Effect of Anterior Chamber Depth on Shear Stress Exerted on Corneal Endothelial Cells by Altered Aqueous Flow after Laser Iridotomy

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PURPOSE. The study hypothesis was that shear stress caused by abnormal aqueous flow is one of the causes of corneal endothelial cell loss after laser iridotomy (LI). The shear stress exerted on the corneal endothelial cells (CECs) in anterior chambers (ACs) of different depths was calculated by a computational fluid dynamics program. The effect of shear stress was also examined on human corneal endothelial cells (HCECs) grown on microscope slides.

METHODS. Three-dimensional models of the AC were constructed, with and without an LI window, and AC depths of 2.8, 1.8, 1.5, and 1.0 mm. The speed of aqueous streaming through the LI window was obtained from animal studies and used to calculate the shear stress exerted on the CECs. Cultured HCECs attached to glass slides were subjected to different magnitudes of shear stress by exposing the cells to different flow rates of the culture solution. The number of cells remaining attached to the slide under each condition was determined.

RESULTS. The shear stresses were 0.14, 0.31, 0.48, and 0.70 dyn/cm² for models with AC depths of 2.8, 1.8, 1.5, and 1.0 mm, respectively. When cultured HCECs were subjected to shear stress within the range calculated by the three-dimensional models, the number of cells remaining attached to the glass slide decreased as the magnitude and duration of the shear stress increased.

CONCLUSIONS. Shear stress exerted on CECs after LI may reach a magnitude high enough to cause cell damage and loss in eyes, especially in those with shallow anterior chambers. (Ophthalmology 2010;51:1956–1964) DOI:10.1167/iovs.09-4280

The first case of irreversible corneal edema after argon laser iridotomy (LI) was reported by Pollack1 in 1984, and five cases of phakic bullous keratopathy after argon LI were reported soon afterward by Schwartz et al.2 Since then, the incidence of LI-induced bullous keratopathy has increased yearly3–5 and is now one of the most common causes for penetrating keratoplasty in Japan.7–9 Although a variety of hypotheses have been made on the cause of this unique form of bullous keratopathy (e.g., excessive laser irradiation,2,3,5 an acute glaucoma episode,2,4,5 and preexisting corneal endothelial abnormalities such as Fuchs’ corneal dystrophy),2,4,5 many cases cannot be fully explained by these factors. Among these, excessive laser irradiation with subsequent thermal damage of the endothelial cells has been considered to be one of the dominant causes. However, it is puzzling that the corneal endothelial cell density decreases progressively over many years without any significant corneal edema or inflammation immediately after LI.2–4

We have demonstrated in animal studies that during miosis, the aqueous humor streams into the anterior chamber (AC) from the posterior chamber through an LI window and strikes the corneal endothelium. During mydriasis, the aqueous humor in the AC is drawn back into the posterior chamber through the LI window.10 From these findings, we hypothesized that the shear stress caused by the abnormal aqueous flow striking the corneal endothelium may be one of the pathogenetic mechanisms for progressive corneal endothelial cell loss. To test this hypothesis, we first constructed three-dimensional AC models of different depths, with or without an LI window, and calculated the speed of aqueous flow as well as the shear stress exerted on the corneal endothelium by using a computational fluid dynamics program. We further examined the effect of shear stress on endothelial cells in vitro experiments. To determine whether the shear stress calculated could have an influence on the corneal endothelium, we exposed human corneal endothelial cells (HCECs) cultured on glass slides to shear stress by exposing them to different flow rates of the culture solution, and examined changes in cell morphology and adhesion.

