

## Evaluating the performance of fine bubble diffused aeration systems in cylindrical aeration tanks by fuzzy c-means algorithm

Er Li \* and Xiangying Zeng

Wuhan Municipal Engineering Design and Research Institute, Changqing Road 40, Jiangnan District, Wuhan 430015, China

\*Corresponding author. E-mail: tjy903@163.com

 EL, 0000-0001-8219-1653

### ABSTRACT

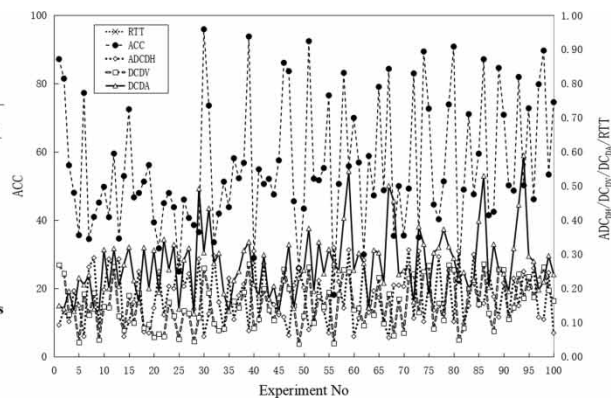
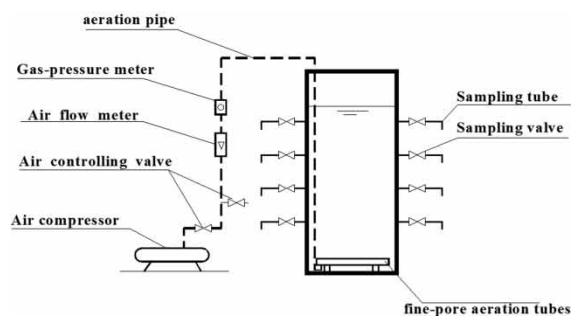
In order to make a comprehensive evaluation of the performance of fine bubble diffused aeration systems in cylindrical aeration tanks, the following parameters are considered: distribution of DO concentration in the horizontal direction of the different water depth of an aeration tank, distribution of DO concentration in the vertical direction of the aeration tank, distribution of DO concentration in all the points of the aeration tank and ratio of total mass increment of DO in the aeration tank to total mass of oxygen in aeration air. The aeration characteristic criterion (ACC) is proposed to synthesize these parameters, and weighted sums of the similarity degrees derived from the extensions of fuzzy c-means (FCM) are used to construct ACC. The results show that compared with total volumetric oxygen transfer coefficient ( $K_L\alpha$ ) and specific standard oxygen transfer efficiency (SSOTE), ACC is demonstrated to be more marked and sensitive for the performance evaluation of the fine bubble diffused aeration systems equipped with fine-pore aeration tubes. Moreover the performance of aeration systems equipped with different layouts of fine-pore aeration tubes is comparatively studied, and their performance from best to worst is ring-type diffuser > square-type diffuser > parallel-lines-type diffuser > cross-type diffuser.

**Key words:** DO concentration uniformity, fuzzy c-means (FCM) algorithm, overall efficiency of oxygen mass transfer, performance evaluation

### HIGHLIGHTS

- The performances of fine bubble diffused aeration systems in cylindrical aeration tanks are evaluated based on the FCM algorithm.
- Comparison of the performance evaluation of aeration systems equipped with different layouts of fine-pore aeration tubes is made based on ACC.
- The performance of aeration systems equipped with different layouts of fine-pore aeration tubes is comparatively studied based on ACC.

### GRAPHICAL ABSTRACT



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## 1. INTRODUCTION

Fine bubble diffused aeration systems equipped with fine-pore aeration tubes have been increasingly used in cylindrical aeration tanks (Duchène *et al.* 2001; Mayer *et al.* 2006; Schierholz *et al.* 2006; Temmerman *et al.* 2015; Wang & Zhang 2017). These aeration systems have several advantages, which have contributed to their extensive development: high oxygenation performances, adaptability to varying oxygen requirements, and a reduction in the production of aerosols (Duchène *et al.* 2001; Gillot *et al.* 2005).

Much research has been carried out on choosing an appropriate means to evaluate aeration performance for these aeration systems. In the past, a simple calculation of overall mass transfer effect was used to evaluate the performance of the aeration system. Kim & Ra (2005), Anjali & Rubina (2015) and Sun *et al.* (2016) calculated the total volumetric oxygen transfer coefficient ( $K_L\alpha$ ) of fine bubble diffused aeration systems in different aeration tanks, Gillot *et al.* (2000) and Andrés *et al.* (2018) studied the specific standard oxygen transfer efficiency (SSOTE) of fine-pore diffusers in activated sludge reactors, and Capela *et al.* (2001) and Gillot *et al.* (2005) defined a transfer number  $N_T$  of fine bubble diffused aeration systems. Although these methods were more direct and simple for evaluating the performance of fine bubble diffused aeration systems, they tended to focus on overall efficiency of oxygen mass transfer and failed to take account distribution uniformity of dissolved oxygen (DO) concentration in aeration tanks (Kim & Ra 2005; Pittoors *et al.* 2014; Anjali & Rubina 2015; Sun *et al.* 2016). The nonuniform distribution of DO concentration in aeration tanks often leads to inconsistency of the biological treatment status of sewage in the different positions of cylindrical aeration tanks and then affects the biological treatment efficiency of them (Temmerman *et al.* 2015; Wang & Zhang 2017; Andrés *et al.* 2018). Therefore there is a need to study a new means for performance evaluation of fine bubble diffused aeration systems, which takes both overall efficiency of oxygen mass transfer and distribution uniformity of DO concentration in cylindrical aeration tanks into consideration. To show distribution of DO concentration in different horizontal and vertical position of a cylindrical aeration tank, the following parameters should be considered: distribution of DO concentration in the horizontal direction of different water depth of a cylindrical aeration tank (DH), distribution of DO concentration in the vertical direction of the aeration tank (DV), distribution of DO concentration in all the sampling points of the aeration tank (DA) and ratio of total mass increment of DO in the aeration tank to total mass of oxygen in aeration air (RTT).

Therefore distribution uniformity of DO concentration in a cylindrical aeration tank involves the above factors and the functional relationships between them are hard to deduce directly by conventional calculation methods. Owing to its ability to show the complex interaction between interrelated factors, fuzzy theory is more appropriate for developing functions considering both overall efficiency of oxygen mass transfer and distribution uniformity of DO concentration compared with other mathematical methods (Arkajyoti & Swagatam 2018; Geranmehr *et al.* 2019; Li *et al.* 2019). In addition fuzzy theory has some advantages. Firstly, fuzzy theory evaluates the aeration performance on the basis of different comparable factors. Secondly, in fuzzy theory, the membership grade function (MGF) between the factors and aeration performance evaluation can be established based on the factor characteristic, and MGF will make the performance evaluation be more objective. Thirdly, fuzzy theory simulates the continuum characteristic of the complicated interplay between multiple factors efficiently. Finally, the evaluation of aeration performance is also a fuzzy concept so the fuzzy algorithm is appealing owing to its high discrimination of the ranks of objects influenced by interacting factors (Janmenjoy *et al.* 2017; Tolentino & Gerardo 2019; Xing & Li 2019).

The k-means algorithm and c-means algorithm are widely used in fuzzy clustering analysis. Although the k-means algorithm has been applied in many of the areas, it still has many disadvantages: (1) The value of k needs to be set in advance, however this is very difficult because it is hard to know in advance how many categories a given data-set should be divided into. (2) An initial division is determined according to the initial cluster center, and the selection of the initial clustering center affects greatly the clustering results. It is hard to make this selection, once the initial value is inaccurate, effective clustering results are hard to obtain. (3) The k-means algorithm has to adjust the sample classification and calculate the adjusted new cluster center constantly. Therefore, its calculation time of the algorithm is very long. Compared with the k-means clustering algorithm, the fuzzy c-means clustering algorithm is an improvement. Their data belonging to a certain class are determined by the membership function, so that each point does not directly pertain to a single cluster center. The number of clusters of c-means algorithm does not need to be set in advance and there is no need to calculate the adjusted new cluster centers constantly. Therefore, its calculation mode is simple and efficient, especially it is capable of dealing with compound multi-variable interactions. Considering the complicated interplay of the factors such as DH, DV, DA and

RTT, fuzzy c-means (FCM) algorithm is fitter for this study than k-means algorithm (Janmenjoy *et al.* 2017; Arkajyoti & Swagatam 2018; Geranmehr *et al.* 2019; Tolentino & Gerardo 2019; Xing & Li 2019).

