Total Corneal Power Estimation: Ray Tracing Method versus Gaussian Optics Formula

Li Wang,1 Ashraf M. Mahmoud,2 Betty Lise Anderson,3 Douglas D. Koch,1 and Cynthia J. Roberts2

PURPOSE. To evaluate with the use of corneal topographic data the differences between total corneal power calculated using ray tracing (TCP) and the Gaussian formula (GEP) in normal eyes, eyes that previously underwent laser in situ keratomileusis/photorefractive keratectomy (LASIK/PRK), and theoretical models.

METHODS. TCP and GEP using mean instantaneous curvature were calculated over the central 4-mm zone in 94 normal eyes, 61 myopic-LASIK/PRK eyes, and 9 hyperopic-LASIK/PRK eyes. A corneal model was constructed to assess the incident angles at the posterior corneal surface for both refracted rays and parallel rays. Corneal models with varying parameters were also constructed to investigate the differences between mean TCP and GEP (4-mm zone), and an optical design software validation was performed.

RESULTS. The TCP values tended to be less than GEP in normal and myopic-LASIK/PRK eyes, with the opposite relationship in some hyperopic-LASIK/PRK eyes having the highest anterior surface curvature. The difference between TCP and GEP was a function of anterior surface instantaneous radii of curvature and posterior/anterior ratio in postrefractive surgery eyes but not in normal eyes. In model corneas, posterior incident angles with parallel rays were greater than those with refracted rays, producing an overestimation of negative effective posterior corneal power; differences in magnitude between TCP and GEP increased with decreasing ratio of posterior/anterior radii of curvature, consistent with clinical results.

CONCLUSIONS. In eyes after refractive surgery, calculating posterior corneal power using the Gaussian formula and its paraxial assumptions introduces errors in the calculation of total corneal power. This may generate errors in intraocular lens power calculation when using the Gaussian formula after refractive surgery. (Invest Ophthalmol Vis Sci. 2011;52:1716–1722) DOI:10.1167/iovs.09-4982

Accurate estimation of the total corneal refractive power is important in the calculation of intraocular lens power. Traditionally, anterior corneal curvature is measured using a keratometry or computerized videokeratography (CVK). To compensate for posterior corneal curvature, keratometers and CVK devices use a standardized index of refraction to convert measurements of anterior corneal curvature to the refractive power of the entire cornea. In most keratometers and CVK devices, a value of 1.3375 is used that is based on the assumption of a single refracting surface. Clinically, this methodology has provided acceptable values for tasks such as intraocular lens calculations in normal, unoperated corneas. However, in eyes that have previously undergone ablative corneal refractive surgery (e.g., excimer laser photorefractive keratectomy [PRK] or laser in situ keratomileusis [LASIK]), the relationship between the front and the back surfaces of the cornea has been altered,1–3 and the use of the standardized index of refraction of 1.3375, which does not account for the altered relationship between the anterior and posterior surfaces, is no longer valid.3

Because of the development of scanning slit and Scheimpflug technology for topographic devices, it is now possible to measure posterior corneal curvature. Total corneal power can be calculated based on measurements of anterior and posterior corneal curvatures and corneal thickness. Methods for calculating total corneal power include ray tracing and the Gaussian optics thick lens formula.4–6

The purposes of the present study were to evaluate in normal corneas and corneas that had undergone LASIK/PRK the differences between values for total corneal power calculated using the ray tracing method (with Snell’s Law refraction at both the anterior and the posterior surfaces) and the Gaussian optics formula and to further explore in theoretical model eyes the factors contributing to these differences.