METHODS

Computational Fluid Dynamics

Computational fluid dynamics is one of the techniques of fluid mechanics that uses numerical methods and algorithms to analyze and solve problems that involve fluid flow. A computational fluid dynamics program (Fluent; Ansys Japan K.K., Tokyo, Japan) was used in the study. The finite-volume method was used to solve the Navier-Stokes continuity equations on an arbitrary target flow domain, and appropriate boundary conditions were assigned. The speed of the thermal current was calculated in the geometrical AC models at various AC depths, where the temperature of the posterior corneal surface was set at 36°C and the temperature of the iris surface was set at 37°C. Then, a virtual LI window was created at the 12 o’clock position of the peripheral iris and values obtained from animal experiments were introduced into the model, to calculate the speed of the aqueous streaming through the LI. Changes in the speed of aqueous flow and in the shear stress on the corneal endothelium were calculated in models of different AC depths and streaming from the LI window.
Geometric Model of the AC

Geometric models of the AC were constructed according to the clinical parameters of the AC cited in published reports and by using Bezier curves. A two-dimensional AC with a depth of 2.8 mm and its parameters are shown in Figure 1A. A three-dimensional AC was constructed by rotating the two-dimensional shape around the ocular axis. In a similar manner, ACs were constructed with AC depths of 1.8, 1.5, and 1.0 mm. (C) The outline of divided three-dimensional AC models with a depth of 2.8 mm (adverse side and flip side) is shown in Figure 1B. In a similar manner, three-dimensional ACs were constructed for AC depths of 1.8, 1.5, and 1.0 mm (Fig. 1C). The 2.8-mm AC depth corresponded to that of a normal eye, 1.8 mm to an eye with a narrow angle, 1.5 mm to an eye with angle-closure glaucoma requiring preventive LI, and 1.0 mm to an eye with an extremely shallow AC during an episode of acute glaucoma requiring LI.

Simulation of Aqueous Streaming in an Eye with an LI Window

We have demonstrated the speed and direction of aqueous flow after LI by particle-tracking velocimetry, using silicone powder as a tracer in rabbit eyes. In that study, the aqueous humor was seen to stream into the AC from the posterior chamber through the LI window during the miosis induced by a light stimulus. The streaming aqueous passed through the LI window beginning at the onset of miosis and continued for the duration of a single miosis as a single pulse, but an aqueous flow through the pupil was not detected. The speed of the aqueous streaming through the LI window in eyes with a diameter of the LI window at 0.56 mm was 9.39 mm/s, determined by the particle-tracking velocimetry technique. From these results, the following parameters were used for the computational fluid dynamics for the study. The diameter of the LI window was set at 0.56 mm, and the position of the LI window was set at the 12 o'clock position of the iris, 1.0 mm away from the AC angle. The initial speed through the LI window was set at 9.39 mm/s. The duration of the analysis time was set to 0.660 second, because that is the average duration of miosis in humans after light stimulation. The attenuation of the speed of aqueous streaming was approximated by a cosine function. Thus, calculations of the speed of aqueous streaming were executed every 0.033 second with the following equation:

\[ V(T) = 9.39 \cos\left(\frac{\pi T}{2 \times 0.660}\right) \]

where \( V \) is the speed of aqueous streaming (in millimeters per second) and \( T \) is time (seconds).

Shear Stress Model Experimental Design

Human corneal endothelial cells (HCECs) were cultured until they reached confluence on a glass slide with a specially designed flow channel. The cells were subjected to various magnitudes of shear stress created by both continuous and intermittent flow of the culture solution. Changes in the morphology of the cells on each slide were observed by optical microscopy after the cells were exposed to shear stress for 4 and 16 hours.

Media and Culture Conditions

All primary and passaged HCECs were cultured in medium consisting of Dulbecco’s modified Eagle’s medium (DMEM) supplemented with 15% fetal bovine serum, 2.5 mg/L amphotericin B (Fungizone; Invitrogen-Gibco, Grand Island, NY), 2.5 mg/L doxycycline (Sigma-Aldrich Co., St. Louis, MO), and 2 μg/mL basic fibroblast growth factor (Invitrogen, Carlsbad, CA). Cultured HCECs were maintained in a humidified incubator at 37°C and 10% CO₂.

