Visual Psychophysics and Physiological Optics

Statistical Eye Model for Normal Eyes

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PURPOSE. To create a binocular statistical eye model based on previously measured ocular biometric data.

METHODS. Thirty-nine parameters were determined for a group of 127 healthy subjects (37 male, 90 female; 96.8% Caucasian) with an average age of 39.9 ± 12.2 years and spherical equivalent refraction of −0.98 ± 1.77 D. These parameters described the biometry of both eyes and the subjects’ age. Missing parameters were complemented by data from a previously published study. After confirmation of the Gaussian shape of their distributions, these parameters were used to calculate their mean and covariance matrices. These matrices were then used to calculate a multivariate Gaussian distribution. From this, an amount of random biometric data could be generated, which were then randomly selected to create a realistic population of random eyes.

RESULTS. All parameters had Gaussian distributions, with the exception of the parameters that describe total refraction (i.e., three parameters per eye). After these non-Gaussian parameters were omitted from the model, the generated data were found to be statistically indistinguishable from the original data for the remaining 35 parameters (TOST [two one-sided t tests]; P < 0.01). Parameters derived from the generated data were also significantly indistinguishable from those calculated with the original data (P > 0.05). The only exception to this was the lens refractive index, for which the generated data had a significantly larger SD.

CONCLUSIONS. A statistical eye model can describe the biometric variations found in a population and is a useful addition to the classic eye models. (Invest Ophtalmol Vis Sci. 2011;52:4525–4533) DOI:10.1167/iovs.10-6705

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Since they were first introduced approximately 150 years ago, eye models have become an indispensable tool in physiological optics. Their simplified layouts of the ocular refractive surfaces allow quick calculations of how light passes through the eye, from which many valuable insights have been obtained over the years.

The most famous eye model was the “No. 1” or “exact” eye proposed by Gullstrand in 1909. Its layout closely approximated that of a real eye, simulating the gradient refractive index of the crystalline lens by means of a shell structure and taking accommodation into account. However, as calculations of light refraction were very time consuming in that period, this model had to be simplified to be of any practical use (Le Grand, Emsley).

With the increased availability of computers from the 1970s onward the practical problems of calculation intensiveness became less important. Advancing technology facilitated researchers in improving their eye models so that they would better match the clinical data. These improvements included a curved retina, aspheric surfaces (Lotmar, Kooijman, Navarro), gradient index crystalline lenses (Liou and Brennan, Siedlecki et al., Goncharov and Dainty), chromatic dispersion (Thibos et al., Navarro), and a consideration of peripheral imaging (Pomerantzeff et al., Escudero-Sanz and Navarro). Furthermore, eye models were proposed by Atchison et al. that included the effects of aging and myopia. For some of these models, quantitative comparisons of optical properties, such as wavefront aberration, modulation transfer function, and Strehl ratio can be found in the literature.

Even though step-by-step these models have come closer to the performance of a real eye, most are still rotationally symmetrical, idealized representations that do not take the wide variations in ocular biometry that exist in the general population into account.

One way to include these variations is in the form of customizable eye models, as proposed by Navarro et al. for phakic eyes and by Rosales and Marcos for pseudophakic eyes. These models incorporate clinically measured biometric data to predict the total wavefront error of an eye, which was found to work well for the pseudophakic eye models, but not always well in the phakic models. This difference in success could be explained by the lack of customized knowledge of the shape and in vivo refractive index of the crystalline lens in these individual eyes. Instead average values for these crystalline lens parameters were used that did not necessarily match physiological values in those eyes. Moreover, customized crystalline lens models calculated by subtracting corneal wavefront aberrations from total wavefront aberrations cannot yet be verified independently. These limitations may be overcome in the very near future with the recent introduction of ocular wavefront tomography and anterior segment OCT, both of which provide very detailed information on the refracting surfaces in the eye and refractive index distribution.

In this work, we chose an alternative method that uses descriptive statistics as a basis to generate a large number of virtual eyes with plausible biometry. These eyes can then serve as a basis for further statistical calculations. Such an approach was first proposed by Thibos et al. as a way to generate sets of realistic wavefronts using Zernike coefficients.

The goal of this work is to continue the basic idea of Thibos et al. in the form of a binocular, statistical eye model for normal eyes.