PATIENTS AND METHODS

Analysis in Clinical Subjects

We obtained institutional review board approval for this study. This research adhered to the tenets of the Declaration of Helsinki. Retrospectively, we reviewed consecutive cases of subjects who visited Baylor College of Medicine during January 2008 to October 2008. Inclusion criteria were patients who underwent no previous corneal or ocular surgery in the normal group or who underwent LASIK at least 3 months previously or PRK at least 6 months previously and patients who had Galilei (Galilei Dual Scheimpflug Analyzer; Ziemer Ophthalmics AG, Port, Switzerland) measurements with good quality (quality check mark displayed on the Galilei maps). Three groups were included: (1) 94 eyes of 58 patients in the normal eye group; the mean (±SD) age was 36 ± 11 years (range, 20–62 years); these subjects were selected from the patients screened for corneal refractive surgery; (2) 61 eyes of 36 patients in the myopic-LASIK/PRK group; the mean age was 38 ± 9 years (range, 21–54 years), and the myopic correction was −3.66 ± 1.66 D (range, −7.58 to −1.00 D); (3) 9 eyes of 5 patients in the hyperopic-LASIK/PRK...
group: the mean age was 52 ± 4 years (range, 45–54 years), and the hyperopic correction was +2.30 ± 1.10 D (range, +1.00 to +4.46 D).

**Ray Tracing Method**

The analyzer we used (Galilei Dual Scheimpflug Analyzer; Ziemer Ophthalmics AG) combines dual-channel Scheimpflug cameras with an integrated Placido disc to measure both anterior and posterior corneal surfaces and corneal thickness. The Galilei calculates the total corneal power (TCP) using ray tracing, which propagates incoming parallel rays and uses Snell’s law to refract these rays through the anterior and posterior corneal surfaces. Power is determined by n/f, based on the calculated focal length (f), which is referenced to the anterior corneal surface, and n is the index of refraction of the aqueous (n = 1.336). TCP values over the central, paracentral, and peripheral zones are displayed. We recorded the average TCP over the central 4-mm area for each eye and used the index of refraction of the aqueous (n = 1.336) to convert ray traced focal length to power.

**Gaussian Formula**

The Gaussian formula calculates Gaussian equivalent power (GEP) by assuming paraxial imaging and combining two lenses separated by the central corneal thickness:

\[
\text{GEP} = F1 + F2 - (d/n)(F1+F2)
\]

where \(F1\) = anterior corneal power, \(F2\) = posterior corneal power, \(d\) = pachymetry, and \(n\) = index of refraction (1.376). In this study, the \(F1\) value was calculated using a paraxial formula \(^6\) by converting the average central instantaneous curvature (central 4-mm zone) displayed on the Galilei in diopeters to anterior power by multiplying by 376/537.5. The \(F2\) value was the posterior average central instantaneous curvature, for which the dioptic value displayed on the Galilei was calculated using the same paraxial formula with both the corneal (1.376) and the aqueous (1.336) indices of refraction. The pachymetric value used was the average over the central 4-mm area, as displayed on the Galilei. As with most corneal topographers, the posterior curvature is converted to diopeters using the same formula as the anterior surface, assuming that incoming rays are parallel. It should also be noted that the GEP is referenced to the second principal plane, which is distinct from the TCP calculation, which is referenced to the anterior corneal surface.

**Data Analysis**

The differences between the TCP and GEP were calculated in the three groups of patients. Student’s t-test was used to compare the TCP and GEP, and correlation analysis was performed to assess the relationship between the differences of TCP and GEP and the anterior instantaneous radii of curvature as well as the posterior/anterior ratio. Statistical analysis was performed using statistical analysis software (SPSS, version 15.0; SPSS, Inc., Chicago, IL), and \(p \leq 0.05\) was considered statistically significant.