Cell Culture Methods

Primary cultures of HCECs were established from normal human corneas obtained from the American Eye Bank. The human tissue was used in strict accordance with the tenets of the Declaration of Helsinki. Primary cultures of HCECs and all subsequent passages were performed by using our published method. We used cultured HCECs at the fifth passage for the experiments.
Culture Slides and Flow Circuit System

We used a collagen IV-coated cell culture slide, with a flow channel and accompanying flow kit. A flow circuit was constructed by connecting a peristaltic pump to a polyvinyl chloride flow tube. HCECs were cultured until they reached confluence on the channel of a glass slide. The cells were subjected to various magnitudes of shear stress created by both continuous and intermittent flow of the culture solution.

Statistical Analyses

Differences in the number of cells remaining attached to the slide under each magnitude or condition of shear stress were analyzed by ANOVA and the Scheffe multiple comparison test. $P < 0.05$ was considered to be statistically significant.

RESULTS

Speed of Thermal Current for Different AC Depths

The velocities of the thermal current in the ACs for different AC depths are shown in the color-coded maps in Figure 3. Each of the maps shows the inner surface of the AC at the central sagittal plane, and also planes at 2.5, 4, and 5 mm from the center to the nasal and temporal planes. The thermal current flowed down the corneal endothelial surface and up the iris surface. The maximum descending speeds of aqueous flow in the central sagittal plane near the corneal endothelium were 0.23, 0.11, 0.076, and 0.037 mm/s at AC depths of 2.8, 1.8, 1.5, and 1.0 mm, respectively. As the AC became shallower, the speed of the aqueous flow became slower in all planes of the AC examined.

Speed of Aqueous Flow through the LI Window during Miosis

The speed of aqueous flow through the LI window was calculated during a single miosis induced by a light pulse. Figure 4 shows the aqueous flow speeds calculated immediately before the aqueous collided with the corneal endothelial surface opposite the LI window from the onset of miosis (time 0) to the end (time 0.660 second). The measurements were taken every 0.033 second in models with AC depths of 2.8, 1.8, 1.5, and 1.0 mm.

The maximum speeds were 0.84, 1.83, 2.79, and 3.90 mm/s in models with AC depths of 2.8, 1.8, 1.5, and 1.0 mm, respectively. The color-coded maps of the central sagittal plane and the nasal and temporal planes 2.5, 4, and 5 mm from the center (Fig. 4) are the spatial distribution of aqueous flow speeds at the time when flow reached a maximum speed.

![Figure 2](image-url) **Figure 2.** Diagram of culture slides and the flow circuit system, a collagen IV-coated cell culture slide, with a flow channel and accompanying flow kit were used. A flow circuit was constructed by connecting a peristaltic pump to a polyvinyl chloride flow tube. HCECs were cultured until they reached confluence on the channel of a glass slide. The cells were subjected to various magnitudes of shear stress created by both continuous and intermittent flow of the culture solution.

![Figure 3](image-url) **Figure 3.** Speed of the thermal current for the different AC depths (ACDs) is shown in the color-coded maps: (a) 2.8, (b) 1.8, (c) 1.5, and (d) 1.0 mm. Each color-coded map shows the inner AC in the central sagittal plane and the nasal and temporal planes 2.5, 4, and 5 mm from the center to the nasal and temporal plane. The thermal current flowed down the corneal endothelial surface and up the iris surface. Thermal current speeds were slower in eyes with shallower ACDs.
Shear Stress on Corneal Endothelial Surface in Eyes with an LI Window

The shear stress on the corneal endothelial surface was calculated during the period of aqueous streaming. Figure 5 shows the maximum shear stress exerted on the corneal endothelial surface opposite the LI window from the onset (time 0) to the end (time 0.660 second) of miosis. The values are plotted for every 0.033 seconds over a 0.660-second period of miosis in models with AC depths of 2.8, 1.8, 1.5, and 1.0 mm. The maximum values of shear stress exerted on the corneal endothelium were 0.14, 0.31, 0.48, and 0.70 dyn/cm² for models with AC depths of 2.8, 1.8, 1.5, and 1.0 mm, respectively. For each AC depth, the spatial distribution of shear stress on the corneal endothelial surface at the time when shear stress reached a maximum is shown in the color-coded maps (Fig. 5). For comparison, the shear stress caused by the descending thermal current in a normal eye with an AC depth of 2.8 mm and no LI window, was 0.0062 dyn/cm² at the center of the corneal endothelial surface.
detached cells (Fig. 7i). At a magnitude of 0.58 dyn/cm², the number of cells attached to the slide had markedly decreased (Fig. 7j).