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METHODS

Subjects
In this work, we used the binocular biometric data of 127 healthy subjects (37 male, 90 female; 123 Caucasian, 4 non-Caucasian) that were measured in the framework of Project Gullstrand, a European multicenter study to determine the correlation between ocular biometry and several psychophysical tests. The subjects included in the Belgian part of the study were mostly employees of the Antwerp University Hospital.

Subjects were between 22.3 and 78.6 years old and without any previous ocular pathology or surgery. The subjects were excluded if they had a refraction outside the range of (−10 to +10 D), a corrected visual acuity less than 20/20, or an ETDRS logMAR chart, or intraocular pressure higher than 22 mm Hg, as measured with a noncontact tonometer (Ocular Response Analyzer [ORA]; Reichert Inc. Buffalo, NY, USA.). Other exclusion criteria were the wearing of hard contact lenses less than 1 month before testing and pregnancy. The study adhered to the tenets of the Declaration of Helsinki and received approval of the ethics committee of the Antwerp University Hospital (Ref. 7/6/24). Signed informed consent was obtained from participating subjects before testing.

Since the Project Gullstrand data did not contain any biometry of the crystalline lens, these values had to be included in the model in a different way. For this purpose, we used the age-related biometry data published by Atchison et al.13 for a group of 66 eyes of 66 emmetropic subjects (32 men, 34 women; 62 Caucasian, 4 non-Caucasian) with a multivariate Gaussian distribution is given by:

\[
x \sim N(a, \Sigma)
\]

where \(a\) is a vector of the mean values of the parameters and \(\Sigma\) is the covariance matrix. The model presented in this article is based on the idea that if a topic pupil size after 5 minutes of dark adaptation. From these data, the following relation was found:

\[
f_x(x) = \frac{1}{(2\pi)^{n/2} \left| C \right|^{1/2}} \exp \left( -\frac{1}{2} (x-M)'C^{-1}(x-M) \right)
\]

with \(M\) a \((1 \times N)\) vector describing the mean values of the parameters and \(C\) a \((N \times N)\) matrix containing the covariance values between the parameters. As there are many known correlations between the different ocular biometric parameters,26–30 this covariance matrix \(C\) should contain only non-0 elements.

Model Parameters
In this work we started from a set of 39 parameters including the subject’s age, the total ocular refraction (\(R_{\text{em}}\), \(R_{\text{em}}\), \(R_{\text{em}}\)) within the form of Thibos’ Fourier power vectors,31 the anterior keratometry (\(K_{\text{A-M}}\), \(K_{\text{A-D}}\), \(K_{\text{A-SIL}}\)), the anterior corneal eccentricity \(E_{\text{A}}\), the posterior keratometry (\(K_{\text{P-M}}\), \(K_{\text{P-D}}\), \(K_{\text{P-SIL}}\)) and eccentricity \(E_{\text{P}}\), the central corneal thickness \(P_{\text{c}}\), the anterior chamber depth \(A_{\text{CD}}\), the anterior and posterior curvature of the crystalline lens (\(R_{\text{LA}}\), \(R_{\text{P}}\)), the crystalline lens thickness \(T\), the crystalline lens power \(P_{\text{L}}\), the ocular axial length \(L\), and the scotopic pupil size \(S_{\text{p}}\). As the Project Gullstrand data did not contain the lens thickness parameter, it was not possible to calculate the crystalline lens power directly using ray tracing or Bennett’s formula.32 Instead the lens power was estimated using the T2 formula,33 an updated version of the SRK/T formula.34 However, rather than the phakic lens power, this procedure provides an estimate of the pseudophakic lens power required to obtain a certain preset refraction after cataract surgery.

To remedy this problem we used the emmetropic group to find the correlation between the crystalline lens power calculated with the Bennett formula (\(P_{\text{Benn}}\)), as well as the pseudophakic lens power using the T2 formula (\(P_{\text{T2}}\)). Through reduced major axis regression we found the following relationship: \(P_{\text{Benn}} = 1.133 P_{\text{T2}} - 1.386 (r^2 = 0.922)\). Inserting \(P_{\text{T2}}\) into this formula allows an estimate of the real crystalline lens power to be made. Even though in individual cases the calculated crystalline lens power may deviate from the actual crystalline lens power, the calculated average and covariance values of the population would be correct.