In order to make a comprehensive performance evaluation of fine bubble diffused aeration systems in cylindrical aeration tanks, the fuzzy c-means (FCM) algorithm was chosen as a means and the aeration characteristic criterion (ACC) based on FCM was put forward as a new indicator in this paper. Considering the effect of the configuration of fine bubble diffusers on the distribution of DO concentration in cylindrical aeration tanks, the studies focus is not on the types but the layouts of fine-pore aeration tubes. The main objective of this paper was to evaluate the performance of fine bubble diffused aeration systems in cylindrical aeration tanks by the fuzzy c-means algorithm, and the aeration performance of different layouts of fine-pore aeration tubes were also compared.

## 2. MATERIALS AND METHODS

### 2.1. DO models of fine bubble diffused aeration system

Based on the oxygen mass transfer model proposed by the NFEN (2004) and ASCE (2007) standards, the basic equation of oxygen mass transfer is:

$$\frac{dC}{dt} = K_L \alpha (C_s - C) = K_L \delta \frac{6G}{d} (C_s - C) \quad (1)$$

where  $C$  is DO concentration in a cylindrical aeration tank ( $\text{ML}^{-3}$ ), and  $C_s$  is oxygen concentration at saturation at  $T^\circ\text{C}$  ( $\text{ML}^{-3}$ ),  $t$  is aeration time (T),  $K_L \alpha$  is total volumetric oxygen transfer coefficient ( $\text{T}^{-1}$ ),  $K_L$  is oxygen transfer coefficient ( $\text{LT}^{-1}$ ),  $\alpha$  is interfacial specific area of fine bubbles in the cylindrical aeration tank ( $\text{L}^{-1}$ ), and its expression can be written as follows, where  $G$  is the air flow rate ( $\text{ML}^{-3}$ ),  $d$  is initial diameter of fine air bubbles entering the aeration tank (L) and

$$\delta = \frac{3P_0}{9800H} \left[ 1 - \left( \frac{P_0}{P_0 + 9800H} \right)^{\frac{1}{3}} \right] \quad (\text{dimensionless}), \text{ where } P_0 \text{ is the standard atmospheric pressure } (\text{ML}^{-1}\text{T}^{-2}), \text{ and } H \text{ is the}$$

water depth of the aeration tank (L). Based on S1 in the Supplementary Material, the following expression can be obtained:

$$C = k_3 G \frac{\frac{3P_0}{9800H} \left[ 1 - \left( \frac{P_0}{P_0 + 9800H} \right)^{\frac{1}{3}} \right] H^{\frac{1}{2}}}{d^{\frac{5}{3}}} \cdot \left( \frac{S_P}{S} \right)^{0.12} \left( \frac{S_P}{S_a} \right)^{-0.06} \left( \frac{D_d}{h} \right)^{0.11} \left( \frac{H}{h} \right)^{0.05} \quad (2)$$

where  $S$  is the surface area of the aeration tank ( $\text{L}^2$ ),  $S_a$  is the total surface area of the zones covered by the diffusers, (aerated area) ( $\text{L}^2$ ),  $S_P$  is the total surface area of the fine-pore aeration tubes ( $\text{L}^2$ ),  $D_d$  is the diameter of the aeration tank (L),  $h$  is the diffuser submergence (L).

### 2.2. The performance evaluation parameters of distribution of DO concentration and overall efficiency of oxygen mass transfer in a cylindrical aeration tank

(1) Distribution of DO concentration in the horizontal direction of different water depth of a cylindrical aeration tank (DH)

It is assumed that there are  $L$  cross-sections set in the vertical direction of a cylindrical aeration tank and  $Q$  sampling points on each cross-section set for measurement of DO concentration. The average value of standard deviation coefficient ( $ADC_{DH}$ , (dimensionless)) of measured DO concentration in the horizontal direction of different water depth of the aeration tank can be written as follows:

$$ADC_{DH} = \frac{1}{L} \left( \frac{\sum_{a=1}^L \sqrt{\frac{1}{Q-1} \sum_{r=1}^Q (C_{ar} - \bar{C}_a)^2}}{\bar{C}_a} \right) \quad (3)$$

where  $C_{ar}$  is the measured DO concentration in the  $r$ th ( $r = 1, 2 \dots Q$ ) sampling point of the  $a$ th cross-section ( $\text{ML}^{-3}$ ), and  $\bar{C}_a$  is the average value of measured DO concentration in all the sampling points of the  $a$ th cross-section.

(2) Distribution of DO concentration in the vertical direction of the aeration tank (DV):

The standard deviation coefficient ( $DC_{DV}$ , (dimensionless)) of measured DO concentration in the vertical direction of the aeration tank can be written as follows:

$$DC_{DV} = \frac{\sqrt{\frac{1}{L-1} \sum_{a=1}^L (\bar{C}_a - C_H)^2}}{C_H} \quad (4)$$

where  $C_H$  is the average value of measured DO concentration in all the sampling points of the aeration tank ( $\text{ML}^{-3}$ ).

(3) Distribution of DO concentration in all the sampling points of the aeration tank (DA)

The standard deviation coefficient ( $DC_{DA}$ , (dimensionless)) of measured DO concentration in all the sampling point of the aeration tank can be written as follows:

$$DC_{DA} = \frac{\sqrt{\frac{1}{LQ-1} \sum_{a=1}^L \sum_{r=1}^Q (C_{ar} - C_H)^2}}{C_H} \quad (5)$$

(4) Ratio of total mass increment of DO in the aeration tank to total mass of oxygen in aeration air (RTT)

The ratio of total mass increment of DO in the aeration tank to total mass of oxygen in aeration air (RTT, (dimensionless)) can be written as follows:

$$ORA = \frac{m_{DO}}{10hm_{O_2}} = \frac{C_H Sh}{10hm_{O_2}} = \frac{C_H S}{3G} \quad (6)$$

$m_{DO}$  is the total mass increment of DO in the aeration tank (M),  $m_{O_2}$  is the mass flow of oxygen in the aeration air stream on per meter of the water depth of the aeration tank, ( $\text{ML}^{-1}$ ), and  $m_{O_2} = 0.3G$ . Obviously the smaller are the values of  $ADC_{DH}$ ,  $DC_{DV}$  and  $DC_{DA}$ , the more uniform is the distribution of DO concentration in the aeration tank. The larger is the value of RTT, the higher is the overall efficiency of oxygen mass transfer.

### 2.3. Fuzzy c-mean algorithm

A data set of  $n$  objects is divided into  $c$  clusters ( $c \leq n$ ) in the general FCM algorithm. The objective function  $J_m$  is constructed as follows, and this division can be obtained from minimizing  $J_m$  by iteration (Aree *et al.* 2001; Liou *et al.* 2003; Janmenjoy *et al.* 2017; Tolentino & Gerardo 2019):

$$J_m(U, V, X) = \sum_{i=1}^c \sum_{k=1}^n \mu_{ik}^m \left\| \vec{x}_k - \vec{v}_i \right\|_A^2, \quad 1 \leq m \leq \infty \quad (7)$$

where  $\vec{x}_k \in R^p$  is an object data  $k$  with  $p$ -dimension;  $X = (\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n) \in R^{p \times n}$  is a matrix of object data; the degree of membership  $\mu_{ik} \in [0, 1]$  is the likelihood of observation  $\vec{x}_k$  belonging to cluster  $i$ ,  $U \in R^{c \times n}$  is a matrix of similarity degrees;  $\vec{v}_i \in R^p$  is the prototype of the  $i$ th cluster ( $i = 1, 2 \dots c$ );  $V \in R^{p \times c}$  is a matrix of cluster centroids;  $\left\| \vec{x}_k - \vec{v}_i \right\|_A^2$  denotes distance functions, and while the covariance matrix of all the observations in the data set is equivalent to the identity matrix  $I$ , the distance metric is equivalent to the Euclidean distance norm, and  $m \in [1, \infty]$  called the fuzziness index, controls the level of dividing fuzziness.