**Comparison of Total Shear Stress at Different AC Depths**

The total amount of shear stress (Fig. 6) exerted on the corneal endothelial cells during a single miosis (0.660-second period of aqueous streaming) was calculated by finding the area under the curve representing the maximum magnitude of shear stress as a function of time (Fig. 5). With an AC depth of 2.8 mm and no LI-induced aqueous streaming (normal), the total shear stress was 0.0041 dyn/cm²·s, whereas in the eyes with LI windows, the total shear stresses were 0.06, 0.13, 0.2, and 0.29 dyn/cm²·s for eyes with AC depths of 2.8, 1.8, 1.5, and 1.0 mm, respectively. These values were 14.7, 31.9, 49.2, and 70.7 times greater than that of the control normal physiological aqueous flow without an LI window (Fig. 6).

**Effect of Shear Stress on Cultured HCECs**

Photomicrographs of cultured HCECs taken after the cells were exposed to shear stress in the different conditions are shown in Figure 7: 7a, 0 dyn/cm² shear stress for 4 hours (control); 7b, 0 dyn/cm² for 16 hours (control); 7c, 0.12 dyn/cm² continuous shear stress for 4 hours; 7d, 0.12 dyn/cm² continuous shear stress for 16 hours; 7e, 0.16 dyn/cm² continuous shear stress for 4 hours; 7f, 0.16 dyn/cm² continuous shear stress for 16 hours; 7g, 0.58 dyn/cm² continuous shear stress for 4 hours; 7h, 0.58 dyn/cm² continuous shear stress for 16 hours; 7i, 0.12 dyn/cm² intermittent shear stress for 4 hours; and 7j, 0.58 dyn/cm² intermittent shear stress for 4 hours. All images were taken at the same magnification.

Cells subjected to 0.16 or 0.58 dyn/cm² of shear stress (Figs. 7e, 7g) appeared longer and narrower than the control cells (Fig. 7a). At a shear stress level of 0.58 dyn/cm² (Fig. 7g), some of the cells were detached from the slide. When the duration of the shear stress was 16 hours (Figs. 7d, 7f, 7h), elongation and detachment of the cells were observed at more sites and in larger areas. As the magnitude of shear stress or the duration of the shear stress increased, the stress had a greater influence on the cells.

When shear stress was applied intermittently at a magnitude of 0.12 dyn/cm² for 4 hours, there were large patches of detached cells (Fig. 7i). At a magnitude of 0.58 dyn/cm², the number of cells attached to the slide glass decreased considerably after 4 hours, and the size of the cells that remained attached to the slide had markedly decreased (Fig. 7j).

**Number of Cells Attached to Slide after Application of Shear Stress**

After exposure of the cells to different magnitudes of shear stress, either continuously or intermittently for 4 hours, the nuclei of the cells were stained with DAPI. Four areas were selected at random, and the number of cells that remained attached to the slide/field was counted. Each experiment was repeated four times, and representative results of an experiment are shown in Figure 8. The mean number of cells ± SE was calculated and plotted from the values in the four areas. The mean number of control cells and the cell counts under each specific flow rate were analyzed by ANOVA, and statistically significant differences were found. Then, the Scheffé multiple comparison test was used to compare the differences between the control condition and each specific flow rate. The mean number of cells for each condition of shear stress was significantly lower than that of the control (control > continuous shear stress 0.12 dyn/cm²; P < 0.05; control > continuous shear stress 0.16 dyn/cm²; P < 0.001; control > continuous shear stress 0.58 dyn/cm²; P < 0.0001; control > intermittent shear stress 0.12 dyn/cm²; P < 0.0001; control > intermittent shear stress 0.58 dyn/cm²; P < 0.0001). When the cells were subjected to continuous shear stress as the magnitude of shear stress increased, the number of cells became fewer.