Since the biometric parameters of left and right eyes are strongly correlated (see the Results section), combining both eyes of the same subject into the calculations may have a considerable influence on the covariance values in matrix \(C\). Although the use of a linear mixed-effects model would account for such correlations, a different approach was chosen here. By including the biometry of right and left eyes into \(M\) and \(C\) separately, one has the opportunity to create a binocular eye model that leaves the correlation between both eyes intact for these parameters. Including this binocular aspect would introduce several interesting options, such as the study of aniseikonia and anisometropia.

As mentioned above, the mean and covariance values for the anterior and posterior curvature of the crystalline lens (\(R_{\text{LA}}\), \(R_{\text{P}}\)) and the crystalline lens thickness \(T\) were taken from the emmetropic data set. With the exception of the scotopic pupil size \(S_{\text{p}}\), the emmetropic data set contained all the model parameters included in the covariance matrix \(C\). Hence most of the covariances between the lens parameters (\(R_{\text{LA}}\), \(R_{\text{P}}, T\)) and the other parameters could be inserted. However, the covariance data set did not include binocular information, so the covariance values between the lens parameters and the other parameters had to be used for both left and right eyes. Covariance values that could not be determined were given the default value of 10⁻³.

In practice, it proved to be necessary to slightly increase the covariance values between the \(K_{\text{A-M}}, P_{\text{c}}, L\) parameters for both eyes, to obtain a more realistic correlation between the eyes. Example of the \(M\) and \(C\) matrices used in this work are given in the Appendix for the monocular version of the model (right eyes only).

One drawback of randomly generating a set of biometric parameters in this fashion is that, even though the values of the individual parameters are realistic and the correlations between them are correct, the parameters defining ocular refraction (i.e., \(K_{\text{A-M}}, P_{\text{c}}, L\)) would not add up to the value of \(R_{\text{em}}\) that was randomly generated by the model. In a healthy real eye on the other hand the refraction calculated from the biometry and the measured refraction would match very closely. This problem can be solved by using ray tracing,35 to calculate the refraction along the meridians of maximum and minimum corneal curvature, from which the resultant spherical and cylindrical refraction may be derived.

The study adhered to the tenets of the Declaration of Helsinki and received approval of the ethics committee of the Antwerp University Hospital.

The biometric equipment used to measure the required parameters were an autorefractometer (ARK-700; Nidek, Gamagori, Japan) for objective refraction; a Scheimpflug camera (Pentacam; Oculus Optikgeräte, Wetzlar, Germany) for anterior and posterior keratometry, pachymetry, and the anterior chamber depth; an ocular biometer (IOL-Master, ver. 2; Carl Zeiss Meditec, Jena, Germany) for axial length; and an aberrometer (iTrace; Tracey Technologies, Houston, TX) for scopic pupil size after 5 minutes of dark adaptation. From these data, the crystalline lens power was estimated using the procedure described below.

In the emmetropic data set, the curvatures of the anterior and posterior lens surfaces were obtained by analyzing Purkinje images,25 and A-scan ultrasonography was used to find the lens thickness.

Multivariate Gaussian Model
The model presented in this article is based on the idea that if a parameter is known to have a Gaussian (or normal) distribution and a set of \(n\) normally distributed parameters \(x\) if the mean values of these parameters and the covariance values between these parameters are known. Then the multivariate Gaussian distribution is given by:

\[
f_x(x) = \frac{1}{(2\pi)^{n/2} \left| C \right|^{1/2}} \exp \left( -\frac{1}{2} (x-M)'C^{-1}(x-M) \right)
\]

where \(M\) a \((1 \times N)\) vector describing the mean values of the parameters and \(C\) a \((N \times N)\) matrix containing the covariance values between the parameters. As there are many known correlations between the differ-
The keratometry of the original data were transformed into Fourier power vectors, as proposed by Thibos et al. The notation \((R_{\text{FM}}, R_{\text{J0}}, R_{\text{J45}}), (K_{\text{AM}}, K_{\text{AJ0}}, K_{\text{AJ45}}), \) and \((K_{\text{RM}}, K_{\text{JM}}, K_{\text{J45}})\) was chosen rather than the more commonly used sphere, cylinder, and axis components, because they form orthogonal sets of additive vector components. The required conversion formulas between both components were published by Thibos et al.