Under the following control conditions, an iterative minimization algorithm is obtained by minimizing the objective function  $J_m$ , which is called the FCM algorithm (Aree *et al.* 2001; Liou *et al.* 2003; Dzung *et al.* 2015; Tolentino & Gerardo 2019):

$$(i) \ 0 \leq \mu_{ik} \leq 1 \forall i, k, \tag{8a}$$

$$(ii) \ \sum_{i=1}^c \mu_{ik} = 1 \forall k, \tag{8b}$$

$$(iii) \ 0 \leq \sum_{k=1}^c \mu_{ik} < n \tag{8c}$$

The following expression of  $\vec{v}_i$  and  $\mu_{ik}$  can be obtained:

$$\vec{v}_i = \frac{\sum_{k=1}^n (\mu_{ik})^m \vec{x}_k}{\sum_{k=1}^n (\mu_{ik})^m}, \ 1 \leq i \leq c \tag{9}$$

$$\mu_{ik} = \begin{cases} \frac{1 / \left[ \left\| \vec{x}_k - \vec{v}_i \right\|_A^{2/m-1} \right]}{\sum_{j=1}^c \left[ \frac{1}{\left\| \vec{x}_k - \vec{v}_j \right\|_A^{2/m-1}} \right]} & \text{for } \left\| \vec{x}_k - \vec{v}_i \right\|_A^2 > 0 \\ 1 & \text{for } \left\| \vec{x}_k - \vec{v}_i \right\|_A^2 = 0 \end{cases} \quad 1 \leq i \leq c, \text{ and } \sum_{i=1}^c \mu_{ik} = 1 \tag{10}$$

Picard iteration is used by the FCM algorithm to obtain the prototypes through the loop determined by Equations (9) and (10), and generates the minimum  $J_m$  for a fixed number of groups  $c$ . An observation,  $\vec{x}_k$ , is assigned to the cluster  $i$  ( $i = 1, 2, \dots, c$ ), while its degree of membership of the particular cluster  $\mu_{ik}$ , is greater than its membership values of all other clusters (Polomčić *et al.* 2017; Arkajyoti & Swagatam 2018; Geranmehr *et al.* 2019).

#### 2.4. The comprehensive performance evaluation model of the aeration system

On the basis of the extended FCM methodology, the aeration characteristic criterion (ACC) can be used to show the performance of fine bubble diffused aeration systems. Due to the similarity performance evaluation calculating the commonality of the observation and being assigned given performance levels orderly, ACC of an observation  $\vec{x}_k$  can be obtained from the accumulated summing up of its similarity degrees to all of the specific performance-ordered levels  $\mu_{ik}$  (for  $i = 1$  to  $c$ ) reset as S2 in the Supplementary Material. The corresponding segmented linear membership functions which are the data point changed from the value of each performance evaluation parameter to the memberships of the level of aeration performance evaluation are shown as follows:

$$f_n(x_n) = \begin{cases} 1 & \text{for } x_n < a_n \\ 2/3 + (1/3) \times \frac{b_n - x_n}{b_n - a_n} & \text{for } a_n \leq x_n \leq b_n \\ 1/3 + (1/3) \times \frac{c_n - x_n}{c_n - b_n} & \text{for } b_n \leq x_n \leq c_n \\ (1/3) \times \frac{d_n - x_n}{d_n - c_n} & \text{for } c_n \leq x_n \leq d_n \\ 0 & \text{for } x_n > d_n \end{cases} \quad n = DH, DV, DA \tag{11a}$$

$$f_m(x_m) = \begin{cases} 1 & \text{for } x_m > a_m \\ 2/3 + (1/3) \times \frac{x_m - b_m}{a_m - b_m} & \text{for } b_m \leq x_m \leq a_m \\ 1/3 + (1/3) \times \frac{x_m - c_m}{b_m - c_m} & \text{for } c_m \leq x_m \leq b_m \quad m = RTT \\ (1/3) \times \frac{x_m - d_m}{c_m - d_m} & \text{for } d_m \leq x_m \leq c_m \\ 0 & \text{for } x_m < d_m \end{cases} \quad (11b)$$

where  $f_{DH}(x_{DH})$ ,  $f_{DV}(x_{DV})$ ,  $f_{DA}(x_{DA})$  and  $f_{RTT}(x_{RTT})$  represent the membership functions of the parameters,  $DH$ ,  $DV$ ,  $DA$  and  $RTT$  respectively.  $a_n \sim d_n$  and  $a_m \sim d_m$  are set values.

The overall score increases with the increment of similarity between the commonality of an observation and the good level of aeration performance (Polomčić *et al.* 2017; Geranmehr *et al.* 2019; Li *et al.* 2019; Tolentino & Gerardo 2019). Based on the number of given aeration performance levels for developing a general formula, the weighting points of the levels  $q_i \in [0, 1]$  (for  $i = 1$  to  $c$ ) are recorded into equal parts. The ACC of observation  $\bar{x}_k$  can be established as follows according to accumulating one set of weighted similarity degrees (Aree *et al.* 2001; Liou *et al.* 2003; Dzung *et al.* 2015; Arkajyoti & Swagatam 2018):

$$ACC = \left( \sum_{i=1}^c \mu_{ik} \times q_i \right) \times 100 \quad (12)$$

For convenience, the factor of 100 is used. Therefore, the value of ACC varies from 0 to 100.

### 2.5. Experimental equipment

The experimental equipment for aeration in clean water in cylindrical aeration tanks is shown in Figure 1. Different groups of experiments were done with the different tanks and layouts of fine-pore aeration. The experimental apparatus of each group of experiment consisted mainly of test tanks with sampling tubes and sampling valves, an air compressor, an air controlling valve, an air flow meter, a gas-pressure meter, and an aeration pipe. The fine-pore aeration tube is a type of soft tubular aerator which is made from nano-polymer material, covered with numerous tiny holes, and it is produced by China’s Jiangli Fishery Machinery Co., Ltd. There are four kinds of layouts of the fine-pore aeration tubes, shown as Figure 2 ring-type diffuser (Figure 2(a)), parallel-lines-type diffuser (Figure 2(b)), cross-type diffuser (Figure 2(c)), and square-type diffuser (Figure 2(d)). The different layouts of the fine-pore aeration tubes in the aeration tank when  $D = 1.2$  m are shown as an example in Figure 3. Different experiments were performed with these different layouts of the fine-pore aeration tubes. The parameters of test tanks and fine-pore aeration tubes are shown on Table 1. The results of the oxygenation measurements

