash. In this study, a significant reduction in the hydraulic conductivity of sands at optimum fly ash content is comparable to those obtained with the fine silts (Kumar & Stewart 2003). This substantiates that sand–fly ash mix with optimum fly ash content has significant benefits for use in embankments, fills, and construction of impermeable groynes.
Hydraulic conductivity using empirical equations

The grain size analysis of the sand-fly ash mix was used to determine the grading characteristics, i.e., $d_{10}$, $d_{50}$, $d_{60}$, and $C_u$ as shown in Table 3.

The $n$ values for each mix were calculated. The obtained parameters were used to calculate $K$ using the four empirical equations, considered in the study, presented in Table 2. An average value of the sorting coefficient ($8.4 \times 10^{-3}$) was used in the Terzaghi equation (Naeej et al. 2017). The value of

![Figure 5 Variation of hydraulic conductivity of sands with different proportions of fly ash.](image)

**Table 3** | Hydraulic conductivity computations using empirical equations

<table>
<thead>
<tr>
<th>Porous media</th>
<th>Fly ash (%)</th>
<th>$d_{10}$ (mm)</th>
<th>$d_{50}$ (mm)</th>
<th>$d_{60}$ (mm)</th>
<th>$C_u$ ($d_{50}$/$d_{10}$)</th>
<th>Porosity</th>
<th>$K_{exp}$ (cm/s) ($10^{-3}$)</th>
<th>$K_{Hazen}$ (cm/s) ($10^{-3}$)</th>
<th>$K_{Terzaghi}$ (cm/s) ($10^{-3}$)</th>
<th>$K_{K}$ (cm/s) ($10^{-3}$)</th>
<th>$K_{Beyer}$ (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beas sand</td>
<td>0</td>
<td>0.090</td>
<td>0.155</td>
<td>0.250</td>
<td>2.778</td>
<td>0.370</td>
<td>7.33</td>
<td>11.31</td>
<td>5.91</td>
<td>9.51</td>
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<tr>
<td></td>
<td>10</td>
<td>0.084</td>
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<td>0.165</td>
<td>1.964</td>
<td>0.356</td>
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<td>20</td>
<td>0.072</td>
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<td>25</td>
<td>0.068</td>
<td>0.134</td>
<td>0.153</td>
<td>2.250</td>
<td>0.350</td>
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<td>5.84</td>
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<tr>
<td></td>
<td>30</td>
<td>0.066</td>
<td>0.128</td>
<td>0.148</td>
<td>2.242</td>
<td>0.348</td>
<td>2.11</td>
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<td>Sutlej sand</td>
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gravitational acceleration in empirical equations was taken as 981 cm/s². The kinematic viscosity, which is an important parameter for the estimation of $K$, has a value of 0.885 mm²/s derived at a water temperature of 27 °C.

The comparison of experimentally measured and empirical equations-based $K$ values indicates that values of $K$ obtained using the Terzaghi equation have a closer agreement with experimentally obtained values as compared to Kozeny–Carman and Hazen equations as shown in Figure 6. The Beyer equation clearly overestimated the $K$ values because it is more convenient for analyzing well-graded mixes (Ishaku et al. 2014). Hazen and Kozeny–Carman equations are based only on the porosity, and $d_{10}$ particle size is less precise than the Terzaghi equation which is based on the entire particle size distribution, porosity, sorting coefficient, and particle shape (Cabalar & Akbulut 2016). The $K$ values estimated by the Terzaghi equation for all mixes were more accurate than other equations considered in the study. Beyer equations, however, overestimated the $K$ values for all mixes, since the equation is not suitable if the particle distribution has a long, flat tail in the fine fraction (Odong 2007).

Various statistical parameters were used to check the performance of the empirical equations with regards to the measured $K$ as shown in Table 4.

The values of SI, RMSE, and MAE ranged from 0 to $\infty$, $I_a$ and $R^2$ from 0 to 1, and bias from $-\infty$ to $\infty$. Lower values of bias, SI, RMSE, and MAE and values closer to 1 for $R^2$ and $I_a$ indicate better agreement between measured and predicted parameters. The bias, SI, RMSE, and MAE for the Terzaghi equation have comparatively lower values whereas the $R^2$ and $I_a$ values are close to 1. Better quantitative agreement in the case of the Terzaghi equation as compared to Hazen, Kozeny–Carman, and Beyer equations substantiates the efficacy of the Terzaghi equation in computing hydraulic conductivity of sand–fly ash mix.

**CONCLUSIONS**

The investigations on the hydraulic conductivity of sand–fly ash mixes were conducted to examine the potential use of fly ash for specific engineering purposes. The $F_t$ and $R_c$ indicate a straight-line variation, signifying the linear regime of flow, i.e., Darcy’s regime. Sand–fly ash mixes show a well-defined moisture density relationship. Hydraulic conductivity

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**Table 4 | Statistical parameters of empirical equations**

<table>
<thead>
<tr>
<th>Statistical parameters</th>
<th>Hazen</th>
<th>Terzaghi</th>
<th>Kozeny–Carman</th>
<th>Beyer</th>
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<tbody>
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<td>Bias</td>
<td>1.58</td>
<td>– 0.59</td>
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<td>SI</td>
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<td>$R^2$</td>
<td>0.81</td>
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<tr>
<td>$I_a$</td>
<td>0.77</td>
<td>0.91</td>
<td>0.88</td>
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<tr>
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<td>MAE</td>
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**Figure 6 | Comparison of measured and predicted $K$ for (a) Hazen, (b) Terzaghi, (c) Kozeny–Carman, and (d) Beyer.**
of sand decreased with the addition of fly ash. MDD, OMC, and $K$ value for Beas, Sutlej, and Ghaggar sands at the optimum fly ash content, i.e., 30, 45, and 40%, respectively, were determined as 12.39 kN/m$^3$, 12%, and $1.21 \times 10^{-3}$ cm/s; 12.1 kN/m$^3$, 15.65% and $1.65 \times 10^{-3}$ cm/s; and 11.38 kN/m$^3$, 16.1%, and $1.29 \times 10^{-3}$ cm/s, respectively. The study indicates that the dry unit weight of the compacted mixes at optimum fly ash content is lower than those of typical compacted fill. The values of $K$ for sand with optimum fly ash content lie in the range of silts. The study establishes the use of fly ash in the construction of impermeable embankments as fill material. Efficacy of the Terzaghi equation in estimating the hydraulic conductivity for sand-fly ash mixes has been established as compared to Hazen and Kozeny–Carman equations, whereas Beyer equations manifestly overestimated the $K$ values. The obtained statistical parameters, i.e., bias, SI, $R^2$, $I_a$, RMSE, and MAE, indicate a high degree of correlation between the Terzaghi equation and experimentally obtained $K$ values.

**DATA AVAILABILITY STATEMENT**

All relevant data are included in the paper or its Supplementary Information.

**REFERENCES**


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Hazen, A. 1892 Some Physical Properties of Sands and Gravels, with Special Reference to their Use in Filtration. Massachusetts State Board of Health, 24th Annual Report, pp. 539–556.


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