### The Crystalline Lens

The radii of curvature \((R_{\text{LA}}, R_{\text{LP}})\), the thickness \(T\), and the power of the crystalline lens \(P_L\) are all randomly generated by the multivariate model. Therefore, if \(P_L\) is calculated from \(R_{\text{LA}}, R_{\text{LP}},\) and \(T\), and a fixed value for the crystalline lens refractive index \(n_L\) is assumed, the calculated value will not necessarily correspond with the generated value for \(P_L\). This mismatch can be eliminated by calculating a value for the

### Table 1. Kolmogorov-Smirnov Test for the Normality of the Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>n</th>
<th>Right Eye</th>
<th></th>
<th>Left Eye</th>
<th></th>
<th>Pearson r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>Age</td>
<td>y</td>
<td>127</td>
<td></td>
<td>39.88 (12.20)</td>
<td>0.177</td>
<td>—</td>
</tr>
<tr>
<td>Refraction</td>
<td>Ref</td>
<td>D</td>
<td>127</td>
<td></td>
<td>-0.98 (2.00)</td>
<td>-0.002</td>
<td>-0.98 (2.14)</td>
</tr>
<tr>
<td>Anterior keratometry</td>
<td>K</td>
<td>D</td>
<td>127</td>
<td></td>
<td>0.04 (0.28)</td>
<td>0.433</td>
<td>0.09 (0.37)</td>
</tr>
<tr>
<td>Posterior corneal eccentricity</td>
<td>Ecc</td>
<td>D</td>
<td>127</td>
<td></td>
<td>-0.04 (0.19)</td>
<td>0.246</td>
<td>-0.02 (0.18)</td>
</tr>
<tr>
<td>Anterior corneal eccentricity</td>
<td>Ecc</td>
<td>D</td>
<td>127</td>
<td></td>
<td>43.29 (1.36)</td>
<td>0.936</td>
<td>43.32 (1.40)</td>
</tr>
<tr>
<td>Posterior keratometry</td>
<td>K</td>
<td>D</td>
<td>127</td>
<td></td>
<td>0.50 (0.28)</td>
<td>0.811</td>
<td>0.31 (0.31)</td>
</tr>
<tr>
<td>Pachymetry</td>
<td>P</td>
<td>127</td>
<td></td>
<td></td>
<td>0.06 (0.25)</td>
<td>0.268</td>
<td>-0.13 (0.24)</td>
</tr>
<tr>
<td>Anterior lens curvature†</td>
<td>RA</td>
<td>mm</td>
<td>66</td>
<td></td>
<td>10.43 (1.40)</td>
<td>0.925</td>
<td>10.43 (1.40)</td>
</tr>
<tr>
<td>Posterior lens curvature†</td>
<td>RA</td>
<td>mm</td>
<td>66</td>
<td></td>
<td>-6.86 (0.85)</td>
<td>0.525</td>
<td>-6.86 (0.85)</td>
</tr>
<tr>
<td>Lens thickness†</td>
<td>T</td>
<td>mm</td>
<td>66</td>
<td></td>
<td>4.07 (0.35)</td>
<td>0.232</td>
<td>4.07 (0.35)</td>
</tr>
<tr>
<td>Lens power</td>
<td>P</td>
<td>D</td>
<td>127</td>
<td></td>
<td>22.99 (2.14)</td>
<td>0.247</td>
<td>23.04 (2.26)</td>
</tr>
<tr>
<td>Pachymetry</td>
<td>Pach</td>
<td>mm</td>
<td>127</td>
<td></td>
<td>0.0545 (0.032)</td>
<td>0.726</td>
<td>0.55 (0.03)</td>
</tr>
<tr>
<td>Anterior chamber depth</td>
<td>ACD</td>
<td>mm</td>
<td>127</td>
<td></td>
<td>2.87 (0.38)</td>
<td>0.964</td>
<td>2.88 (0.38)</td>
</tr>
<tr>
<td>Axial length</td>
<td>L</td>
<td>mm</td>
<td>127</td>
<td></td>
<td>23.67 (1.12)</td>
<td>0.745</td>
<td>23.64 (1.16)</td>
</tr>
<tr>
<td>Scotopic pupil size</td>
<td>S</td>
<td>mm</td>
<td>127</td>
<td></td>
<td>6.51 (1.12)</td>
<td>0.805</td>
<td>6.43 (1.15)</td>
</tr>
</tbody>
</table>

* KS, one-sample Kolmogorov-Smirnov test. \( P < 0.01\). Bold indicates a significant difference.
† Taken from the emmetropic data; the same values were used for left and right eyes.