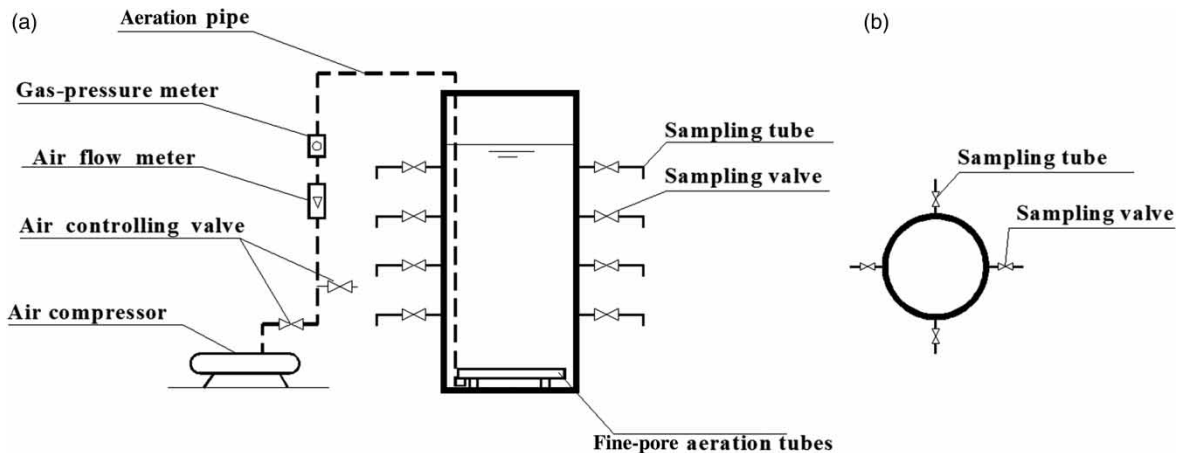
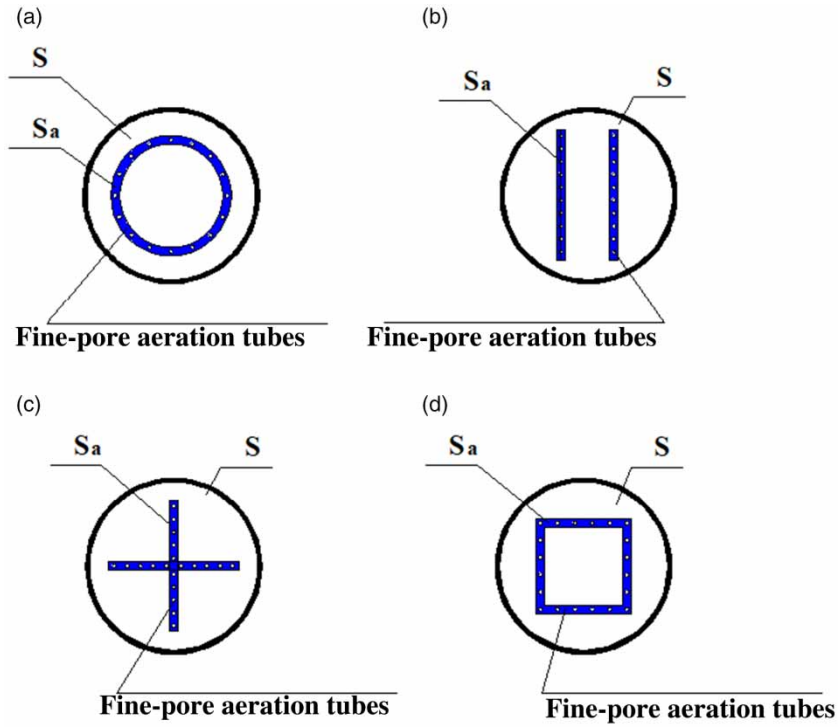
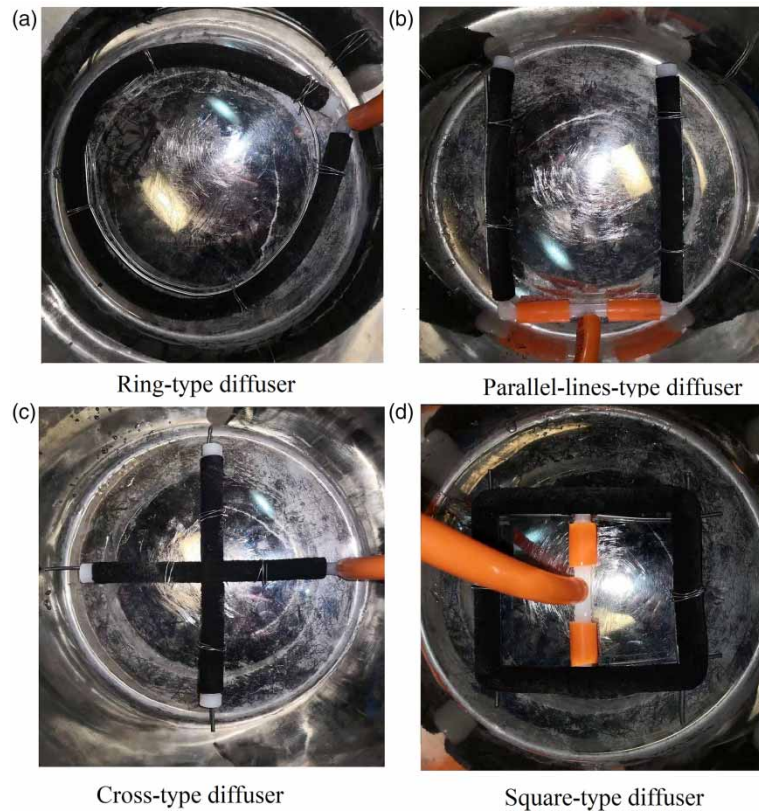


Figure 1 | Schematic diagram of the experimental equipment in test cylindrical aeration tanks.



**Figure 2** | Different layouts of fine-pore aeration tubes in test cylindrical aeration tanks.



**Figure 3** | The different layouts of fine-pore aeration tubes in the aeration tanks ( $D = 1.2$  m).

**Table 1** | Summary of the range of the relevant parameters in aeration

Parameter	$D_d$ (m)	$H$ (m <sup>2</sup> )	$h$ (m <sup>2</sup> )	$s$ (m <sup>2</sup> )	$S_a$ (m <sup>2</sup> )	$S_p$ (m <sup>2</sup> )	$G$ (m <sup>3</sup> /h)
Range	1–2	2–4	1.5–3.5	15.9–78.5	13–70.9	2.7–12.6	2.5–12
Parameter	$d_0$ (mm)	$S_p/S$	$S_p/S_a$	$D_d/h$	$H/h$		
Range	0.03–1.0	0.06–0.67	0.06–0.83	0.5–0.8	1.15–1.6		

are expressed at standard conditions: (1) initial DO concentration = 0 mg/L, (2) water temperature = 20 °C, (3) atmospheric pressure = 1,013 hPa. The aim of the tests, performed according to a standardized procedure (NFEN 2004; ASCE 2007), are to evaluate the performance of fine bubbles diffused aeration systems based on both distribution uniformity of DO concentration and overall efficiency of oxygen mass transfer in test cylindrical aeration tanks.

The sampling valves on each cross-section are electrically controlled. In the process of aeration, the water sample of four sampling points in each cross-section is obtained at the same time every 30 seconds, and the values of DO were determined by a portable dissolved oxygen meter.

### 3. RESULTS AND DISCUSSION

#### 3.1. Membership functions of the aeration performance evaluation

Set  $X = \{x_{DH}, x_{DV}, x_{DA}, x_{RTT}\}$  as a four-dimension sampling space of aeration performance evaluation. The piecewise linear membership functions of the critical variables are established. Four critical breakpoints: 0, 0.33, 0.67 and 1 are set based on the four specific standard levels of aeration performance evaluation. The average values of each parameter in each level of aeration performance are defined according to the degree of aeration performance evaluation (shown in Table 2).

#### 3.2. Determination of optimal fuzziness index $m$

Different groups of synthetic data sets are used for all of the measurements  $\vec{f}_k$  (for  $k=1$  to  $n$ ). There are 101 subsets of measurements varying from  $\vec{f}_0 = (0, 0, 0, 0)$  to  $\vec{f}_{101} = (1, 1, 1, 1)$  in every group, are employed to calculate the similarity degrees and ACC. According to FCM algorithm, the redundant specific aeration performance ordered levels are unnecessary since they weaken the validity of ACC (Aree *et al.* 2001; Liou *et al.* 2003; Dzung *et al.* 2015; Polomčić *et al.* 2017; Arkajyoti & Swagatam 2018). According to previous research, although given performance-ordered level  $c$  can be set as different values, the conclusions of aeration performance evaluation based on them are consistent with each other (Aree *et al.* 2001; Liou *et al.* 2003; Arkajyoti & Swagatam 2018; Lohani *et al.* 2018). For simplicity,  $c$  is set as 2 in this work. The fuzziness index,  $m$ , is set with 6/5, 2, 3, 5 and 12. The notations employed in the study for synthetic data are exhibited in Table 3. The subsets of  $\vec{e}_0 = (0, 0, 0, 0, 0, 0)$  and  $\vec{e}_{101} = (1, 1, 1, 1, 1, 1)$  represent the extremely ‘bad’ and extremely ‘good’ performance-ordered level respectively. Based on Equation (14), the similarity degrees,  $\mu_{0k}$  and  $\mu_{1k}$  between object  $\vec{x}_k$  to the two levels