When the cells were subjected to intermittent shear stress, significantly fewer remained attached to the slide than with continuous shear stress of an identical magnitude (continuous shear stress 0.12 dyn/cm² > intermittent shear stress 0.12 dyn/cm²; P < 0.05; continuous shear stress 0.58 dyn/cm² > intermittent shear stress 0.58 dyn/cm²; P < 0.001).

**DISCUSSION**

The aqueous humor plays an important role in maintaining the homeostasis of the corneal endothelial cells and other structures in the anterior segment of the eye. Any changes in the dynamics of the flow of the aqueous humor can have profound effects on the corneal endothelial cells and may be associated with a variety of ocular disorders. Unfortunately, it is very difficult to observe aqueous flow in humans unless an AC inflammatory reaction is present or a tracer is intentionally introduced into the AC. Thus, computational fluid dynamics...
However, it is not possible to include all the in vivo physiological or pathologic conditions in the computational fluid dynamics technique. The computational fluid dynamics technique requires some degree of simplification if a condition is to be analyzed properly. Therefore, it is necessary to consider whether the results obtained from the computational fluid dynamics simulation are consistent with the physiological values to be applicable to the in vivo situation.

In our computational fluid dynamics model, we used shapes of the AC based on clinical data, and we used these shapes for the in vivo measurements of aqueous flow speed found in experiments on rabbits, to ensure the precision of our calculations.

The temperature difference between the surface of the iris and the corneal endothelium is the most important factor that must be set to ensure a consistency of the data obtained by the model and the eye in situ. However, there are currently no published data on the in vivo temperature of the corneal endothelial surface. In our model, the iris surface temperature was assumed to be the same as internal body (rectal) temperature of 37°C, and the corneal endothelial surface temperature was set at 36°C, based on preliminary calculations of thermal current speed made using our 2.8 mm AC depth model (Fig. 9), experimental thermal current speeds found in particle tracking velocimetry experiments on rabbit eyes, and thermal current speeds from mathematical models in earlier reports.

There may be some concern that the flow of the aqueous caused by the thermal currents is affected by the flow through the pupil and drainage of aqueous from the trabecular meshwork. In the literature, the speed of aqueous flow from the pupil was reported to be 0.0017 mm/s by Maurice and 0.002 mm/s by Kumer et al. Fitt and Gonzalez reported that the maximum speed of aqueous flow from the pupil was 0.0075 mm/s and that the thermal convection caused by the temperature difference between the iris surface and corneal endothelial surface is greater than that produced by any other physical mechanism (e.g., lens phakodonesis or rapid eye movements).

In addition, the aqueous flow due to thermal convection caused by 1°C difference between the iris surface and corneal endothelial surface was 0.16 to 0.21 mm/s (Fig. 9), and the aqueous streaming from the LI window was 9.89 mm/s which is >100 to 1000 times faster than that of aqueous flow through the pupil. For our results, the flow of aqueous through the pupil was omitted to simplify the computational fluid dynamics model.

We found that the shear stress exerted on the corneal endothelium by aqueous streaming from the LI window was greater in the eyes with shallower AC depth. The total shear stress (the area under the shear stress versus time curve in Fig. 5) exerted on the corneal endothelial surface by aqueous streaming during a single miosis cycle was greater as the AC depth decreased (Fig. 6). In the extreme case in which the AC depth was 1.0 mm, the total shear stress in eyes with an LI was 70 times greater than that produced by physiological thermal currents. Therefore, in clinical cases in which the peripheral AC depth does not become sufficiently large after LI, such eyes may be at high risk of having considerable shear stress.

Miosis commonly occurs in daily life in response to changes in light levels. Because increases in aqueous streaming occur repeatedly after each episode of miosis in eyes with an LI, the corneal endothelial cells can be expected to be more strongly influenced by shear stress because the stress results from intermittent rather than continuous flow.