However, this process leads to a refraction with a Gaussian distribution (see the Results section), which does not correspond with reality. It is therefore necessary to filter the generated data in such a way that both distributions will match each other more closely. The filtering was achieved by dividing both the original and the random refraction data of the right eyes into bins of 1 D according to their refraction. Next, eyes were removed randomly from the bins of the generated data until the overall distribution matched that of the original data. This reduced the amount of usable generated data by a factor of 4 to 5.

After exclusion of the refractive parameters (i.e., \(R_{\text{FM}}, R_{\text{J0}}, R_{\text{J45}}\), for each eye), 33 parameters were included in the following calculations.

### Transposing Keratometry and Refraction into Fourier Power Vectors

Before the calculation of the covariance matrix \(C\), the refraction and the keratometry of the original data were transformed into Fourier

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**Figure 1.** Distribution of the spherical equivalent refraction calculated with the SRK/T formula using the unfiltered generated data in comparison with a smoothed distribution of the original data. Generated data included 5000 right eyes; original data included 127 right eyes.

**Figure 2.** Distribution of the positions of the posterior corneal surface (1), the anterior and posterior crystalline lens surfaces (2, 5), and the retina (4) along the optical axis with respect to the corneal apex. Generated data included 1000 right eyes; original data included 127 right eyes. The peaks corresponding with the posterior cornea could not be shown in this scale. Instead the maximum peak values were given (respectively, 86% and 89%).
refractive index $n_L$ that balances out all these lens parameters by means of the following equation, which was derived from the thick-lens formula:\textsuperscript{36}

$$n_L = \frac{n(T+A)+0.001P_Rn_LR}{\sqrt{[n(T+A)+0.001P_Rn_LA]}^2-4n^2A}$$

(2)

with $n = n_A = n_V = 1.336$, respectively, the refractive indices of the aqueous ($n_A$) and vitreous ($n_V$) and $A = T - R_{LA} + R_{LP}$.

**Parameters Derived from Generated Data**

Based on these randomly generated biometry data, other parameters can be derived, such as the vitreal depth $V$:

$$V = L - ACD - T - Pachy.$$  

(3)

The conic constants of the anterior and posterior corneal surfaces can be calculated from the corneal eccentricity as follows:\textsuperscript{37}

$$\text{Conic} = 1 - Ecc^2$$

(4)

and the corneal radii of curvature are given by

$$R_{CA} = 337.2/K_A \text{ and } R_{CP} = (1337.2 - 1378)/K_P.$$  

(5)

Finally, the aniseikonia of a pair of eyes can be calculated using the formulas proposed by Gobin et al.\textsuperscript{38}

**Statistics**

All comparisons between original and generated data were done with the Student’s t-test. However, as the lack of a statistically significant difference does not necessarily mean the equivalence of both populations, the TOST procedure\textsuperscript{39,40} (two one-sided t tests) was also performed. This procedure defines a certain range of acceptance ($-\theta_ - +\theta$) around the difference between the means of both populations and compares this with a 99% confidence interval. In case of equivalence of both populations this 99% confidence interval should completely fall within the range of acceptance. Note that this range of acceptance is not equivalent to what would be clinically acceptable (all calculations performed with MatLab 6; The MathWorks, Natick, MA, and Excel 2003, Microsoft Corp., Redmond, WA).

**RESULTS**

**Verification of the Conversion from Pseudophakic to Crystalline Lens Power**

In an effort to confirm the conversion from pseudophakic to crystalline lens power, described earlier in the model parameters section, the conversion was applied to previously published biometric data for a group of 117 myopic and emmetropic...
eyes (44 men, 73 women; 99 Caucasian, 18 non-Caucasian; refraction range, −12.38 to 0.75 D; age range (18–36 years). Comparing the crystalline lens power, calculated with the Bennett formula, with lens power obtained from the conversion of the T2 formula, generated a high correlation coefficient \( r = 0.806 \). We therefore felt it safe to use the conversion in the following.

**Mean Values of the Original Data**

The mean values and standard deviations of the parameters used in the model are given in Table 1 for both left and right eyes.

**Normality of Parameters**

First a Kolmogorov-Smirnov (KS) test was performed on each of the parameters to ensure that the initial data follow Gaussian distributions. As shown in Table 1, no Gaussian distribution was found for several of the refraction parameters of both left and right eyes (\( \text{Ref}_{\text{L}}, \text{OD}; \text{Ref}_{\text{R}}, \text{OS}; \text{Ref}_{\text{R}, \text{OS}} \)). Note that a significance level of \( P < 0.01 \) was used instead of the customary \( P < 0.05 \) to avoid the effects of an inflation caused by the large number of KS tests performed (Bonferroni correction).