**Theoretical Analysis**

**Model with Average Parameters in Normal Eyes.** A corneal model was constructed using the mean values found in the normal eyes included in this study (anterior radius of curvature, \(r1 = 7.7\) mm; posterior radius of curvature, \(r2 = 6.3\) mm; and central pachymetry = 0.56 mm). The incident angles at the posterior corneal surfaces were calculated in the ray tracing method by refracting incoming parallel rays at the anterior corneal surface using Snell’s law. The differences in incident angles between these refracted and parallel rays were analyzed. Furthermore, values for effective posterior corneal power (EPP) were calculated using the ray traced angle of incidence on the posterior surface and the refracted angle through the posterior surface. Therefore, EPP is the ray traced power of the posterior surface using nonparallel rays refracted by the anterior surface that have been propagated through the corneal thickness. This power is referenced to the posterior surface with \(n1 = 1.376\) and \(n2 = 1.336\). EPP values were then compared to values for posterior corneal powers used in the Gaussian formula (GPP), which were determined by the topographer using the paraxial approximation (\(GPP = (1336 - 1376)/r2\), where \(r2 = \) posterior corneal radius of curvature), which is based on the assumption of parallel rays approaching the posterior corneal surface.

**Model with Varying Parameters.** A set of theoretical corneas with two spherical surfaces representing the anterior and posterior corneal surfaces was constructed. The anterior corneal radius of curvature ranged from 6.5 mm to 10.0 mm, in 0.25-mm steps. The ratio of posterior to anterior radii of curvature ranged from 0.7 to 0.9, in 0.025 steps. Central pachymetry ranged from 450 \(\mu\)m to 550 \(\mu\)m, in 25-\(\mu\)m steps. Rays of light were propagated through both surfaces assuming indices of refraction as follows: air = 1.0, cornea = 1.376, and aqueous = 1.336. Average TCP and GEP within the central 4-mm zone were calculated for each posterior/anterior ratio and pachymetry. These average values were calculated using the same zone as that used in the clinical patients. The differences between TCP and GEP (TCP - GEP) were analyzed as functions of ratio of posterior/anterior radius of curvature, pachymetry, and anterior corneal power.

The same sets of theoretical corneas were implemented in optical design software (ZEMAX; ZEMAX Development Corp., Bellevue, WA). The surfaces were spherical. The pupil (aperture stop) measured 2 mm in radius. The value for pachymetry was assumed in ZEMAX to be apex to apex (e.g., measured along the axis), the thickness was therefore not uniform. The input was a set of rays traveling parallel to the optical axis and filling the pupil. The focal point was calculated to be where the radial spot size was minimized, using nonparaxial ray calculations. The effective focal length (EFL) referred to air is reported by ZEMAX, referenced to the second principal plane. The power computed from the EFL is Power = 1/EFL(meters).

**RESULTS**

**Clinical Subjects**

Anterior and posterior instantaneous radii of curvature values are shown in Table 1. The mean ratio of posterior/anterior instantaneous radii of curvature was 0.82 in normal eyes, 0.76 in myopic-LASIK/PRK eyes, and 0.86 in hyperopic-LASIK/PRK eyes. Values for TCP calculated using ray tracing and for GEP calculated with the Gaussian formula are shown in Table 2. TCP tended to be less than GEP in normal and myopic-LASIK/PRK eyes, with the opposite relationship in some hyperopic-LASIK/PRK eyes having the highest anterior surface curvature. In general, the absolute differences between the TCP and GEP

<table>
<thead>
<tr>
<th>Anterior (mm)</th>
<th>Posterior (mm)</th>
<th>Ratio (posterior/anterior)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal eyes (n = 94)</td>
<td>7.69 ± 0.24 (7.25–8.28)</td>
<td>6.27 ± 0.25 (5.71–7.04)</td>
</tr>
<tr>
<td>Myopic-LASIK/PRK eyes (n = 61)</td>
<td>8.29 ± 0.34 (7.46–9.08)</td>
<td>6.34 ± 0.26 (5.60–6.81)</td>
</tr>
<tr>
<td>Hyperopic-LASIK/PRK eyes (n = 9)</td>
<td>7.46 ± 0.14 (7.30–7.68)</td>
<td>6.40 ± 0.17 (6.20–6.65)</td>
</tr>
</tbody>
</table>