**Table 2** | The defined key-points of the membership functions

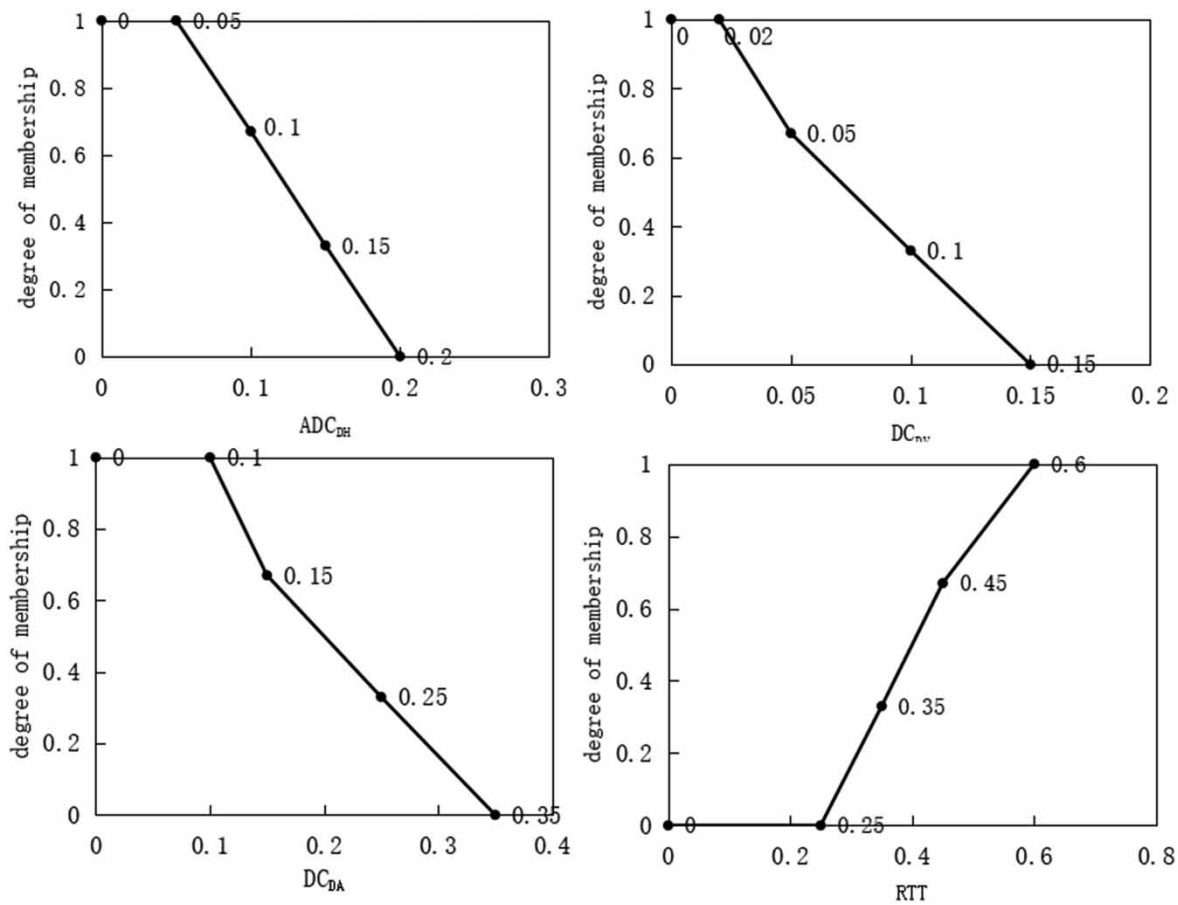
Membership degree ( $f_x$ )	0	1/3	2/3	1
$ADC_{DH}$	Under $a_{DH} = 0.05$	$b_{DH} = 0.1$	$c_{DH} = 0.15$	Above $d_{DH} = 0.2$
$DC_{DV}$	Under $a_{DV} = 0.02$	$b_{DV} = 0.05$	$c_{DV} = 0.1$	Above $d_{DV} = 0.15$
$DC_{DA}$	Under $a_{DA} = 0.1$	$b_{DA} = 0.15$	$c_{DA} = 0.25$	Above $d_{DA} = 0.35$
$RTT$	Above $a_{RTT} = 0.6$	$b_{RTT} = 0.45$	$c_{RTT} = 0.35$	Under $d_{RTT} = 0.25$

The rating curves of the four criteria variables resulting from the membership functions are shown in Figure 4.



**Table 3** | The notations employed in the study for synthetic data

Notation	Definition
$\vec{e}_{0/n-1}, \vec{e}_{1/n-1}, \dots, \vec{e}_{n-2/n-1}$ and $\vec{e}_{n-1/n-1}$	$n$ specific aeration performance ordered levels
$\vec{\mu}_{0/n-1}, \vec{\mu}_{1/n-1}, \dots, \vec{\mu}_{n-2/n-1}$ and $\vec{\mu}_{n-1/n-1}$	The similarity between the synthetic data and their identified performance levels
$\vec{q}_{0/n-1}, \vec{q}_{1/n-1}, \dots, \vec{q}_{n-2/n-1}$ and $\vec{q}_{n-1/n-1}$	The weighting points in accordance with aeration performance ordered levels

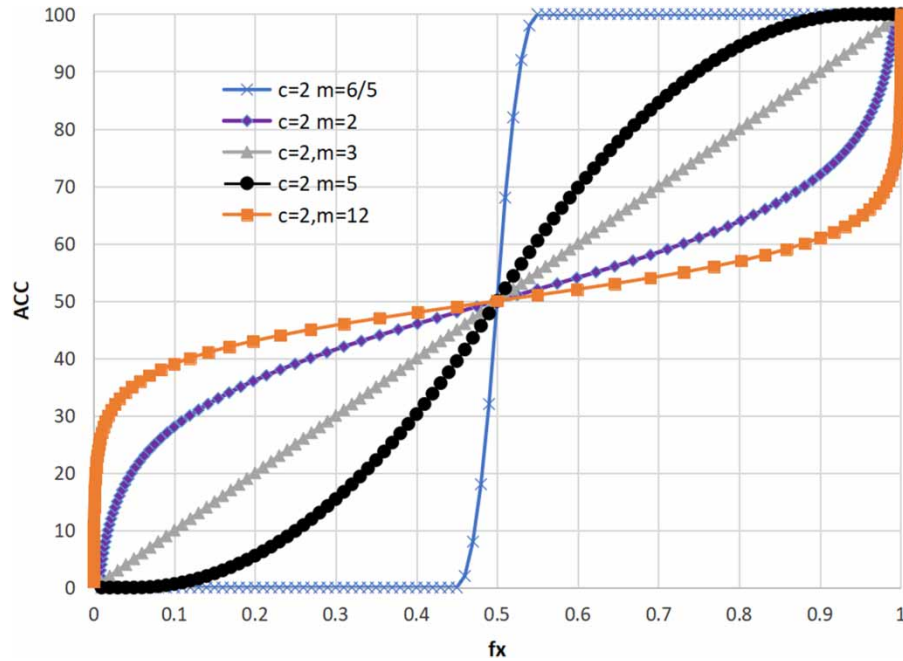


**Figure 4** | The rating curves of the employed parameters:  $DH$ ,  $DV$ ,  $DA$  and  $RTT$ .

from interval  $[0, 1]$  can be written as follows:

$$\mu_{0k} = \frac{1 / \left\| \vec{f}_k - \vec{e}_0 \right\|_A^{2/(m-1)}}{\left( 1 / \left\| \vec{f}_k - \vec{e}_0 \right\|_A^{2/(m-1)} \right) + \left( 1 / \left\| \vec{f}_k - \vec{e}_1 \right\|_A^{2/(m-1)} \right)} \tag{13}$$

$$\mu_{0k} = \frac{1 / \left\| \vec{f}_k - \vec{e}_0 \right\|_A^{2/(m-1)}}{\left( 1 / \left\| \vec{f}_k - \vec{e}_0 \right\|_A^{2/(m-1)} \right) + \left( 1 / \left\| \vec{f}_k - \vec{e}_1 \right\|_A^{2/(m-1)} \right)} \tag{14}$$