As the KS values of the refraction parameters (\( \text{Ref}_{\text{L}, \text{OD}} \)) are low (i.e., around or below \( P = 0.01 \)), we decided not to include the refraction parameters in the model. With this in mind, we will assume in the following that a multivariate Gaussian function will provide an adequate base for our model.

For most of the parameters in Table 1 strong correlations are seen between left and right eyes (i.e., most Pearson correlation coefficients \( r > 0.5 \)). The only exceptions to this are parameters \( \text{Ref}_{\text{L}, \text{OD}} \) and \( K_{\text{L}, \text{OS}} \), for which no strong correlation was expected.

**Effect of Refraction Filtering**

By inserting the mean and covariance matrices \( M \) and \( C \) into formula,\(^1\) we generated a random data set, which we then used to calculate the refraction of each generated eye by means of the SRK/T formula. As shown in Figure 1, this process results in a Gaussian distribution, which does not match the distribution of the original data. After the data of the right eyes are filtered, both distributions are identical (Fig. 1).

**Comparison of Original and Generated Data**

Figure 2 shows the position distributions of the posterior cornea surface, the anterior and posterior lens surfaces, and the retina, with respect to the corneal apex for both the original and the generated data. The data sets match very well.

As an illustration, Figure 3 shows the spherical equivalent refraction, the mean anterior keratometry, the anterior chamber depth, and the crystalline lens power plotted as a function of axial length. It can be seen that both the original and generated data sets match each other very well. The same is found for any other combination of parameters, including subject age. A small monocular sample of these generated data can also be found in the Appendix.

When comparing the original and generated datasets using unpaired \( t \) tests, no statistically significant differences were found for any of the 33 parameters (Table 2). This finding was confirmed by the TOST procedure, which demonstrated that for all parameters, the original and generated data are equivalent, using a 99% confidence interval (Table 2).

Table 3 compares the mean and standard deviation of each of the generated parameters, with the original data in the form of an averaged eye model. Again, no significant differences were found between both data sets.

**Refractive Index of the Crystalline Lens**

The mean crystalline lens refractive index \( n_L \) for the generated data was 1.432 ± 0.013 (1000 right eyes), and that of the emmetropic data was 1.451 ± 0.010 (66 eyes). The mean values of both data sets were not significantly different (unpaired \( t \)-test: \( P = 0.760 \)); however, the SD of the generated data were significantly larger than that of the original data (Levene test, \( P = 0.007 \); Fig. 4).

**Anisometropia and Aniseikonia**

The anisometropia of the original data was not normally distributed (Kolmogorov-Smirnov test, \( P < 0.001 \)), whereas the

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**Table 2. Results of the TOST Procedure for Equivalence between Generated Data and the Original Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Right Eye</th>
<th>Left Eye</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P^* )</td>
<td>( \text{RoA}, [-\Theta, +\Theta] )</td>
<td>99% CI</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td>y</td>
<td>0.254</td>
<td>−9.73, 9.73</td>
</tr>
<tr>
<td>( K_{\text{R}, \text{OD}} )</td>
<td>D</td>
<td>0.898</td>
<td>−1.08, 1.08</td>
</tr>
<tr>
<td>( K_{\text{R}, \text{OS}} )</td>
<td>D</td>
<td>0.527</td>
<td>−0.22, 0.22</td>
</tr>
<tr>
<td>( Ecc_{\text{L}} )</td>
<td>D</td>
<td>0.503</td>
<td>−0.14, 0.14</td>
</tr>
<tr>
<td>( K_{\text{L}, \text{OD}} )</td>
<td>D</td>
<td>0.786</td>
<td>−0.18, 0.18</td>
</tr>
<tr>
<td>( K_{\text{L}, \text{OS}} )</td>
<td>D</td>
<td>0.829</td>
<td>−0.06, 0.06</td>
</tr>
<tr>
<td>( Ecc_{\text{L}} )</td>
<td>D</td>
<td>0.493</td>
<td>−0.05, 0.05</td>
</tr>
<tr>
<td>( R_{\text{A}} )</td>
<td>mm</td>
<td>0.563</td>
<td>−0.22, 0.22</td>
</tr>
<tr>
<td>( R_{\text{P}} )</td>
<td>mm</td>
<td>0.889</td>
<td>−1.12, 1.12</td>
</tr>
<tr>
<td>( T )</td>
<td>mm</td>
<td>0.824</td>
<td>−0.68, 0.68</td>
</tr>
<tr>
<td>( P_{\text{L}} )</td>
<td>D</td>
<td>0.065</td>
<td>−0.28, 0.28</td>
</tr>
<tr>
<td>( Patchy )</td>
<td>mm</td>
<td>0.642</td>
<td>−2.09, 2.09</td>
</tr>
<tr>
<td>( ACD )</td>
<td>mm</td>
<td>0.961</td>
<td>−0.03, 0.03</td>
</tr>
<tr>
<td>( L )</td>
<td>mm</td>
<td>0.865</td>
<td>−0.30, 0.30</td>
</tr>
<tr>
<td>( S_{\text{P}} )</td>
<td>mm</td>
<td>0.294</td>
<td>−0.89, 0.89</td>
</tr>
</tbody>
</table>