Values are mean ± SD (range).
tended to increase with increasing anterior instantaneous radii of curvature in hyperopic-LASIK/PRK eyes with TCP > GEP and to decrease in myopic-LASIK/PRK eyes with TCP < GEP, whereas normal eyes showed no relationship with anterior surface curvature (Fig. 1). The Pearson correlation coefficient values were 0.064 (P = 0.069) in myopic-LASIK/PRK eyes, and −0.313 (P = 0.412) in hyperopic-LASIK/PRK eyes. If the post-refractive surgery eyes were grouped together, the Pearson correlation coefficient value was −0.504 (P < 0.001). Note that without the single outlier in the normal population, the range of difference is <1 D in normal eyes and approximately 1.5 D in eyes after refractive surgery. The differences between TCP and GEP were also a function of posterior/anterior ratio in eyes after refractive surgery, whereas no relationship was found in normal eyes (Fig. 2). Differences were greatest at the lowest ratios in myopic LASIK/PRK eyes.

Theoretical Analysis

Model with Average Parameters in Normal Eyes. With r1 of 7.7 mm, r2 of 6.3 mm, and pachymetry of 0.56 mm, at the posterior corneal surface, the incident angles with parallel rays were greater than the incident angles with rays refracted by the anterior corneal surface. The difference in incident angles between the parallel rays and the refracted rays increased with increasing distance from the center. The differences between EPP and GPP decreased with increasing anterior corneal radius of curvature (decreasing curvature) and increased with increasing distance from the center of the cornea (Fig. 3).

Model with Varying Parameters. As the ratio of posterior/anterior radius of curvature decreased, the magnitude of the absolute differences between TCP and GEP increased. The average differences for anterior corneal radii of curvature from 6.5 mm to 10.0 mm (anterior corneal powers from 57.9 D to 37.6 D) ranged from −0.54 D for a ratio of 0.7 to −0.45 D for a ratio of 0.9 (Fig. 4). As central corneal thickness increased, the differences between TCP values and GEP values decreased; assuming a constant ratio for posterior/anterior radius of curvature of 0.8 and an anterior radius of curvature of 7.5 mm, the differences ranged from −0.46 D for thickness of 0.45 mm to −0.63 D for thickness of 0.55 mm (Fig. 5). As anterior corneal radius of curvature increased, the differences between TCP and GEP decreased (Fig. 6).

The result of the ZEMAX validation is shown in Figure 7 and indicates that in theoretical surfaces, both GEP and TCP show excellent correlation with the ZEMAX reference. The differences between the intercepts of the two formulas lie in their distinct references, with GEP and ZEMAX referenced to the second principal plane, whereas TCP is referenced to the anterior corneal surface.

DISCUSSION

Accurate estimation of corneal refractive power is critical in the calculation of intraocular lens power. Because it is possible to obtain measurements of posterior corneal curvature, total corneal power can be determined using either the Gaussian optics thick lens formula or ray tracing. Traditionally, the Gaussian formula has been used to calculate the equivalent corneal power. However, the dual Scheimpflug topographer used in this study also calculates total corneal power using the ray tracing method. To the best of our knowledge, this is the first study to compare the differences between values for total

### Table 2. TCP Using the Ray Tracing Method and the GEP Calculated with the Gaussian Formula

<table>
<thead>
<tr>
<th></th>
<th>TCP (D)</th>
<th>GEP (D)</th>
<th>Difference (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal eyes</td>
<td>42.27 ± 1.33 (39.26–44.96)</td>
<td>42.71 ± 1.33 (39.65–45.29)</td>
<td>-0.44 ± 0.20 (-0.89 to 0.72)</td>
</tr>
<tr>
<td>Myopic-LASIK/PRK eyes</td>
<td>38.65 ± 1.82 (34.48–42.86)</td>
<td>39.20 ± 1.72 (35.47–43.40)</td>
<td>-0.55 ± 0.29 (-1.37 to 0.08)</td>
</tr>
<tr>
<td>Hyperopic-LASIK/PRK eyes</td>
<td>44.41 ± 1.11 (42.82–45.64)</td>
<td>44.33 ± 0.87 (43.02–45.64)</td>
<td>0.08 ± 0.47 (-0.84 to 0.71)</td>
</tr>
</tbody>
</table>

Values are mean ± SD (range).
corneal power calculated using ray tracing and the Gaussian formula.