**Figure 5** | ACC curves for the different data sets with different fuzzy index ( $m$ ).

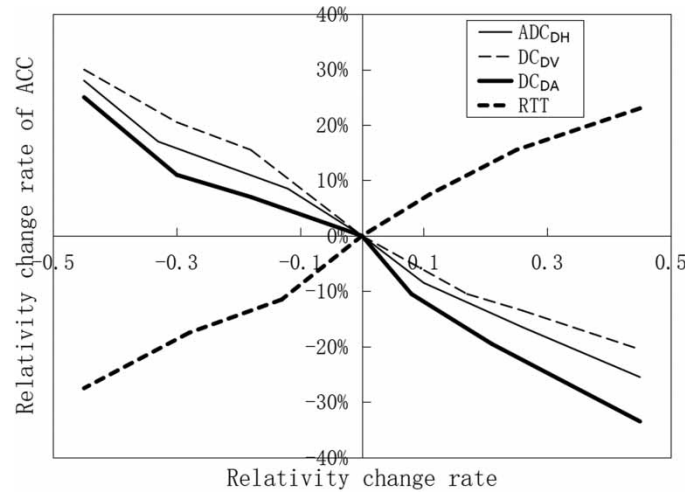
According to Equation (12), ACC of the object  $\vec{x}_k$  can be calculated using the following formula:  $ACC_k = (\mu_{0k} \times 0 + \mu_{1k} \times 1) \times 100$ . The ACC curves for the 101 synthetic data sets on the basis of two specific performance levels with five varying fuzzy index  $m$  are shown in Figure 5. Essentially, the ACC increases with the increasing value of the defaulted data set,  $\vec{f}_k$ , so the performance of the observation can be studied. According to Figure 5, when either  $m = 2$  or  $m = 5$ , the curves have imperfect linearity, when  $m = 12$ , for the values of ACC concentrate on 50, the largest fuzzy appears, and when  $m = 6/5$ , The ACC values are either 100 or 0, and exhibit crisp, only when  $m = 3$ , the most perfectly linear relationship exists. When  $m \rightarrow 1$ , either  $\mu_{ik} \rightarrow 0$  or  $\mu_{ik} \rightarrow 1$ , the clusters tend to be crisp and while  $m \rightarrow \infty$ ,  $\mu_{ik} \rightarrow 1/c$  can be yielded. Therefore, the range of  $2 \leq m \leq 5$  seems to be preferred for optimizing the performance of ACC. In addition, when  $m = 1$ , the partition is hard, and for  $m > 1$ , the partition is fuzzy and increasing  $m$  causes the partition to become fuzzy (Aree *et al.* 2001; Liou *et al.* 2003; Dzung *et al.* 2015; Janmenjoy *et al.* 2017; Polomčić *et al.* 2017; Geranmehr *et al.* 2019; Tolentino and Gerardo 2019; Xing & Li 2019).

### 3.3. Sensitivity analysis

Based on the above analysis, fuzziness index  $m = 3$  and two standard performance levels, perfect 'bad' and perfect 'good', are used in the aeration performance assessment. In order to obtain the specific transferred vector, the vector of the observation was set with  $\vec{x} = (x_{DH}, x_{DV}, x_{DA}, x_{RTT}) = (0.12, 0.08, 0.21, 0.39)$  where  $\vec{f} = (f_{DH}, f_{DV}, f_{DA}, f_{RTT}) = (0.5, 0.5, 0.5, 0.5)$ . Each of the four parameters changed in its range while other parameters remained initial. The relative sensitivity of ACC is shown in Figure 6. The result shows that the relative changes of ACC are from  $-31\%$  to  $+27\%$  when  $ADC_{DH}$ ,  $DC_{DV}$ ,  $DC_{DA}$  and  $RTT$  varies from  $-45\%$  to  $+45\%$  respectively. The values of ACC decrease with the increment of  $ADC_{DH}$ ,  $DC_{DV}$  and  $DC_{DA}$ , and increase with the increment of  $RTT$ . The relative changes of ACC for the four criteria variables match their membership functions respectively. Moreover it can be concluded from Figure 6 that ACC can sharply distinguish the change of performance with the variations of the four parameters ( $ADC_{DH}$ ,  $DC_{DV}$ ,  $DC_{DA}$  and  $RTT$ ).

### 3.4. Compared with other conventional characteristic criteria

(1) Total volumetric oxygen transfer coefficient at 20 °C ( $K_L \alpha_{20}$ )



**Figure 6** | The relative sensitivity analysis of ACC.

The expression of  $K_L\alpha_{20}$  can be written as follows (Anjali & Rubina 2015; Andrés *et al.* 2018):

$$K_L\alpha_{20} = 1.57GS^{-1.175}S_p^{0.14}S_a^{0.10}h^{-0.15} \quad (15)$$

(2) Specific standard oxygen transfer efficiency per meter (*SSOTE* in %/m of diffuser submergence), (Kim & Ra 2005; Schierholz *et al.* 2006; Li *et al.* 2008; Temmerman *et al.* 2015):

$$SSOTE = \frac{K_L\alpha C_{\infty,20}^*V}{10hm_{O_2}} \quad (16)$$

where the oxygen concentration at saturation  $C_{\infty,20}^* = 8.84h^{0.15}$  (Capela *et al.* 2001; Gillot *et al.* 2005), and Equation (16) can be transformed into the following expression based on Equation (15).

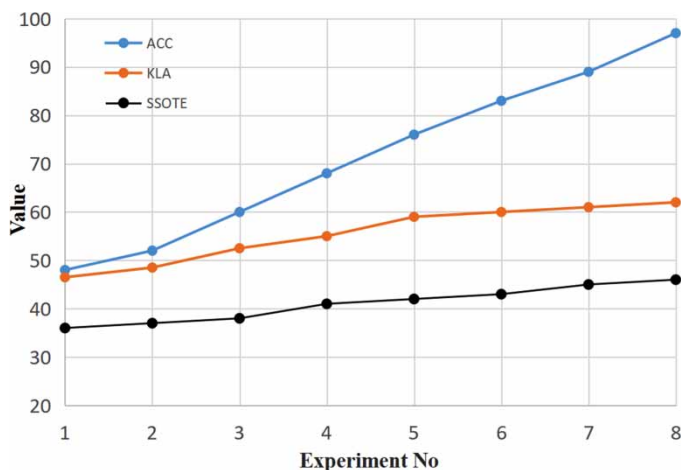
$$SSOTE = 4.90S^{-1.175}S_p^{0.14}S_a^{0.10}h^{-0.02} \quad (17)$$

Experiments (nos. 1–8) were performed for comparison of ACC with characteristic criteria such as  $K_L\alpha_{20}$  and *SSOTE* using different layouts of fine-pore aeration tubes. The relevant parameters in aeration of each experiment are shown in Table 4. Figure 7 shows the values of these three criteria in each experiment, respectively, and the y-axis in Figure 7 is the value of these criteria (the units of ACC,  $K_L\alpha_{20}$ , and *SSOTE* are dimensionless,  $\times 10^{-1} h^{-1}$  and  $\times 10^{-1} m^{-1}$  respectively).

Commonly the performance of fine bubble diffused aeration systems improves with the increase in ACC,  $K_L\alpha_{20}$  and *SSOTE*. It can be concluded from Figure 7 that the results of qualitative evaluation on the different layouts of fine-pore aeration tubes

**Table 4** | The relevant parameters in each aeration experiments

Experiment no.	Layout of fine-pore aeration tubes	$D$ (m)	$h$ (m <sup>2</sup> )	$H$ (m <sup>2</sup> )	$G$ (m <sup>3</sup> /h)	$d_0$ (mm)	Length of fine-pore aeration tube (m)
1	Ring-type diffuser	1.2	3	2.5	7	0.2	2
2	Parallel-lines-type diffuser						
3	Square-type diffuser						
4	Cross-type diffuser						
5	Ring-type diffuser	1.5	3.5	3.0	9	0.25	3
6	Parallel-lines-type diffuser						
7	Square-type diffuser						
8	Cross-type diffuser						



**Figure 7** | Comparison of ACC,  $K_L\alpha_{20}$ , and SSOTE in different aeration experiments.

by ACC and the other characteristic criteria such as  $K_L\alpha_{20}$  and SSOTE are in good accordance. Figure 7 shows that the deviation of the value evaluated with ACC in the different experiments is larger than that with  $K_L\alpha_{20}$  and SSOTE. Therefore, ACC is more marked and sensitive to show the difference in the aeration performance evaluation than  $K_L\alpha_{20}$  and SSOTE. Hence, it obvious that ACC is more fit for the performance evaluation of the fine bubble diffused aeration system with different layouts of fine-pore aeration tubes in aeration tanks.