Shear stress can be either advantageous or disadvantageous for maintaining the homeostasis of different types of cells, depending on the magnitude and properties of the shear stress. At physiological levels, shear stress has a protective effect on vascular endothelial cells, because it inhibits apopto-

![FIGURE 7. Photomicrographs of cultured HCECs taken after the cells were exposed to shear stress. (a) 0 dyn/cm² shear stress for 4 hours (control), (b) 0.12 dyn/cm² continuous shear stress for 4 hours, (c) 0.16 dyn/cm² continuous shear stress for 4 hours, (d) 0.12 dyn/cm² intermittent shear stress for 4 hours, (e) 0.16 dyn/cm² continuous shear stress for 4 hours, (f) 0.58 dyn/cm² continuous shear stress for 4 hours, (g) 0.58 dyn/cm² intermittent shear stress for 4 hours, (h) 0.58 dyn/cm² continuous shear stress for 4 hours, (i) 0.58 dyn/cm² intermittent shear stress for 4 hours, (j) 0.58 dyn/cm² intermittent shear stress for 4 hours, As the magnitude of shear stress or the duration of the stress increased, elongated cells and cell detachment were observed more frequently. At 0.12 dyn/cm² intermittent shear stress for 4 hours, large patches of detached cells were visible. At 0.58 dyn/cm² intermittent shear stress for 4 hours, the number of cells attached to the slide was considerably lower after 4 hours and the size of the cells remaining attached to the slide had markedly decreased. Scale bar, 100 µm.](image-url)
On the other hand, vascular smooth muscle cells exposed to nonphysiological shear stress levels show reduced proliferation and increased apoptosis. Therefore, shear stress not only leads to the physical detachment of cells, but may also cause apoptosis and wound-healing disorders and may be a factor in the disruption of the homeostasis of corneal endothelial cells.

According to the estimates from our model, the maximum shear stress exerted by aqueous streaming through the LI window was 0.70 dyn/cm². This shear stress is far greater than that which caused morphologic changes and cell detachments in our flow experimental model. Moreover, in the cultured cells, intermittent flow had a greater effect than continuous flow. The reason for this is unclear, but it has been shown that when vascular endothelial cells are subjected to identical magnitudes of shear stress, cells that are subjected to changes in flow were more likely to show morphologic changes or responses on a cellular or molecular level than are the cells that are exposed to constant flow.

Some discrepancies may arise when in vitro data are applied to in vivo situations, because the attachment of cells to a culture slide or to Descemet’s membrane should not be the same. However, it should still be remembered that prolonged periods of stress on the corneal endothelium caused by aqueous streaming can lead to corneal endothelial cell decompensation.

Irreversible corneal edema developed in the upper region of the cornea where the LI was performed, but in some cases it developed in the lower region of the cornea far from the site of the LI. We did not study the relationship between the area of LI and initial corneal edema, because in most of the cases of bullous keratopathy caused by LI, the patients had diffuse edema at the initial visit to our clinic.

In our model, the flow of the aqueous back through the LI window into the posterior chamber during mydriasis, aqueous flowing through the pupil, and drainage of aqueous through the trabecular meshwork were not simulated. Therefore, we did not examine how these combined factors affected the aqueous streaming in and out through the LI window in eyes with shallow ACs and slow thermal currents. To construct a computational fluid dynamics model that will allow investigation of these factors, clinical data regarding changes in the

**FIGURE 8.** After the cells were exposed to different magnitudes of shear stress either continuously or intermittently for 4 hours, the number of cells remaining attached to the slide was determined. Four individual areas were selected randomly, and the number of cells attached to the slide glass per field was counted. The mean number of cells ± SE of four areas were graphed. The mean number of cells for each condition of shear stress was significantly lower than that of the control. When the cells were subjected to intermittent shear stress, significantly fewer cells remained attached to the slide than that with continuous shear stress of an identical magnitude (*P < 0.05; **P < 0.001; ***P < 0.0001; Sheffe multiple comparison test).