Our results showed that ray tracing calculated lower values for corneal power than did the Gaussian formula for post-myopic LASIK eyes and normal eyes (mean differences of $-0.55$ D and $-0.44$ D, respectively) and a slightly higher mean difference of $+0.08$ D for post-hyperopic LASIK eyes. One source for the differences between TCP and GEP is the distinct reference, with TCP referenced to the anterior surface of the cornea and GEP to the second principal plane, in front of the cornea. In normal eyes, the differences between TCP and GEP are independent of anterior surface curvature and posterior-anterior ratio. However, after refractive surgery, the differences between TCP and GEP are a function of both posterior/anterior ratio and anterior surface curvature. This is likely due to the dramatically altered surface profile after refractive surgery, which changes the region over which paraxial calculations are appropriate. Consistent with theoretical predictions, the lower posterior/anterior radius of curvature ratio in the myopic group was associated with the greatest absolute differences between TCP and GEP, resulting from error in the use of paraxial topography-driven values for $F_2$ in the GEP formula. Interestingly, theoretical surfaces predicted that greater anterior surface curvature would result in the greatest absolute difference between TCP and GEP. However, this was not consistent with clinical results, which showed that the greatest absolute differences were at the lowest anterior surface curvature in the myopic group. This leads to the conclusion that the posterior/anterior ratio has a stronger impact on the magnitude of the difference in TCP and GEP than anterior surface curva-

**Figure 2.** Differences between the TCP with the ray tracing from the Galilei and GEP using the Gaussian formula as a function of ratio of posterior/anterior instantaneous radius of curvature. In post-refractive surgery eyes, the differences in magnitude between the TCP and GEP increased with decreasing ratio. The Pearson correlation coefficient value was 0.654 ($P < 0.001$).

**Figure 3.** Differences between EPP determined by the ray tracing method from the Galilei and the posterior corneal power calculated using the GPP as functions of anterior corneal radii of curvature and the distance from the center of the cornea ($r_2 =$ posterior corneal radius of curvature).
ture and that the paraxial region of both the anterior and the posterior surfaces interact to determine the size of the valid paraxial region, especially after refractive surgery.

Figures 1 and 2 provide insight into the source of error in calculating intraocular lens (IOL) power after refractive surgery. Although the average differences between TCP and GEP were similar in myopic subjects and normal subjects, the variability was much higher in the postrefractive surgery subjects. Without the single outlier in the normal group, the variability would have been approximately half that of either the myopic or the hyperopic subjects. In addition, there was a significant relationship between the TCP-GEP difference and both the posterior/anterior ratio and anterior surface curvature in eyes after refractive surgery. These relationships are absent in normal eyes. The distribution of the error function in the normal population confirms what clinical experience has shown: there would be acceptable accuracy with IOL calculations that use an empiric formula with an assumed posterior surface. However, the distribution of the error function of both the hyperopic and the myopic subjects unfortunately also confirms clinical experience that, because of the variability in these populations and the significant slope with changing anterior curvature and posterior/anterior ratio, standard IOL calculation formulas are not sufficiently accurate for these eyes. Therefore, we believe that, in eyes that have undergone LASIK/PRK, the use of values for total corneal power calculated with ray tracing will prove to be superior to corneal power calculations based on the anterior curvature alone or the GEP.

**Figure 4.** Differences between the TCP using the ray tracing method from the Galilei and GEP with the Gaussian formula as a function of ratio of posterior/anterior radius of curvature with a constant central pachymetry of 0.5 mm.