### 3.5. Measurement of ACC

Different experiments were done with the different layouts of fine-pore aeration tubes such as ring-type diffuser, parallel-lines-type diffuser, square-type diffuser, and cross-type diffuser. The relevant parameters in aeration of each experiment are shown in Table 5. The ACC values of these experiments are shown in Figure 8 (the units of ACC are dimensionless). The result shows that when the minimum or maximum value of ACC appears (i.e. the worst or best aeration performance exists), the minimum or maximum values of  $ADC_{DH}$ ,  $DC_{DV}$ ,  $DC_{DA}$  and  $RTT$  do not occur correspondingly. Therefore, ACC is a comprehensive characteristic criterion for aeration performance evaluation incorporating all of the four factors. Its value is not affected by one of them, but influenced by their comprehensive interaction.

### 3.6. Comparison of performance evaluation of fine bubble diffused aeration systems equipped with different layouts of fine-pore aeration tubes based on ACC

Comparative experiments were performed to evaluate the performance of fine bubble diffused aeration systems equipped with different layouts of fine-pore aeration tubes such as ring-type diffuser, parallel-lines-type diffuser, square-type diffuser, and cross-type diffuser in different aeration conditions.

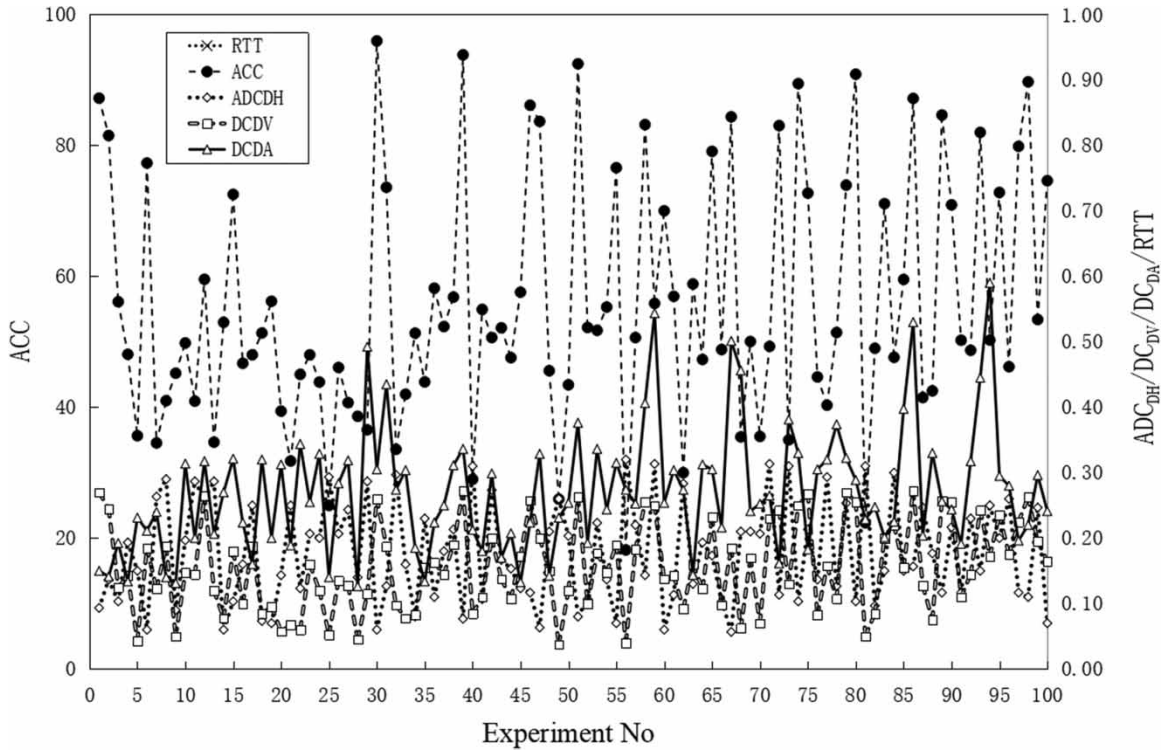
#### (1) Comparative study on the same length of fine-pore aeration tubes

Comparative experiments (nos. 1–10) were done. In each experiment, the performance of the fine bubble diffused aeration systems equipped with the above four layouts of fine-pore aeration tubes was evaluated respectively. The relevant parameters in each experiment are shown in Table 6, and the whole length of the aeration tubes were all 1.5 m in every experiment. Figure 9 shows the ACC values of each experiment.

It can be concluded from Figure 9 that based on the value of ACC, the performance of fine bubble diffused aeration systems equipped with different layouts of fine-pore aeration tubes from best to worst in that order is ring-type diffuser > square-type

**Table 5** | The relevant parameters in each aeration experiment

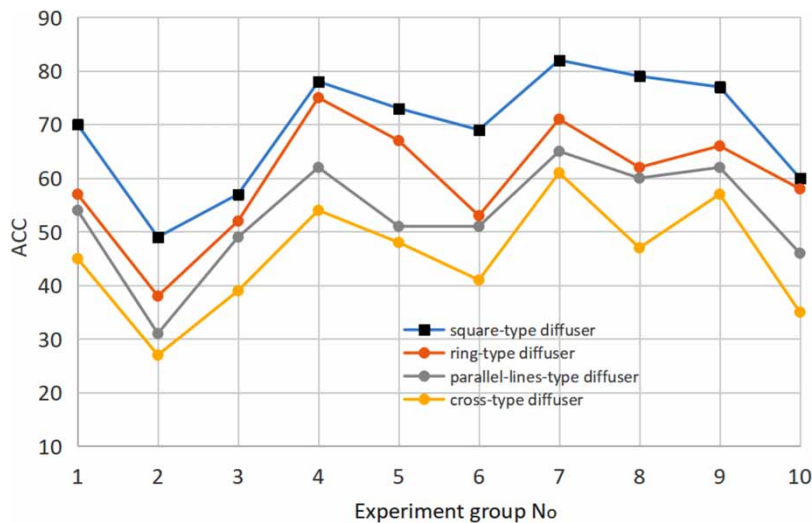
$D$ (m)	$h$ ( $m^2$ )	$H$ ( $m^2$ )	$G$ ( $m^3/h$ )	$d_0$ (mm)	Length of fine-pore aeration tube (m)
1.2, 1.5, 1.8	2.5, 3, 3.5, 4	2.0, 2.5, 3, 3.5	2.5, 3.5, 4.5, 5.5, 6, 6.5, 7	0.05, 0.1, 0.20, 0.25.	2, 3, 4



**Figure 8** | The values of ACC and  $ADC_{DH}$ ,  $DC_{DV}$ ,  $DC_{DA}$  and RTT in different experiments.

**Table 6** | The relevant parameters in aeration of each experiment

Experiment no.	1	2	3	4	5	6	7	8	9	10
$D$ (m)	1	1.2	1.2	1.5	1.5	1.8	1.8	1.8	2	2
$H$ (m)	2.2	2.4	2.6	2.7	2.8	2.5	2.6	2.8	3	3
$h$ ( $m^2$ )	2	2.2	2.4	2.5	2.4	2.2	2.4	2.6	2.7	2.7
$G$ ( $m^3/h$ )	2.8	3.9	4.5	5.5	7.3	8.2	8.8	9.6	10.5	11
$d_0$ (mm)	0.15	0.25	0.33	0.36	0.45	0.27	0.41	0.53	0.62	0.75