\( n = 127 \) pairs of eyes for both sets.

* Unpaired \( t \) test.
aniseikonia was normally distributed ($P = 0.147$). However, both distributions of the generated data appear to roughly match that of the original data (Fig. 5).

**DISCUSSION**

The results show that a statistical eye model can generate a large amount of realistic biometric data that are indistinguishable from the original data, while maintaining all correlations between the various biometric parameters, including subject age. The generated data can serve a large number of purposes, such as statistical analysis, intraocular lens design, and optical calculations.

The main advantage of this model is that it takes the large physiological variability between subjects into account in a very simple and elegant manner. This in contrast with the majority of eye models in the literature that use only one single set of biometry values to represent an idealized eye.

The manner in which the refraction is defined in this model is also physiological, in the sense that the generated ocular biometry is used to calculate the total refraction of the eye, much in the same way as in a real eye. This choice for this approach resulted from the non-Gaussian distribution of the refraction.

This method may also be used to calculate the matrices $M$ and $C$ for other population studies and to compare these directly with the current data. This method may also be used to calculate the matrices $M$ and $C$ for other population studies and to compare these directly with the current data.

In a way, the model may also present a method of data compression, since the $N \times 33$ matrix containing the original data is reduced to the $33 \times 1$ vector $M$, the $33 \times 33$ matrix $C$, and the mean refraction distribution. For larger original data sets (i.e., a large $N$), this advantage becomes more apparent. However, it is valid only if all parameters included are normally distributed.

There are also a number of limitations to the method that should be kept in mind. First, model data generated in this manner are only as good as the original data that they are based on. One example of this is the imbalance between the sexes in our original data, with 70% female and 30% male subjects. As female subjects are known to have shorter eyes than males, as well as steeper anterior corneal radii, higher crystalline lens powers, and a higher refractive index of the crystalline lens,13,29,41 some of the parameters generated by the model may have slightly different values than if a balanced male:female population had been used. Another example is the
average spherical equivalent refraction of $-0.98$ D in the original data, which is more myopic than the emmetropia or slight hyperopia that other studies reported in the literature.\cite{29,41} This discrepancy may be the result of a difference in demographics of the various study populations and will cause the model to generate slightly longer eyes than when more hyperopic eyes are included.

The mean and SD values of the generated data were found to be equivalent to the original data for all parameters. As the SD depends largely on the sample size used, it is therefore important to use an original data set of sufficient size.

Second, the original data for this work did not contain any phakometric data (i.e., on the parameters $R_{LA}$, $R_{LP}$, and $T$). The absence of such data was remedied by including the mean and covariance values from the age-related biometry data of the emmetropic data set.\cite{13} As this process involved combining two separate data sets, one of which contained only emmetropes, we may have introduced an error into the model. However, as the emmetropic set contained almost all the parameters used in this work, we believe the error due to combining these two data sets is negligible. Any error due to using phakometric data from a group of emmetropes to represent the phakometry of a group containing both emmetropes and ametropes is probably also negligible, as the correlation between the lens thickness $T$ and refraction has been shown to be either nonsignificant\cite{14,42} or very weak.\cite{27,43} For this reason we also assumed that $R_{LA}$ and $R_{LP}$ would not show any significant changes as a function of refraction. However, this conclusion remains to be confirmed.

A possible issue with the emmetropic data set is that it contained data of only one eye per subject, whereas the model presented in this article is binocular. This should also not pose a problem, since for all ocular biometric parameters there is a very strong correlation between both eyes (Table 1, last column).