**FIGURE 9.** In a computational fluid dynamics model with an AC depth of 2.8 mm, the corneal endothelial surface temperature was set at different levels, and preliminary calculations of thermal current speed were performed. The graph shows the descending thermal current speed near the corneal endothelium at different corneal endothelial surface temperatures.
The fluid dynamics program (Fluent; Ansys) solves the conservation of energy equation for incompressible flows involving heat transfer or compressibility, an additional equation for conservation of energy can be solved.

Aqueous humor dynamics appear to play a role not only in post-LI disorders, but also in many other disorders, such those occurring after cataract surgery and in glaucomatous eyes. The continuing development of applications of computational fluid dynamics has the potential to contribute to the understanding of the pathogenesis of a variety of anterior segment disorders.

APPENDIX

Mathematical Equations for Computational Fluid Dynamics

Continuity and Momentum Equations. For all target flows, the fluid dynamics program (Fluent; Ansys) solves the conservation equations for mass and momentum. For flows involving heat transfer or compressibility, an additional equation for conservation of energy can be solved.

Mass Conservation Equation. The equation for conservation of mass can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_m \quad (1)$$

Equation 1 is the general form of the mass conservation equation and is valid for incompressible as well as compressible flows, where $S_m$ is the mass source term.

Momentum Conservation Equation. Conservation of momentum is described by

$$\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) = -\nabla p + \nabla \cdot \tau + \rho \vec{g} + \vec{F} \quad (2)$$

where $p$ is the static pressure, $\rho$ is the density, $\tau$ is the stress tensor, and $\vec{g}$ and $\vec{F}$ are the gravitational body force and external body forces, respectively.

The stress tensor $\tau$ is given by equation 3:

$$\tau = \mu \left[ (\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} \nabla \cdot \vec{v} \right] \quad (3)$$

where $\mu$ is the molecular viscosity and $I$ is the unit tensor.

Energy Conservation Equation. The fluid dynamics program (Fluent; Ansys) solves the conservation of energy equation in the following form:

$$\frac{\partial}{\partial t} (\rho E) + \nabla \cdot \left[ \vec{v} (\rho E + p) \right] = \nabla \cdot \left[ k_{\text{eff}} \nabla T + \sum_j b_j \vec{j}_j + (\tau_{\text{eff}} \cdot \vec{v}) \right] + S_h \quad (4)$$

where $k_{\text{eff}}$ is the effective conductivity and $\vec{j}_j$ is the diffusion flux of species $j$. The first three terms on the right side of equation 4 represent energy transfer due to conduction, species diffusion, and viscous dissipation, respectively. $S_h$ represents a volumetric heat source term. In equation 4, $E$ is given by:

$$E = b - \frac{\rho}{\rho} + \frac{v^2}{2}$$

where the sensible enthalpy $b$ is defined for incompressible flows as

$$b = \sum_j y_j b_j + \frac{\rho}{\rho}$$

$y_j$ is the mass fraction of species $j$ and $b_j$ is expressed as

$$b_j = \int_{\text{int}} r c_{p,j} dT$$

where $T_{\text{eff}}$ is 298.15 K.

Boussinesq Model. This model treats density as a constant value in all solved equations except for the buoyancy term in the momentum equation:

$$(\rho - \rho_0)g = -\rho_0 \beta (T - T_0)g \quad (5)$$

where $\rho_0$ is the (constant) density of the flow, $T_0$ is the operating temperature, and $\beta$ is the thermal expansion coefficient. Equation 5 is obtained by using the Boussinesq approximation $p = \rho_0 (1 - \beta \Delta T)$ to eliminate $\rho$ from the buoyancy term. This approximation is accurate as long as changes in the actual density are small.

Physical Parameters

The physical parameters of the aqueous humor at normal body temperature were estimated from previous reports18–22 to be: density, 994 kg/m$^3$; specific heat, 4178 J/kg · K; thermal conductivity, 0.6241 W/m · K; dynamic viscosity, 0.000746 kg/m · s; and volume expansion coefficient, 0.000353 1/K.

References