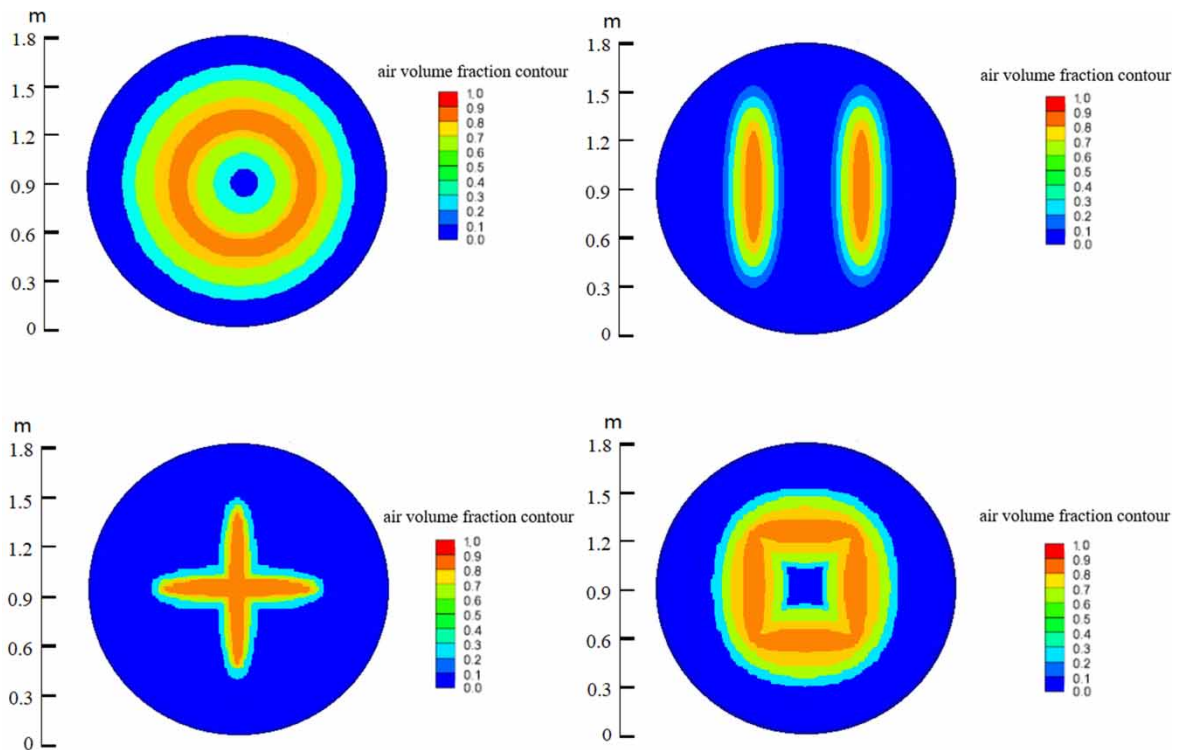
**Figure 9** | The values of ACC in comparative experiments on the same length of fine-pore aeration tubes.

diffuser > parallel-lines-type diffuser > cross-type diffuser. Compared with other layouts of fine-pore aeration tubes, the ring-type diffuser can use the thrust to rotate the water flow in aeration tanks to generate a stable vortex flow field, so that fine bubbles can contact with the water fully and this leads to high overall efficiency of oxygen mass transfer and good uniform distribution of DO concentration in aeration tanks (Duchène *et al.* 2001; Kim & Ra 2005). In the case of the square-type diffuser, it can generate thrust and make a minor horizontal push flow, however, the push flow cannot make fine bubbles and water mixes well due to the lack of turbulent force, and this results in the worse distribution uniformity of DO concentration in the aeration tank and lower overall efficiency of oxygen mass transfer compared with that caused by the ring-type diffuser. Compared with the ring-type diffuser and the square-type diffuser, the parallel-lines-type diffuser and the cross-type diffuser tend to lead to low overall efficiency of oxygen mass transfer and non-uniform distribution of DO concentration in aeration tanks owing to the absence of vortex flow formed by fine bubbles (Gillot *et al.* 2000; Schierholz *et al.* 2006; Fayolle *et al.* 2010). Especially for the cross-type diffuser, its fine-pore aeration tubes are arranged in a cross shaped, and the surface area of aeration tanks is not completely covered by the fine bubbles generated by the diffuser, so it has the worst overall efficiency of oxygen mass transfer and distribution uniformity in these four layouts (Gillot *et al.* 2000; Schierholz *et al.* 2006).

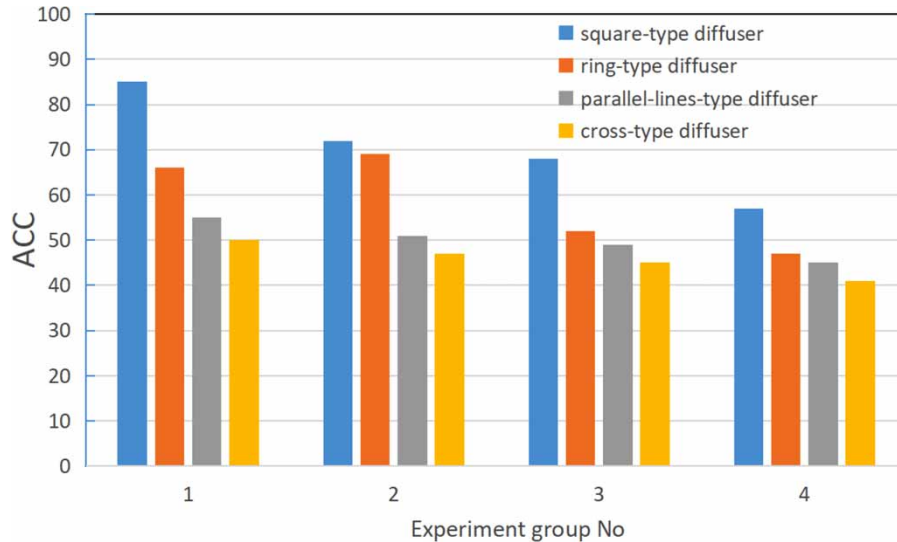
The above conclusion can be proved by the CFD models shown in Figure 10. Based on the air volume fraction contour in the profile of the same vertical depth of aeration tank equipped with different layouts of fine-pore tubes, Figure 10 indicates that the coverage area of fine bubbles generated by ring-type diffuser is the largest, followed by square-type diffuser and parallel-lines-type diffuser, and cross diffuser is the smallest. Therefore, the contact area between the bubble and water of the ring-type diffuser is also the largest, so its oxygen mass transfer efficiency is the highest, and that of the cross-type diffuser is the lowest. Similar results can be obtained in the other profile of vertical depth of aeration tank.

## (2) Comparative study on the different length of fine-pore aeration tubes

Four groups of comparative experiments were carried out. In each group of experiments, the performance evaluation of the fine bubble diffused aeration systems equipped with the four layouts of fine-pore aeration tubes was conducted. The parameters in each group of experiments were the same as follows:  $D = 1.8\text{m}$ ,  $H = 2.5\text{m}$ ,  $h = 2.3\text{m}$ ,  $G = 8\text{m}^3/h$ ,



**Figure 10** | The air volume fraction contour in the profile of the same vertical depth (vertical depth = 1.5 m) of aeration tank equipped with different layouts of fine-pore tubes in Experiment 6.



**Figure 11** | The values of ACC comparative experiments with different lengths of fine-pore aeration tubes.

$d_0 = 0.26\text{mm}$  and the length of fine-pore aeration tubes in experiment group No.1 to No.4 were 3 m, 2.5 m, 2 m and 1.5 m respectively. The values of ACC are shown in Figure 11. Hence, the fine bubble diffused aeration systems performance evaluation on the basis of ACC can be concluded as: ring-type diffuser is the best followed by square-type diffuser and parallel-lines-type, and the performance of cross-type diffuser is the worst. Figure 11 shows the same results as those from Figure 9. Moreover, in the same air flow rate of aeration, the longer is the length of fine-pore aeration tubes, the better is the aeration performance. The reason is that the increment of fine-pore aeration tubes length leads to a decrease in air flow rate of aeration per unit length of tubes and the size of fine bubbles decreased correspondingly, and this results in the increase in the specific surface area of gas-liquid mass transfer (Schierholz *et al.* 2006; Zhang *et al.* 2009).

#### 4. CONCLUSION

- (1) Compared with total volumetric oxygen transfer coefficient ( $K_L\alpha$ ) and specific standard oxygen transfer efficiency (SSOTE), the aeration characteristic criterion (ACC) is demonstrated to be marked and sensitive for the performance evaluation of fine bubble diffused aeration systems equipped with fine-pore aeration tubes.
- (2) Comparison of performance evaluation of fine bubble diffused aeration systems equipped with different layouts of fine-pore aeration tubes is made based on ACC, and the result of performance evaluation from best to worst in order is ring-type diffuser > square-type diffuser > parallel-lines-type diffuser > cross-type diffuser.
- (3) The fuzzy c-means algorithm is used as a new means to evaluate the performance of fine bubble diffused aeration systems in cylindrical aeration tanks. The model based on fuzzy c-means (FCM) algorithm can enhance the influence of DO distribution uniformity on the aeration performance evaluation. When the aeration characteristic criterion is used, not only the overall efficiency of oxygen mass transfer of fine bubble diffused aeration systems but also the distribution uniformity of DO concentration in the horizontal and vertical direction of aeration tank should be taken into account.

#### DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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