Finally, the standard deviation of the calculated refractive index of the crystalline lens $n_L$ was significantly higher than that of the emmetropic data set. This may be the result of the compounding of the standard deviations of the $R_{LA}$, $R_{LP}$, and $T$ parameters used to calculate $n_L$. This discrepancy between both distributions could be resolved by adding a second filtering of the generated eyes using $n_L$ as the filtering parameter.

In a later stage, the current model may be expanded further to include other ocular biometric parameters of interest, such as the tilt and centration of the various optical interfaces of the eye or the shape of their surfaces in the form of Zernike polynomials. However care must be taken of the way such additions are introduced into a statistical model, as illustrated (e.g., by McLellan et al.\cite{44} for randomized signs of Zernike coefficients). Moreover, if the binocular model described above is expanded, care must be taken that parameters are included only if they have a high correlation between left and right eyes, which would exclude parameters such as (e.g., the Stiles Crawford effect\cite{45}) or transverse chromatic aberrations.\cite{46,47}

**Acknowledgments**

The authors thank Nadia Zakaria, Jeroen Claey, and Greet Vandeweyer for support in collecting the data and Kristien Wauters for help with the statistics.

**Appendix**

The matrices $M$ and $C$ required for generating monocular data for right eyes are given in Tables A1 and A2. These are to be inserted into equation 1. Note that the generated data $f_D(x)$ may have complex values, because $C$ is composed of contributions from two separate data sets. In that case \text{sign}(\text{Re}[f_D(x)]) \cdot |f_D(x)|$ may be used instead without any significant difference in the result.

An example of randomly generated emmetropic biometry data is also given in Table A3. Note the increase in the lens thickness $T$ with age and the decreases in anterior chamber depth $ACD$ and pupil size $S_p$, all of which are well known age-related physiological changes.\cite{13}

**Table A1. Mean Matrix $M$ for Monocular Model in Right Eyes Only**

<table>
<thead>
<tr>
<th>Age</th>
<th>$K_{A,M}$</th>
<th>$K_{A,o}$</th>
<th>$K_{A,30}$</th>
<th>$K_{A,45}$</th>
<th>$Ecc_A$</th>
<th>$K_{F,M}$</th>
<th>$K_{F,30}$</th>
<th>$K_{F,45}$</th>
<th>$Ecc_F$</th>
<th>$R_{LA}$</th>
<th>$R_{LP}$</th>
<th>$T$</th>
<th>$Pacby$</th>
<th>$ACD$</th>
<th>$L$</th>
<th>$S_p$</th>
<th>$P_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>39.878</td>
<td>43.294</td>
<td>0.297</td>
<td>0.060</td>
<td>0.403</td>
<td>-6.265</td>
<td>-0.166</td>
<td>-0.003</td>
<td>0.151</td>
<td>10.427</td>
<td>-6.864</td>
<td>4.070</td>
<td>0.545</td>
<td>2.870</td>
<td>23.667</td>
<td>6.505</td>
<td>22.994</td>
</tr>
</tbody>
</table>
### Table A2. Covariance Matrix for Monocular Model in Right Eyes Only

<table>
<thead>
<tr>
<th>Age</th>
<th>Anterior keratometry</th>
<th>Posterior keratometry</th>
<th>Phakometry</th>
<th>Lens Power</th>
</tr>
</thead>
<tbody>
<tr>
<td> </td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
</tr>
<tr>
<td> </td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
</tr>
<tr>
<td> </td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
</tr>
<tr>
<td> </td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
<td>$k_{nm}$</td>
</tr>
</tbody>
</table>

### Table A3. Twenty-Examples of Generated Intraocular Eyes

<table>
<thead>
<tr>
<th>Age</th>
<th>Spb</th>
<th>Cyl</th>
<th>Axes</th>
<th>K1, K2, K3</th>
<th>K1, K2, K3</th>
<th>Ect, PKM, P,J45</th>
<th>Ect, PKM, P,J45</th>
</tr>
</thead>
<tbody>
<tr>
<td> </td>
<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
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</tr>
<tr>
<td> </td>
<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
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</tr>
<tr>
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<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
<td>$s_{nm}$</td>
</tr>
</tbody>
</table>

Data are for eyes with spherical equivalent in the range -0.5 to +0.5 D; Sp: calculated sphere; Cyl: calculated cylinder; Axis: calculated axis.

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