**Figure 5.** Differences between the TCP using the ray tracing method from the Galilei and GEP with the Gaussian formula as a function of pachymetry with a constant ratio of posterior/anterior radius of curvature of 0.8.
In studies using an automatically rotating Scheimpflug camera (Pentacam; Oculus, Wetzlar, Germany) to measure normal corneas, the equivalent corneal power calculated using the Gaussian formula was consistently lower than the simulated keratometry (SimK) obtained from various devices by 1.2 to 1.3 D (Table 3). Using optical coherence tomography (OCT), in normal eyes, the total corneal power calculated by the summation of the anterior and posterior corneal powers underestimated the Atlas SimK (Humphrey Atlas; Carl Zeiss Meditec, Jena, Germany) by 1.13 D. The contribution of corneal thickness in the Gaussian formula is around 0.1 D, indicating that the Gaussian formula using the OCT would have underestimated the SimK by approximately 1.23 D. These reported differences between the SimK and the equivalent corneal power calculated with the Gaussian formula are consistent with our finding of 1.30 D using the Galilei (Table 3).

The SimK is an estimation of total corneal power based on anterior corneal curvature and keratometric index of refraction, by modeling the cornea as a single refracting surface. Norrby\(^1\) pointed out that the commonly used index of refraction of 1.3375 gives the power at the posterior vertex of the cornea, and an index of 1.3315 proposed by Olsen\(^1\) gives the power at the second principal plane, which is approximately 0.8 D less than at the posterior vertex. Estimated corneal power is further reduced by about another 0.5 D when the recently reported lower posterior/anterior ratio of 0.813 is

\[ y = 0.9998x + 0.007 \]
\[ R^2 = 1 \]

\[ y = 0.9827x + 0.2123 \]
\[ R^2 = 0.9999 \]
used\(^{12}\) instead of the Gullstrand ratio of 0.883 (6.8/7.7). However, because of variation in the population in the ratio of posterior to anterior corneal radius of curvature, especially in eyes after corneal refractive surgery, a single index of refraction is not sufficient, and the accuracy of SimK in estimating the total corneal power is poor.

This study had several limitations: a small number of eyes were included in the hyperopic-LASIK/PRK group; spherical surfaces were used in the theoretical models; in normal corneas, especially corneas after myopic or hyperopic LASIK/PRK, corneal surfaces are not spherical; the relative accuracy of using ray tracing for the prediction of IOL power must be validated in the clinical setting; and the TCP calculated using the ray tracing method is the power at the anterior vertex of the cornea, and the GEP using the Gaussian formula is the power at the second principal plane. The second principal plane of the cornea is approximately 0.05 mm in front of the anterior corneal vertex,\(^{15}\) which produces a power difference of <0.1 D. This magnitude of difference is small in comparison with the mean differences of ≥0.4 D between TCP and GEP found in healthy clinical subjects and those after myopic refractive surgery. It is important to note that, to the best of our knowledge, posterior corneal power is not accurately represented in any corneal topographer or anterior segment imaging device because radius of curvature is converted to diopters using a paraxial formula that does not account for a Snell’s law refraction, as has been described for the anterior surface.\(^{8}\) In addition, the rays propagating to the posterior surface have already been refracted by the anterior surface; therefore, the “effective” posterior power will be less than what is calculated using parallel incident rays and a paraxial formula.

In conclusion, this study demonstrated that the Gaussian formula overestimated total corneal power in most clinical subjects and in theoretical models. The paraxial assumption inherent in the Gaussian formula generates variable errors in eyes after refractive surgery. The errors vary according to anterior corneal curvature, ratio of posterior/anterior radii of curvature, distance from the center of the cornea, and corneal thickness. Ray tracing does not rely on paraxial optics and is the better method with which to calculate total corneal refractive power.

### References